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REFLECTION OF PLANE WAVE IN NON-LOCAL ROTATING TWO TEMPERATURE MEDIA WITH INITIAL STRESS

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In this research article, the governing relation and equations are derived in context of rotation, initial stress, and two temperature for the non-local thermoelastic solid. These equations are solved for the plane wave in non-local thermoelastic solid material with initial stress, rotation and two temperature, resulting the existence of three waves (*P*, *SV* and *T*). It has been observed that these waves are affected by the initial stress and nonlocality parameter. Using appropriate boundary conditions, the reflection coefficient are calculated for the *P* waves. The physical properties of the aluminium material are considered for the numerical simulations and the phase velocity, attenuation coefficient and reflection coefficient are plotted graphically with respect to frequency. It has been observed that the nonlocality parameter, initial stress, rotation and two temperature has significant effect on phase speeds, attenuation coefficient and reflection coefficient.

Key words: Generalized thermoelasticity, plane wave, two temperature, initial stress, reflection coefficients.

1. Introduction

In classical elasticity, the deformation of a material is characterized by Hooke's law, which asserts that the stress exerted on a material is directly proportional to the resulting strain. However, this relationship does not account for the changes in dimensions that occur due to temperature variations. Thermoelasticity is the study of the relationship between temperature and elastic deformation in material. It is a branch of solid mechanics that combines the theories of thermodynamics and elasticity. In this field, the response of a material due to changes in temperature and stress is analyzed. When a material undergoes thermal loading, it experiences variations in temperature, leading to thermal expansion or contraction. At the same time, it also experiences stress due to external loading. Thermoelasticity explains how these two phenomena are inter-dependent and can affect the overall behaviour of the material.

The generalization of thermoelasticity involve the extension of the theory of thermoelasticity, to include additional factors such as thermal stresses, thermal expansion and thermal conductivity. These factors have a significant impact on the behaviour of material under various thermal loads. Dhaliwal and Sherief [1] studied the generalized thermoelasticity for anisotropic media and proved the uniqueness theorem. Youssef [2] studied the theory of generalized thermoelasticty with two temperature. Hetnaeski and Ignaczak [3] discussed the generalized themoelasticity. Sherief and Anwar [4] discussed the problems in generalized thermoelasticity. Agarwal [5] studied the generalized thermoelasticity on the plane waves. Puri [6] studied the plane wave propagation in generalized thermoelasticity. Kiani and Eslami [7] discussed the generalized thermoelasticity using Lord Shulman (L-S) theory of an isotropic layer. Sharma *et al.* [8] examined the reflection of thermoelastic waves from rigidly established, isothermal, and insulated boundaries within a half-space framework, employing various theoretical approaches. They also analyzed the reflection coefficient in contrast to the incident coefficient for different types of waves.

In thermoelasticity, initial stress refers to the stress in a material before applying any external loads or any change in temperature. This type of stress can originate from various sources including residual stress from

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the manufacturing process, thermal expansion or contraction, or chemical reaction and it can be analysed using various mathematical models. El-Naggar [9] investigated how initial stress, rotation, voids, and magnetic fields influence plane waves in the context of generalized thermoelasticity. Othman and Song [10] examined the reflection of a plane wave originating from an elastic half-space under initial stress, without any energy dissipation. Dey and Addy [11, 12] discussed the reflection of plane elastic wave under the influence of initial stress in the medium at the free surface and interface between two semi-finite elastic solid media. Ailawalia *et al.* [13] investigated the effect of hydrostatic initial stress and rotation in Green-Naghdi thermoealstic half space using the normal mode analysis techniques. Singh and Chakraborty [14] examined the influence of initial stress and magnetic fields on wave reflection at the boundary of a solid half-space. Poonia [15] explored the propagation of elastic waves and their interactions at an imperfect interface between two different media: a micropolar elastic solid half-space and a fluid-saturated porous solid half-space. Shams [16] investigated the nonlinear theory of thermoelasticity to assess the impact of initial stress on plane waves in incompressible materials.

