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STUDY OF LOVE WAVES IN A HETEROGENEOUS VISCOPOROELASTIC LAYER OVERLYING A VISCOPOROELASTIC HALF-SPACE

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This research examines to study the Love-type surface waves move through a heterogeneous layer of viscoporoelastic material that rests on a viscoporoelastic half-space. Both regions are different in their characteristics namely like heterogeneities, density fluctuations, porosity, and viscosity. Because of these variations, the problem is more pertinent and practical for intricate subsurface conditions that are frequently encountered in geophysical and civil engineering applications. In order to investigate the wave behavior, the basic equations of motion for viscoporoelastic media are developed, considering the combined influence of said characteristics. Applying the boundary conditions and corresponding equations of motion, an expression for the dispersion relation is produced. The solutions of corresponding equations of motion and the prescribed boundary conditions give the dispersion relation. Numerical visualization demonstrates how phase velocity and attenuation change with different medium-depth ratios. For Geophysical and Engineering applications, these findings provide important insights about the viscoporoelastic characteristics that influence seismic wave behavior.

Key words: Love waves, heterogeneity, phase velocity, wavenumber, attenuation.

1. Introduction

The Earth contains layers of elastic materials, each of which has a different stiffness and thickness. The propagation of earthquake waves provides a wide range of information regarding the physical characteristics of the medium, the seismic source, and the Earth's interior. These waves are mostly knocking for the devastation caused by earthquakes. Love waves are a kind of surface waves that move across a medium's surface. Their propagation depend on the characteristics of the medium's layers.Compared to other wave types, Love waves are more susceptible to architectural intricacies. Seismic waves propagate differently based on the Earth's heterogeneous composition, which includes a hard stratum, medium porosity, and stiff interface. Therefore, Applied Mathematics and Engineering disciplines have given a lot of focus to study the wave propagation in homogenous media. In this connection, authors have examined how seismic waves travel through layered structures that are surrounded by various irregular barriers. Chakraborty [1] found that the body wave's velocity varies according to the propagation direction, and wave speed is affected by the non-homogeneity of the substanceand the layer porosity. Romeo [2] investigated transient shear horizontal waves that appear when viscoelastic solids meet at boundaries through analysis of their admissibility with Laplace transform asymptotic expansion techniques. Kundu *et al.* [3] studied that the medium transmits two Love wave fronts, one influenced by differences in pore volume division and the other by the rigidity modulus of the elastic matrix. Heterogeneity effects and initial stress behavior are studied for Love type waves in a strained hybrid fiber-strengthened layer with viscoelastic half-space [4]. The said paper [4] focuses at how

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wave behavior and dispersion are affected by surface corrugation and fiber reinforcement. Understanding wave propagation in designed composite structures is made easier by their findings. The propagation of Love-type surface waves in a heterogeneous viscoelastic medium was investigated by Dewangan and Sahu [5]. Their research examines the effects of viscoelastic characteristics and material heterogeneity on the waves phase velocity. Making a theoretical investigation, Kakar [6] studied two heterogeneous layers that supported transmission of Love waves in viscoelastic half-spaces from point source surface actions at the boundary. Billon and Baroudi [7] examine Love wave propagation through viscoelastic waveguides containing complex fluids. The attenuation and phase velocity of Love waves were derived as a function of frequency, fluid thickness, and waveguide thickness. The authors of the paper [8] established that Love wave transmission occurs in fluid-saturated homogenous porous materials which form at the semi-infinite boundary point source. Their research explores the effects of porosity on the wave dispersion properties also the findings aid in simulating the behavior of waves in porous geological formations. Ke *et al.* [9] analyzed characteristics of waves associated with inhomogeneous and saturated with fluid. The study focuses on how wave dispersion is affected by material property scaling. Their results are pertinent to the investigation of seismic response in rock and layered soil. The authors of the paper [10] examined the propagation of Love waves across a porous half-space supported by dispersed boundary conditions in a strained homogeneous layer. Dispersion relations that take into consideration elaborate boundary requirements were derived by the authors. The impact of boundary imperfections on phase velocity and wave attenuation is highlighted by their research. For seismic analysis in geotechnical and civil engineering applications, the findings are important. In the paper [11], Kumhar obtained the dispersion expression utilizing mathematical methods under well-defined boundary conditions. Attenuation and dispersion phenomena have been studied within layered media structures and demonstrating poroelastic and poroviscoelastic properties by Alam *et al.* [12]. Utilizing the analysis of Biot's theory [13], the resultant Hill's differential equation is solved using Valeev's approach in combination with the Laplace transform explored by Venu Gopal *et al.* [14]. Poonem *et al.* [15] conducted research on the impact of heterogeneity and initial stress on longitudinal wave propagation.The propagation of Love waves in the presence of a non-local elastic material was examined by Pramanik and Manna [16]. Eringen's modification methods and Fourier transformation are used in the paper [17] to obtain the scattering equation and displacement field for the Love wave. Applying the perturbation method, Kumar and Saini [18] determined displacement fields in porous layers when considering parabolic irregularity. In this paper, the Fourier transform and Green's function are used to determine the inner deformation inside the Earth model.

Bibi *et al.* [21] examined the plane wave propagation in terms of thermoelastic rotating medium via porous medium. The pertinent results have implications for geophysical investigation in complicated media and advanced material design. Ali *et al.*[22] calculated and showed the graphical determination of wave amplitude ratio using free boundary conditions. The following research [23] incorporates wave propagation analysis with electromagnetic, thermal, and poroelastic interactions. The authors took diffusion and rotation into account when deriving analytical equations for the reflection and transmission coefficients. The study emphasizes how nonlocal thermoelasticity and the Hall current affect wave properties. Ali *et al.*[24] studied the mathematical model on Hall current influence on thermoelastic wave-behavior under fractional order viscoelastic porous rotating diffusive medium. The impact of Hall current on the propagation and reflection of the thermoelastic waves via a viscoelastic nonlocal fractional order rotating porous isotropic solid was considered by Azhar *et al.*[25]. The authors of the aforementioned studies have not taken viscosity, heterogeneity into consideration. The present paper studies Love wave propagation in viscoporoelastic layers resting on viscoporoelastic half-spaces. An attempt has been made to highlight the response of viscoporoelasticity and heterogeneity on surface wave propagation through analytic study.

This is how the remaining part of the work is structured. Section 2 presents the problem's formulation and solution. Section 3 gives frequency equations and boundary conditions. Section 4 discusses particular cases, whereas Section 5 described numerical findings. Section 6 provides a conclusion.

2. Formulation of the problem

Consider a viscoporoelastic layer with a finite width H placed on a semi-infinite half-spaceas given in the Fig.1. Figure 1 illustrates that the X-axis represents the primary direction of wave propagation. The Z-axis is the depth direction, showing the transition from the finite layer to the half-space. The Y-axis extends perpendicularly to the XZ-plane.

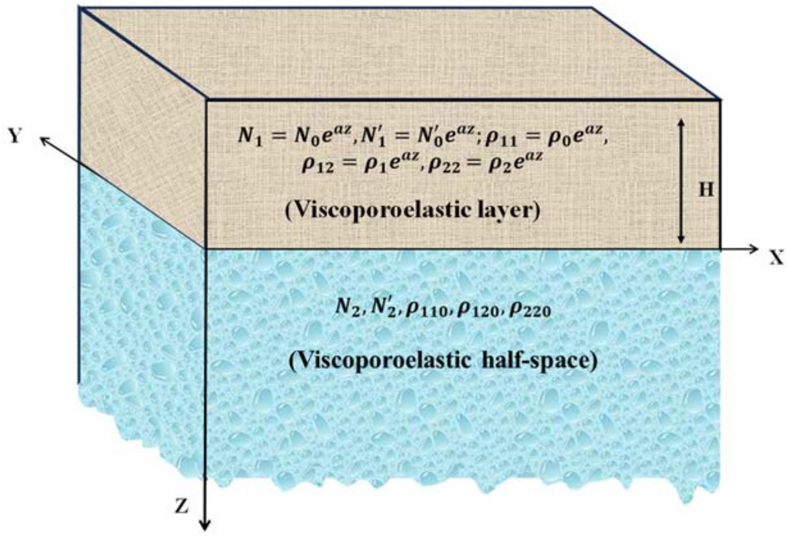

Fig.1.Diagrammatic depiction for the problem's geometry.