In thermoelasticity, rotation refers to the change in the orientation of a body or material due to external loads or temperature changes. When a body is subjected to an external load, it experiences deformation and rotation. The behaviour of a material undergoing rotation can be described using the theory of elasticity. In the context of thermoelasticity, this theory is extended to include the effects of temperature changes on the material's behaviour. To analyze the behaviour of material with rotation in thermoelasticity, mathematical models are used that incorporate the effect of both deformation and temperature changes. These models can be used to predict the behaviour of material under various loads and temperature conditions. Ailawalia and Narah [17] conducted a study on the deformation of a rotating generalized thermoelastic solid that is situated beneath an infinite thermoelastic fluid, examining the effects of various forces acting at the interface while considering the influence of gravity. Singh and Yadav [18] investigated the reflection of plane waves in a transversely isotropic thermoelastic medium, taking into account the effects of rotation and magnetic fields, and analyzed the reflection coefficients of different wave types. Singh *et al.* [19] explored how rotation impacts the reflection of plane waves within a thermoelastic medium. Othman and Song [20] employed various models to examine the reflection of thermoelastic waves influenced by magnetic fields and rotation. Abo-Dahab *et al.* [21] analyzed the effects of rotation, voids, and magnetic fields under a single relaxation time on the reflection of plane waves at stress-free surfaces. Singh *et al.* [22] investigated Rayleigh waves in the context of generalized thermoelasticity, considering the effects of magnetic fields and initial stress. Singh *et al.* [23] formulated the study of isotropic and homogeneous generalized thermoelastic media under hydrostatic initial stress within the framework of Lord–Shulman theory. The governing equations derived from this research were solved analytically to determine the dimensional velocities in the *x-y* plane. Two-temperature thermoelasticity is a theory that extends classical thermoelasticity to account for the fact that heat can be conducted at different rates in different directions within a material. Youssef and El-Bary [24] studied the generalized thermoelasticity using two temperature parameter in context of thermal conductivity. Othman *et al.* [25] discussed the plane wave reflection using three theories with rotation, two temperature and gravity.

The nonlocal theory of thermoelasticity is a mathematical framework that extends classical thermoelasticity to include the effect of nonlocal interactions between different parts of a material [26]. In the nonlocal thermoelastic theory, the nonlocality effect are incorporated into the equation of motion. This is done by introducing a nonlocal parameter that characterises the degree of nonlocality in the material. Lotfy [27] studied the transient waves caused by a line heat source with a stable internal heat source inside isotropic homogenous thermoelastic perfectly conducting half-space permeate into a uniform magnetic field. ~~Asli Tabak *et al.*~~ Ailawalia [28] investigated the mechanical and thermal response of a functionally graded medium (FGM) subjected to initial stress within the framework of Green-Naghdi type III thermoelasticity, specifically for the case without energy dissipation. The application of generalized thermoleasticity ranges from wave propagation in thermoelastic materials applications in science and technology, including atomic physics, industrial engineering, thermal power plants, underwater constructions, pressure vessels, aircraft, chemical pipes, and metallurgy. Variations in gravity, temperature differences, quenching processes and other factors were studied by Hobiny *et al.* [29], Zeeshan *et al.* [30] . Das *et al.* [31] studied the propagation of plane wave in a nonlocal thermoelastic medium using the Eringen's elasticity model and Green-Lindsay type-III model. Initial stress has numerous practical applications across various engineering disciplines such as wave propagation and were studied by Asli Tabak *et al.* [32], Feng *et al.* [33, 34]. Mahmoud *et al*. [35, 36] and Ahmed Salih *et al.* [37] gave the important studies on non local effect expressing it uses in modern engineering and scientific applications where traditional local models fail to capture essential physical behaviours, particularly at small scales where size-dependent effects become significant. The nonlocal theory has application in various fields such as wave propagation in small-scale structures. The theory can account for wave dispersion and energy attenuation over distances that are comparable to the characteristic length scales of the material. This theory can be used to analyze the thermal stresses in materials subjected to instant heating or cooling, where the response may be influenced by nonlocal effects.

In the present work, we consider the governing equations of non local thermoelasticity with initial stress, rotation and two temperature. These equation are solved in *x-y* plane which shows the existence of three waves. The velocity equation and reflection coefficient using the appropriate boundary conditions for these waves are also calculated. The physical quantity of the aluminium material has been taken for the numerical simulation. The phase velocity, attenuation coefficient and Reflection coefficient are plotted graphically which represents the effect of various parameter.

2. Basic equation

Following [18], [38]-[40] the governing equations along with the constitutive equation for homogeneous rotating isotropic media, which incorporates the two-temperature model and a nonlocal parameter within the framework of the Lord-Shulman model, are presented as follows:
(i) Equation of motion

$$\left(1-e^2\nabla^2\right)\rho_0\left\{\ddot{u}_i+(2\Omega\times\dot{u})_i+\left(\Omega\times(\Omega\times u)\right)_i\right\}=\left(1-e^2\nabla^2\right)\sigma_{ij,j}\,, \tag{2.1}$$

(ii) Stress components

$$\left(1-e^2\nabla^2\right)\sigma_{ij}=-p(\delta_{ij}+w_{ij})+\lambda e_{kk}\delta_{ij}+2\mu e_{ij}-\frac{\alpha}{K_T}\Theta\delta_{ij}\,, \tag{2.2}$$