Equations of motion for an isotropic poroelastic solid can be stated as follows [13]:

$$N\nabla^2 \vec{u} + \nabla(Ae + Q\varepsilon) = \frac{\partial^2}{\partial t^2}\left(\rho_{11}\vec{u} + \rho_{12}\vec{U}\right),$$

$$\nabla(Qe + R\varepsilon) = \frac{\partial^2}{\partial t^2}\left(\rho_{12}\vec{u} + \rho_{22}\vec{U}\right). \tag{2.1}$$

In Eq.(2.1), $\vec{u} = (u, v, w)$ and $\vec{U} = (U, V, W)$ are displacement vectors of solid and fluid, respectively, e and ε are the dilatations of solid and fluid respectively, and $P = A + 2N$, Q and R are all poroelastic constants. ρ_{11}, ρ_{12} and ρ_{22} are mass coefficients. The solid stresses σ_{ij} and fluid pressure s are given by [13]:

$$\sigma_{ij} = 2Ne_{ij} + (Ae + Q\varepsilon)\delta_{ij}, \quad (i, j = 1, 2, 3),$$

$$s = Qe + R\varepsilon. \tag{2.2}$$

In Eq.(2.2), δ_{ij} is the Kronecker delta function. The analysis focuses on wave transmission in the half-space followed by the layer through separate sections.

2.1. Wave propagation in upper viscoporoelastic heterogeneous layer

For the Love waves, the displacements components for upper viscoporoelastic layer are $u_1 = w_1 = 0$, $v_1 = v_1(x,z,t)$, $U_1 = W_1 = 0$, $V_1 = V_1(x,z,t)$. In the heterogeneous viscoporoelastic layer, Eq.(2.1) becomes [5,13],

$$\frac{\partial}{\partial x}(\sigma_{yx}) + \frac{\partial}{\partial z}(\sigma_{yz}) = \frac{\partial^2}{\partial t^2}(\rho_{11}v_1 + \rho_{12}V_1),$$

$$0 = \frac{\partial^2}{\partial t^2}(\rho_{12}v_1 + \rho_{22}V_1). \tag{2.1.1}$$

Where $\sigma_{yx} = \left(N_1 + N_1^{'}\frac{\partial}{\partial t}\right)\frac{\partial v_1}{\partial x}$, $\sigma_{yz} = \left(N_1 + N_1^{'}\frac{\partial}{\partial t}\right)\frac{\partial v_1}{\partial z}$, here $N_1, N_1^{'}$ are considered to be solely dependent on depth and provided by $N_1 = N_0 e^{az}$, $N_1^{'} = N_0^{'} e^{az}$, where N_0, $N_0^{'}$ are the constant values of N_1, $N_1^{'}$ at the interface and $'a'$ is a constant whose size is the opposite of its length. Substituting $v_1 = v_1(z)e^{i(\omega t - kx)}$ in Eq.(2.1.1), one obtains:

$$\frac{d^2 v_1}{dz^2} + \frac{1}{\overline{N_1}}\frac{d\overline{N_1}}{dz}\frac{dv_1}{dz} + \left[\frac{1}{\overline{N_1}}\left(\rho_{11} - \frac{\rho_{12}^2}{\rho_{22}}\right)\omega^2 - k^2\right]v_1 = 0 \tag{2.1.2}$$

where $\overline{N_1} = N_1 + i\omega N_1^{'}$, substituting $v(z) = \dfrac{X_1(z)}{\sqrt{\overline{N_1}}}$ in Eq.(2.1.2) yields

$$\frac{d^2 X_1}{dz^2} + \beta^2 X_1 = 0, \tag{2.1.3}$$

where

$$\beta^2 = \frac{1}{H^2}\left[\frac{1}{\overline{N_0}}\left(\rho_{11} - \frac{\rho_{12}^2}{\rho_{22}}\right)\left(\frac{c}{\beta_1}\right)^2 (kH)^2 - \frac{(aH)^2}{4} - (kH)^2\right] \text{ and } \overline{N_0} = N_0 + i\omega N_0^{'}.$$

The solution of Eq.(2.1.3) is $X_1 = C_1\cos(\beta z) + C_2\sin(\beta z)$, here C_1 and C_2 are constants.
From the Eqs.(2.1.2) and (2.1.3), one obtains

$$v_1 = \frac{1}{\sqrt{\overline{N_0}}}\left(e^{az}\right)^{\frac{-1}{2}}\left[C_1\cos(\beta z) + C_2\sin(\beta z)\right] e^{i(\omega t - kx)}. \tag{2.1.4}$$

The non-zero stress component in this case is obtained as

$$(\sigma_{yz})_1 = \left(N_1 + i\omega N_1^{'}\right)\frac{\left(e^{az}\right)^{\frac{-1}{2}}}{\sqrt{\overline{N_0}}}\left[\frac{-a}{2}\left(C_1\cos\beta z + C_2\sin\beta z\right) - \beta\left(C_1\sin\beta z + C_2\cos\beta z\right)\right] e^{i(\omega t\text{-}kx)}.$$

2.2. Lower viscoporoelastic half-space

Due to the similar discussion as stated in earlier case, the displacement components here are $u_2 = w_2 = 0,\ v_2 = v_2(x,z,t),\ U_2 = W_2 = 0,\ V_2 = V_2(x,z,t)$. Because of nature of the half-space there are variants in stress terms, consequently equations of motion for the lower viscoporoelastic half-space is [5,13],

$$\left(N_2 + N_2' \frac{\partial}{\partial t}\right)\left(\frac{\partial^2 v_2}{\partial x^2} + \frac{\partial^2 v_2}{\partial z^2}\right) = \frac{\partial^2}{\partial z^2}(\rho_{110} v_2 + \rho_{120} V_2),$$

$$0 = \frac{\partial^2}{\partial t^2}\ (\rho_{120} v_2 + \rho_{220} V_2). \tag{2.2.1}$$

N_2 is the shear modulus and N_2' is viscoelastic parameter for N_2.

Substituting $v_2 = v_2(z)\ e^{i(\omega t - kx)}$ in Eq.(2.2.1), yields

$$\frac{d^2 v_2}{dz^2} - p^2 v_2 = 0 \tag{2.2.2}$$

where $p = \left[k^2 - \frac{\omega^2}{\overline{N_2}}\left(\rho_{11} - \frac{\rho_{12}^2}{\rho_{22}}\right)\right]^{\frac{1}{2}},\ \overline{N_2} = N_2 + i\omega N_2'.$

The solution of $v_2(z) = C_3 e^{pz} + C_4 e^{-pz}$, here C_3, C_4 are arbitrary constants. Using the condition that $v_2 \to 0$ as $z \to \infty$, then the above solution becomes

$$v_2(z) = C_4 e^{-pz}\ e^{i(\omega t - kx)}. \tag{2.2.3}$$

The non-zero stress component here is

$$(\sigma_{yz})_2 = -C_4 p\left(N_2 + i\omega N_2'\right) e^{-pz + i(\omega t - kx)}.$$

3. Boundary conditions and frequency equation

The assumed boundary conditions are given below:
(i) The top most layer surface is stress-free, i.e.

$$\left(\sigma_{yz}\right)_1 = 0. \tag{3.1}$$

(ii)At the interface, displacements are assumed to be continuous

$$v_1 = v_2 \quad \text{at} \quad z = 0. \tag{3.2}$$

(iii)The boundary is continuously stressed, i.e.

$$\left(\sigma_{yz}\right)_1 = \left(\sigma_{yz}\right)_2. \tag{3.3}$$

The following frequency equation is obtained from the system of homogeneous equations in arbitrary constants C_1, C_2, C_3 as a result of the above boundary conditions:

$$\left[A_{ij}\right]=0\ ;\quad i,\ j=1,2,3, \tag{3.4}$$

here

$$A_{11}=-B_1\sin z_1+\frac{\cos z_1}{2H\sqrt{aH}},\ A_{12}=-B_1\cos z_1-\frac{\cos z_1}{2H\sqrt{aH}},\ A_{13}=0,\ A_{21}=1,\ A_{22}=0,$$

$$A_{23}=-\sqrt{\overline{N}_0},\ A_{31}=0,\ A_{32}=\frac{B_1\overline{N}_1}{\sqrt{\overline{N}_0}},\quad A_{33}=P\overline{N}_2,\ P=\left[k^2-\frac{\omega^2}{\overline{N}_2}\left(\rho_{110}-\frac{\rho_{120}^2}{\rho_{220}}\right)\right]^{\frac{1}{2}},$$

$$B_1=\frac{1}{H^2}\left[\frac{1}{\overline{N}_0}\left(\rho_{110}-\frac{\rho_{120}^2}{\rho_{220}}\right)m^2\left(kH\right)^2-\left(\frac{aH}{2}\right)^2-\left(kH\right)^2\right],\ z_1=B_1(aH),$$

$$\overline{N}_0=N_{01}+i\omega N_{10},\ \overline{N}_1=N_{02}+i\omega N_{20},\ \overline{N}_2=N_{03}+i\omega N_{30}.$$