(iii) Heat conduction equation

$$\rho_0 C_E\left(1+\tau_0\frac{\partial}{\partial t}\right)\frac{\partial\Phi}{\partial t}+\frac{\alpha T_0}{K_T}\left(1+\tau_0\frac{\partial}{\partial t}\right)e_{kk}=K\nabla^2\Theta\,, \tag{2.3}$$

where,

$$\Theta=\Phi-a^*\{\Phi_{,ii}\}\,. \tag{2.4}$$

3. Formulation of problem

We consider the two dimensional displacement vector $u=(u_1(x,y,t),u_2(x,y,t),0)$ in the half space. Using Eqs (2.1) and (2.2) we get,

$$\left(1-e^2\nabla^2\right)\left[\ddot{u}_1-\Omega^2u_1+2\Omega\dot{u}_2\right]=B_T^2\frac{\partial u_1}{\partial x^2}+B_S^2\frac{\partial u_1}{\partial y^2}+\left(B_T^2-B_S^2\right)\frac{\partial u_2}{\partial x\partial y}-\beta\frac{\partial\Theta}{\partial x}\,, \tag{3.1}$$

$$\left(1-e^2\nabla^2\right)\left[\ddot{u}_2-\Omega^2 u_2+2\Omega\dot{u}_1\right]=B_T^2\frac{\partial u_2}{\partial y^2}+B_S^2\frac{\partial u_2}{\partial x^2}+\left(B_T^2-B_S^2\right)\frac{\partial u_1}{\partial x\partial y}-\beta\frac{\partial\Theta}{\partial y}, \tag{3.2}$$

and
$$C_E\left(1+\tau_0\frac{\partial}{\partial t}\right)\frac{\partial\Phi}{\partial t}+\beta T_0\left(1+\tau_0\frac{\partial}{\partial t}\right)\frac{\partial}{\partial t}\left[\frac{\partial u_1}{\partial x}+\frac{\partial u_2}{\partial y}\right]=\frac{K}{\rho_0}\nabla^2\Theta. \tag{3.3}$$

Where, $B_s^2=\dfrac{\lambda+2\mu}{\rho_0},\quad B_T^2=\dfrac{\mu-p}{\rho_0},\quad \beta=\dfrac{\alpha}{K_T}.$

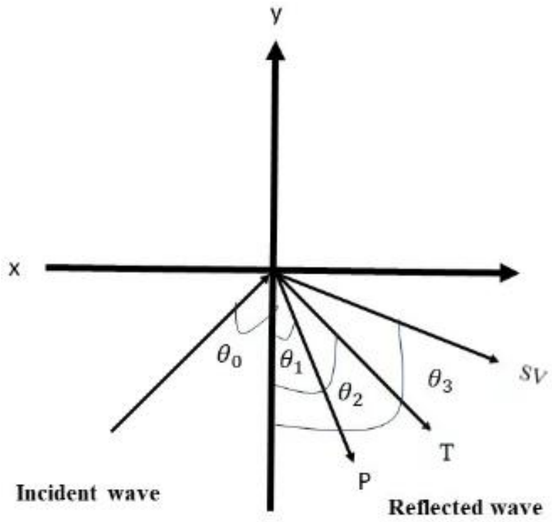


Fig.1. Diagram of the problem.

Making use of Helmholtz principle, the potentials u_1, u_2 can be written as,

$$u_1=\frac{\partial q}{\partial x}-\frac{\partial\psi}{\partial y},\quad u_2=\frac{\partial q}{\partial y}+\frac{\partial\psi}{\partial x}. \tag{3.4}$$

Using (2.3) and (3.4) in Eqs (3.1)-(3.3), we get

$$\left(1-e^2\nabla^2\right)\left[\frac{\partial^2 q}{\partial t^2}-\Omega^2 q\right]=B_T^2\nabla^2 q-\beta\left[\Phi-a^*\nabla^2\Phi\right], \tag{3.5}$$

$$\left(1-e^2\nabla^2\right)\left[-\frac{\partial^2\psi}{\partial t^2}+\Omega^2\psi\right]=-B_S^2\nabla^2\psi, \tag{3.6}$$

$$C_E\left(1+\tau_0\frac{\partial}{\partial t}\right)\frac{\partial\Phi}{\partial t}+\beta T_0\left(1+\tau_0\frac{\partial}{\partial t}\right)\frac{\partial}{\partial t}\left(\nabla^2 q\right)=\frac{K}{\rho_0}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\left[\Phi-a^*\nabla^2\Phi\right]. \tag{3.7}$$

Here it is seen that Eqs (3.5) and (3.7) are dependent on q and Φ whereas Eq.(3.6) is independent in ψ .