Equation (3.4) is a frequency equation with complex values are as follows:

$$\left|\,a_{ij}\right|+i\left|\,b_{ij}\right|=0,\quad i,j=1,2,3, \tag{3.5}$$

where

$$a_{11}=-B\sin(BH)+\frac{\cos(BH)}{2H\sqrt{aH}},\quad a_{12}=-B\cos(BH)+\frac{\sin(BH)}{2H\sqrt{aH}},\quad a_{13}=0,\ a_{21}=1,$$

$$a_{22}=0,\ a_{23}=\sqrt{N_{01}},\ a_{31}=0,\quad a_{32}=0,\ a_{33}=\frac{-N_{03}}{H^2}\left[\left(kH\right)^2-\left(\frac{c}{\beta_1}\right)^2\left(\rho_{110}-\frac{\rho_{120}^2}{\rho_{220}}\right)\right],$$

$$B=\left(kH\right)^2\left(\rho_{11}-\frac{\rho_{12}^2}{\rho_{22}}\right)\left(\frac{c}{\beta_1}\right)^2-\left(kH\right)^2\left(1-\left(\frac{\omega N_{10}}{N_{01}}\right)^2\Big/N_{01}\right)-\left(\frac{aH}{4}\right)^2,$$

$$b_{11}=-B_1\sin(B_1H)+\frac{\cos(B_1H)}{2H\sqrt{aH}},\qquad b_{12}=-B_1\cos(B_1H)-\frac{\sin(B_1H)}{2H\sqrt{aH}},$$

$$b_{13}=0,\qquad b_{21}=1,\quad b_{22}=0,$$

$$b_{23}=\sqrt{N_{01}}\left[1+\left(\frac{\omega N_{10}}{N_{01}}\right)^2\right]^{\frac{1}{4}}\sin\left(\frac{1}{2}\tan^{-1}\left(\frac{\omega N_{10}}{N_{01}}\right)\right),\qquad b_{31}=0,$$

$$b_{32} = B_1 \sqrt{\frac{N_{02}}{N_{01}}} \left(\frac{1+\left(\frac{\omega N_{20}}{N_{02}}\right)^2}{1+\left(\frac{\omega N_{10}}{N_{01}}\right)} \right)^{\frac{1}{4}} \sin\left(\frac{\alpha-\beta}{2}\right),$$

here

$$\alpha = \tan^{-1}\left(\frac{\omega N_{20}}{N_{02}}\right), \ \beta = \tan^{-1}\left(\frac{\omega N_{10}}{N_{01}}\right),$$

$$b_{33} = \frac{1}{H^2}\left[(-kH)^2 N_{03} - \left(\frac{c}{\beta_1}\right)^2 \left(\rho_{110} - \frac{\rho_{120}^2}{\rho_{220}}\right)\right],$$

$$B_1 = (kH)^2 \left(\rho_{11} - \frac{\rho_{12}^2}{\rho_{22}}\right)\left(\frac{c}{\beta_1}\right)^2 - (kH)^2 \frac{1-\left(\frac{\omega N_{10}}{N_{01}}\right)^2}{N_{01}} - \left(\frac{aH}{4}\right)^2.$$

4. Particular cases

Case (i): (Non-viscous half-space)

In this case, assume $N_1^{'} = 0$ as $z \to \infty$, then Eq.(3.5) simplifies to:

$$\tan\beta H\left(1+\frac{p\overline{N}_2}{2H\sqrt{aH}}\right) = \frac{\beta}{2H\sqrt{aH}}, \text{ here } p = \left[k^2 - \frac{\omega^2}{\overline{N}_2}\left(\rho_{11} - \frac{\rho_{12}^2}{\rho_{22}}\right)\right]^{\frac{1}{2}},$$

$$\beta^2 = \frac{1}{H^2}\left[\frac{1}{\overline{N}_0}\left(\rho_{11} - \frac{\rho_{12}^2}{\rho_{22}}\right)c^2 (kH)^2 - \frac{(aH)^2}{4} - (kH)^2\right].$$