4. Solution of the problem

We consider a propagating plane wave moving in the positive direction along the x-axis within the framework of rotating isotropic homogeneous non-local thermoelasticity in a solid half-space. The wave solution may be assumed as [20]:

$$(q,\psi,\Phi)=(\bar{q},\bar{\psi},\bar{\Phi})\exp\{ik(x\sin\theta+y\cos\theta-Vt)\}\,. \tag{4.1}$$

Here $\bar{q},\bar{\psi},\bar{\Phi}$ are considered as constants.
Using (4.1), the Eqs (3.5) and (3.7) have a non trivial solution, if

$$A_1V^4+A_2V^2+A_3=0\,. \tag{4.2}$$

Where,
$$A_1=C_E\left(\tau_0+\frac{i}{\omega}\right)\left(1+e^2k^2\right),$$

$$A_2=C_E\left(\tau_0+\frac{i}{\omega}\right)\left(1+e^2k^2\right)\frac{\Omega^2}{k^2}-\frac{K}{\rho_0}\left(1+a^*k^2\right)\left(1+e^2k^2\right)-C_E\left(\tau_0+\frac{i}{\omega}\right)B_T^2+\beta^2T_0\tau_0^*\left(1+a^*k^2\right),$$

$$A_3=\frac{K}{\rho_0}\left[B_T^2\left(1+a^*k^2\right)-\frac{\Omega^2}{k^2}\left(1+a^*k^2\right)\left(1+e^2k^2\right)\right].$$

The roots of Eq.(4.2) are,

$$V_1^2,V_2^2=\frac{-A_2\pm\sqrt{A_2^2-4A_1A_2}}{2A_1}.$$

Here, V_1 represents the velocity of P-wave, V_2 represents the velocity of T wave.
The SV wave velocity is calculated as $V_3^2=\left(\dfrac{B_S^2}{1+e^2k^2}-\dfrac{\Omega^2}{k^2}\right)$ by solving the Eq.(3.6).
Here $\omega=kV$ (where k is the wave number and V is the velocity) is known as circular frequency of the wave. If $V_i^{-1}=v_i^{-1}+i\omega^{-1}q_i$, $(i=1,2,3)$ then the real parts v_1, v_2, v_3 are considered to be phase velocity of P, SV and T wave and imaginary part is considered as attenuation coefficient.
Following [41] the phase velocity, specific loss $\left(\dfrac{\Delta W}{W}\right)_i$ and attenuation coefficient can be represented as below,

$$v_i=\frac{(\Re(V_i))^2+(\Im(V_i))^2}{\Re(V_i)},\qquad S_i=\left(\frac{\Delta W}{W}\right)_i\qquad Q_i=\frac{(\Re(V_i))^2+(\Im(V_i))^2}{\Re(V_i)}. \tag{4.3}$$

Where, $\Re(V_i)$, $(\Im(V_i)$ $\Re\left(V_i\right)$, $\Im\left(V_i\right)$ are the real and imaginary part of V_i respectively.

Special cases:
Case 1: In the absence of nonlocal parameter $\left(e=0\right)$ in Eq.(4.2) we get

$$A_1V^2 + A_2V^2 + A_3 = 0,$$

where, $$A_1 = C_E\left(\tau_0 + \frac{i}{\omega}\right),$$

$$A_2 = C_E\left(\tau_0 + \frac{i}{\omega}\right)\frac{\Omega^2}{k^2} - \frac{K}{\rho_0}\left(1 + a^*k^2\right) - C_E\left(\tau_0 + \frac{i}{\omega}\right)B_T^2 + \beta^2 T_0\left(\tau_0 + \frac{i}{\omega}\right)\left(1 + a^*k^2\right),$$

$$A_3 = \frac{K}{\rho_0}\left[B_T^2\left(1 + a^*k^2\right) - \frac{\Omega^2}{k^2}\left(1 + a^*k^2\right)\right].$$

Case 2: In the absence of rotation $(\Omega = 0,\ e = 0)$ in Eq.(4.2) we get

$$A_1V^2 + A_2V^2 + A_3 = 0,$$

where, $$A_2 = -\frac{K}{\rho_0}\left(1 + a^*k^2\right) - C_E\left(\tau_0 + \frac{i}{\omega}\right)B_T^2 + \beta^2 T_0\left(\tau_0 + \frac{i}{\omega}\right)\left(1 + a^*k^2\right),$$

$$A_3 = \frac{K}{\rho_0}\left\{B_T^2\left(1 + a^*k^2\right)\right\}.$$

Case 3: In the absence of two temperature parameter $\left(\Omega = 0,\ e = 0,\ a^* = 0\right)$ in Eq.(4.2) we get

$$A_1V^2 + A_2V^2 + A_3 = 0,$$

where, $$A_2 = -\frac{K}{\rho_0} - C_E\left(\tau_0 + \frac{i}{\omega}\right)B_T^2 + \beta^2 T_0\left(\tau_0 + \frac{i}{\omega}\right),$$

$$A_3 = \frac{K}{\rho_0}\left(B_T^2\right).$$

A_1 is same as calculated in case 1.

5. Reflection from a free surface

Figure 1 illustrates the existence of three reflected waves resulting from the incidence wave, therefore we examine the reflected waves *P*, *SV* and *T* where the incidence wave forms an angle of incidence θ_0. The potentials of both the incident and reflected waves in the half-space are expressed as [10].

$$q = A_0 \exp\{ik_1(x\sin\theta_0 + y\cos\theta_0 - Vt\} + \sum_{i=1}^{2} A_i \exp\{ik_i(x\sin\theta_i - y\cos\theta_i - Vt\}, \tag{5.1}$$

$$\Phi = \xi_1 A_0 \exp\{ik_1(x\sin\theta_0 + y\cos\theta_0 - Vt\} + \sum_{i=1}^{2} \xi_i A_i \exp\{ik_i(x\sin\theta_i - y\cos\theta_i - Vt\}, \tag{5.2}$$

$$\Psi = B_1 \exp\{ik_3(x\sin\theta_3 - y\cos\theta_3 - Vt\}, \tag{5.3}$$

where $$\xi_i = \frac{\left(k_i^2 V_i^2 + \Omega^2\right)\left(1 + e^2 k_i^2\right)}{\beta\left(1 + a^* k_i^2\right)}.$$

For stress-free surface we take the required boundary conditions are

$$\left(1 - e^2\nabla^2\right)\sigma_{yy} = -p, \qquad \left(1 - e^2\nabla^2\right)\sigma_{yx} = 0, \qquad \frac{\partial\Theta}{\partial y} = 0. \tag{5.4}$$

The potentials given in Eqs (5.1)-(5.3) must satisfy the boundary conditions (5.4).
The wave numbers along with the angles of reflection for the waves that are reflected can be expressed using Snell's Law as:

$$\frac{\sin\theta_0}{V_0} = \frac{\sin\theta_i}{V_i}, \qquad k_i V_i = \omega, \qquad \left(i = 1, 2, 3\right). \tag{5.5}$$

Where, $\theta_0 = \theta_{+1}$ for P wave and $\theta_0 = \theta_{+3}$ for SV wave.

For P waves

Using Eqs (5.1)-(5.3) and (5.4) we get the three non-homogeneous system of equations in the form of

$$\sum_{i,j=1}^{3} c_{ij} Z_j = b_i, \tag{5.6}$$

where, $$c_{1j} = -\lambda k_j^2 - 2\mu k_j^2 \cos^2\theta_j - \frac{\alpha}{K_T}\xi_j\left(1 + a^* k_j^2\right), \qquad j = (1,2).$$

$$c_{13} = 2\mu k_3^2 \sin\theta_3\cos\theta_3, \qquad b_1 = c_{12}, \qquad c_{2j} = \mu k_j^2 \sin 2\theta_j, \qquad j = (1,2).$$

$$c_{23} = -\left(\mu + \frac{p}{2}\right)k_3^2\left(\sin^2\theta_3 - \cos^2\theta_3\right), \quad b_2 = -c_{22},$$

$$c_{3j} = -i\xi_j \cos\theta_j\left(1 + a^* k_j^2\right), \;\; j = (1,2), \qquad c_{33} = 0, \; b_3 = -c_{32}.$$

Here, $$Z_1 = \frac{A_1}{A_0}, \; Z_2 = \frac{A_2}{A_0}, \;\; Z_3 = \frac{B_1}{A_0},$$

where, the reflection coefficients are represented by Z_i.

6. Numerical results and discussion

The values of parameter for the aluminium material taken from [23] mentioned below have been used to find the numeric results are as follows,

$E = 6.910^{11}\, dyne/cm^2,\ K_T = 0.5,\ \sigma = 0.33,\ T_0 = 20°C,\ \rho_0 = 2.7\, gm/cm^3,\ C_E = 0.236\ cal/gm°C,\ \tau_0 = 0.05\, s$
$K = 0.0492\ cal/cm\ s°C,\ e = 0.5$.

The value of Lame's constant are as follows,

$$\lambda = \frac{E\sigma}{\eta(1+\sigma)(1-2\sigma)}, \quad \mu = \frac{E}{2\eta(1+\sigma)}.$$

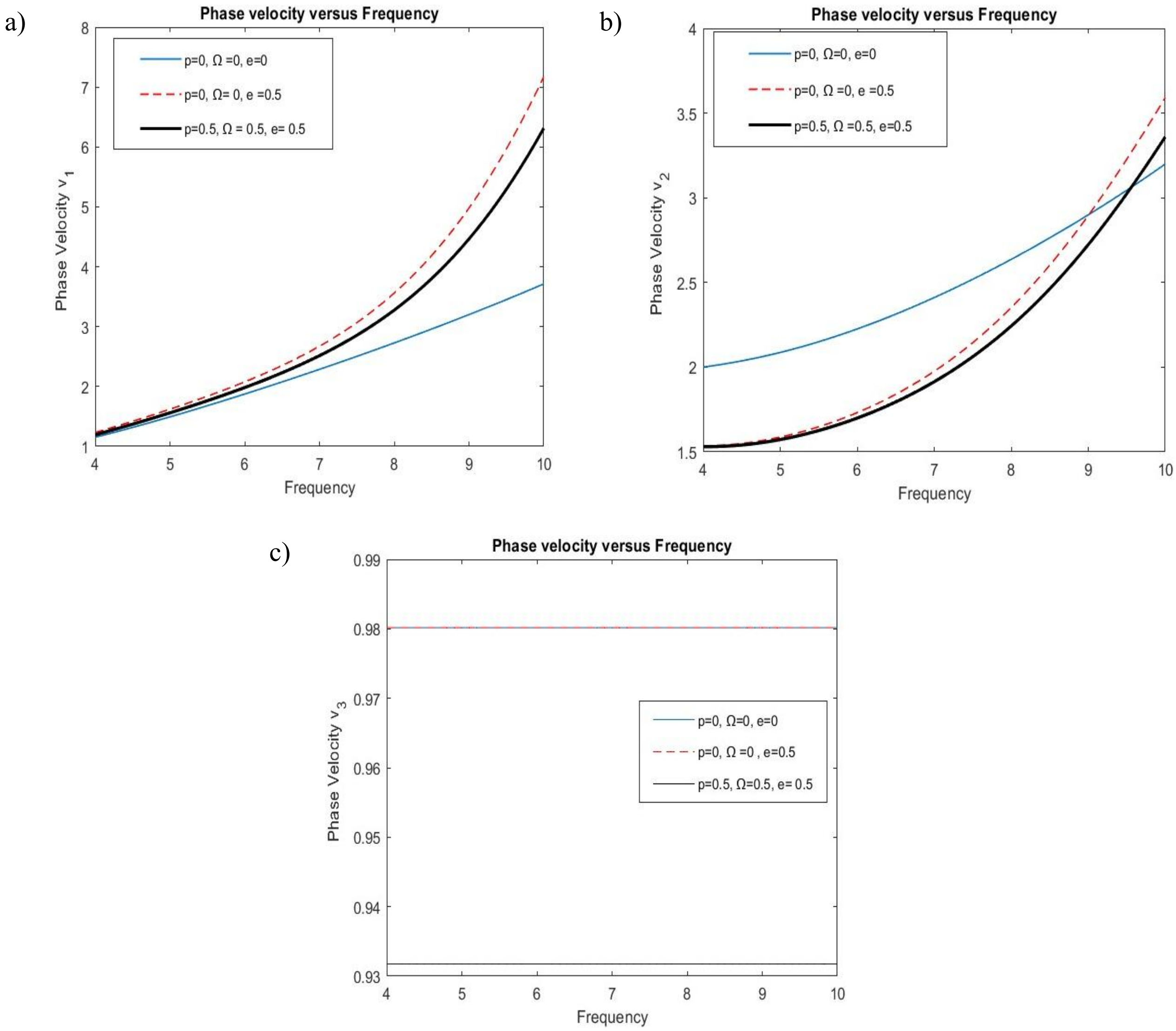
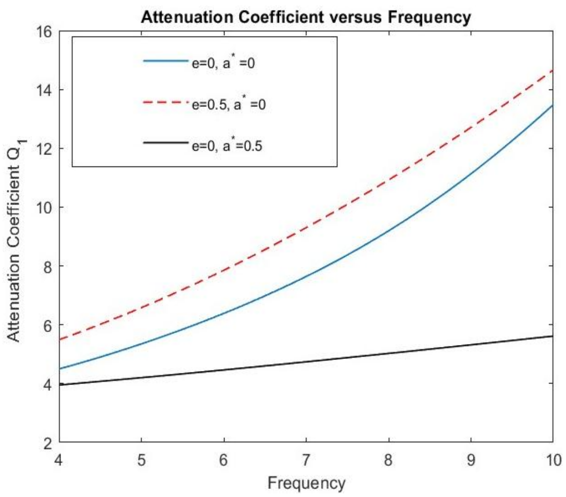


Fig.1a-c. Phase velocity versus frequency.

Figure 1a-c illustrate that the velocity of *P*-waves as a function of frequency under varying nonlocal parameter, initial stress, and rotation. For Fig.1a The phase velocity v_1 increases for all the cases as with the increase in frequency. As it represents that the phase velocity slowing increase in absence of the initial stress, rotation and non local. When $e = 0.5$ the causes the phase velocity to increase faster. When all parameters are non-zero, the phase velocity increases even more rapidly, indicating a stronger effect of these parameters on phase velocity. Figure 1b shows that the velocity v_2 increases as frequency increases for all cases. When all parameter are zero, the phase velocity starts higher and rises steadily. When $e = 0.5$ the phase velocity starts lower but overtakes the but overtakes the blue line after about frequency 8, indicating that no local parameter influences the rate of increase of v_2. In presence of all parameter the phase velocity starts the lowest but grows fastest, surpassing the others near the higher frequencies. Figure 1c shows that the phase velocity v_3 remains constant with respect to frequency, but have a significant impact of nonlocal, initial stress and rotation.

a) 

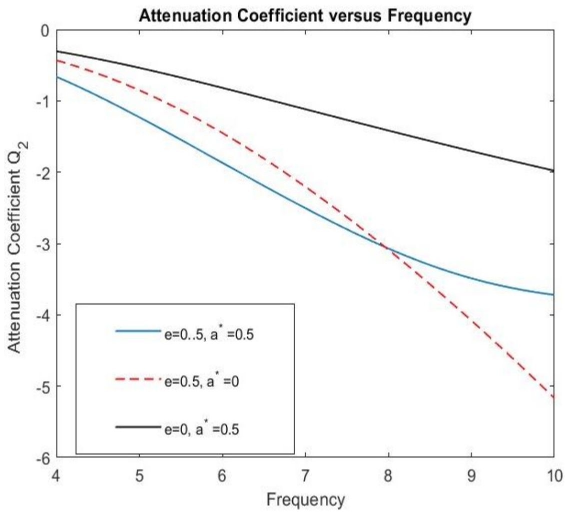
b) 

Fig.2a-b. Attenuation coefficient against frequency.

Figure 2a-b shows the attenuation coefficient of *P*-waves across the frequency spectrum. For Q_1 in Fig.2a attenuation coefficient increases with frequency for all cases. When non local parameter preshows the highest attenuation, increasing steeply as frequency increases, whereas it has moderate increase when non local is present. When two temperature are available, it shows the smallest increase and remains relatively low and almost flat compared to the others. Attenuation increasing with frequency, that waves lose energy faster. The non local parameter has a major impact on increasing attenuation, especially at higher frequencies, while as the two temperature a has a smaller, more constant effect. For figure Q_2 the attenuation coefficient shows decreasing trend for all the cases. The attenuation coefficient Q_2 is more negative at higher frequencies, meaning higher frequency signals experience stronger attenuation.

Figure 3a-c presents the reflection coefficient of *P*-waves versus frequency at a free surface with the effect of two temperature and non local parameter. In Fig.3a the reflection coefficient R_1 describes how much of the wave energy is reflected back at a boundary or medium interface. Reflection depends strongly on frequency and system parameters (*e* and *a**).When $e = 0.5$, $a^* = 0$ have strong reflection at lower frequencies, but drops steeply at higher frequencies. In presence of non local and two temperature parameter ($e = 0.5$, $a^* = 0.5$) reflection is moderate, peaks in the middle frequency range, then decreases. When $e = 0$, $a^* = 0.5$ reflection is weaker overall and more stable across frequencies.

For the reflection coefficient R_2 there's a constant change, when either of the two factors i.e non local and two temperature is absent but for $e = 0$ and $a^* = 0.5$ there is a steep decrease in the reflection coefficient with respect to frequency showing stress shifts resonance frequencies undermines overall reflection levels, demonstrating that preloading can amplify energy return at targeted frequencies. In the Fig.2c the resonating energies are variating between *0.8* to *2.0* with the shifts in the frequencies demonstrating non local and two temperature parameter smooth's extreme oscillations at high frequencies, indicating improved energy absorption and damping.

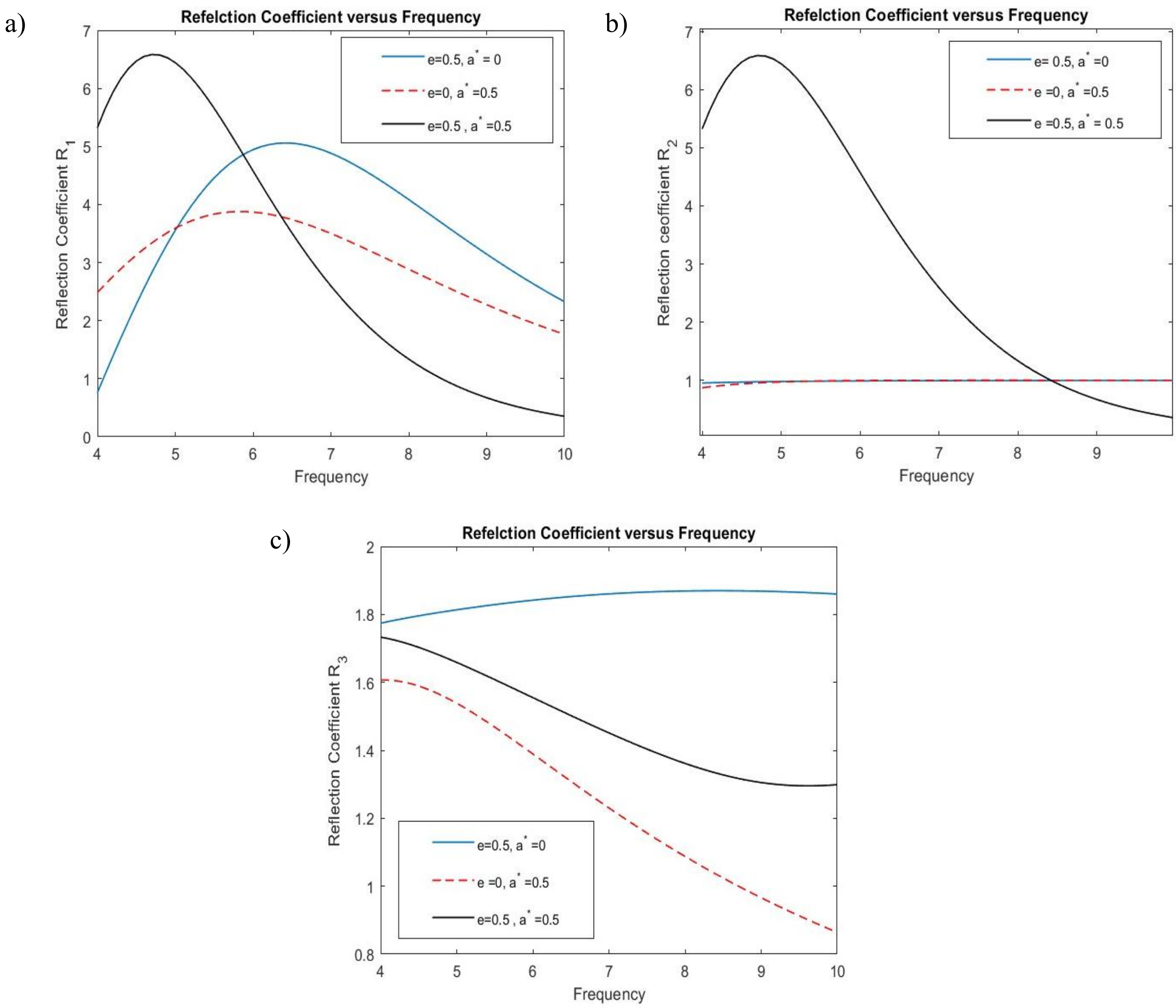


Fig.3a-c. Reflection coefficient versus frequency.

7. Conclusion

The paper provides a comprehensive analysis of plane wave propagation and reflection in a rotating, two-temperature thermoelastic solid half-space with initial stress, employing the Eringen nonlocal elasticity model alongside the Lord-Shulman theory. The study solves the governing equations for such a system, explicitly accounting for the effects of nonlocality parameter, rotation, initial stress, and two-temperature on wave propagation. The reflection coefficient for the place wave are calculated using the appropriate boundary condition and the effect of the various parameters are observed.

Key findings

1. **Existence and behaviour of three wave:** The mathematical formulation and solutions reveal the presence of three wave in the medium. The phase velocities, attenuation coefficients, and specific losses for these waves are all strongly influenced by the nonlocal parameter, initial stress, rotation, and two-temperature effects.
2. **Effect of initial stress, rotation, and two-temperature:** The combined influence of initial stress, rotation, and two-temperature theory introduces dispersion and alters the wave reflection characteristics at boundaries. These effects become particularly important in advanced materials or structures where such conditions exist naturally or can be engineered.
3. **Influence of nonlocal parameter and material properties:** As the nonlocal parameter increases, physical phenomena such as phase velocity, reflection coefficient and attenuation coefficient all experience significant changes.

Data availability: The data used have been cited in [23].

Conflict of interest: The authors declare that they have no conflicts of interest.

Nomenclature

a^*	– two temperature parameter
$e_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right)$	– strain tensor
e_{kk}	– cubical dilatation
C_E	– specific heat
K	– thermal conductivity
K_T	– isothermal compressibility
p	– initial stress parameter
t	– absolute temperature
T_0	– reference uniform temperature of the medium
$w_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right)$	– component of small rotation tensor
α	– volume coefficient of thermal expansion
δ_{ij}	– Kronecker delta
$\Theta = T - T_0$	– small temperature increment
λ, μ	– Lame's constant
ρ_0	– material density
σ_{ij}	– stress tensor
τ_0	– relaxation time
$\left[\Omega \times (\Omega \times u)\right]$	– time dependent part of the centripetal acceleration
Ω	– angular velocity
$2\Omega \times \dot{u}$	– Time dependent part of the Coriolis acceleration

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