Case (ii): Assume that the fluid parameters are absent i.e., $N_i^{'}, \rho_{i2}, \rho_{i20} = 0, i = 1,2,$ then the problem is reduces to the classical elasticity [5]. In this case, Eq.(3.5) reduces to

$$\left|c_{ij}\right| = 0, \quad i, j = 1,2,3$$

where

$$c_{11} = -B\sin(BH) + \frac{\cos(BH)}{2H\sqrt{aH}}, \quad c_{12} = -B\cos(BH) + \frac{\sin(BH)}{2H\sqrt{aH}}, \quad c_{13} = 0,$$

$$c_{21} = 1, \quad c_{22} = 0, \quad c_{23} = -\sqrt{N_{01}}, \quad c_{31} = 0, \quad c_{32} = 0,$$

$$c_{33} = \frac{-N_{03}}{H^2}\left[(kH)^2 - \left(\frac{c}{\beta_1}\right)^2 (\rho_{110}) \right],$$

$$B = \frac{1}{H^2}\left[\frac{1}{\overline{N}_0}(\rho_{110})\ m^2 (kH)^2 - \left(\frac{aH}{2}\right)^2 - (kH)^2 \right].$$

5. Numerical results

For the computational procedure, the following resources are utilized. The material values of the upper viscoporoelastic layer and lower viscoporoelastic half-space are considered as that of paper [19] and paper [5]. The values for N_1' and N_2' are taken from [5].Table 1 provides values for two viscoporoelastic solids. These values are used in the frequency equations to derive the implicit relationship between the wave characteristics. To calculate phase velocity $\left(\frac{c}{\beta_1}\right)$ and the attenuation $\left(Q^{-1}\right)$ the following are used [20].

$$\left(\frac{c}{\beta_1}\right) = \frac{\text{Solution of real part of Eq.(3.5)}}{\text{Wavenumber}}, \quad Q^{-1} = \frac{2\ \text{(Solution of imaginary part of Eq.(3.5)}}{\text{Solution of real part of Eq.(3.5)}}.$$

Table 1. Poroelastic Solid Parameters

Material parameters	N_1'	N_1	$\rho_{11}\left[kg/m^3\right]$	$\rho_{12}\left[kg/m^3\right]$	$\rho_{22}\left[kg/m^3\right]$
Mat-I	6.7×10^1	0.276×10^{10}	1.926×10^3	-0.002×10^3	0.215×10^3
Material parameters	N_2'	N_2	$\rho_{110}\left[kg/m^3\right]$	$\rho_{120}\left[kg/m^3\right]$	$\rho_{220}\left[kg/m^3\right]$
Mat-II	7.1×10^{10}	0.922×10^{10}	1.903×10^3	0	0.226×10^3

Values are calculated in a MATLAB environment using bisection technique, and the outcomes are illustrated in Figs2-7. Figure 2 depicts the relationship between wavenumber (kH) and phase velocity $\left(c/\beta 1\right)$.From the graph the wavenumber increases proportionally to rising phase velocity $\left(\frac{c}{\beta_1}\right)$ due to wave behavior modifications from heterogeneities (aH).The wave's speed of propagation is influenced by various characteristics of the medium. When heterogeneity is present, the wave probably behaves differently, i.e. rising wavenumber cause the phase velocity to increase. For three distinct levels of heterogeneity, attenuation rises with increasing wavenumber, as illustrated in Fig.3. This implies that waves with higher wavenumber experience more energy loss. The wave's propagation is affected by the heterogeneity medium, which intensifies attenuation. The degree of heterogeneity affects the connection between wavenumber and attenuation. For fixed values of $\left(\frac{\omega N_1'}{N_1}\right)$ as shown in Fig.4, as the wavenumber (kH) increases, so does the phase velocity $\left(\frac{c}{\beta_1}\right)$.For the given values of $\left(\frac{\omega N_1'}{N_1}\right)$, the phase velocity rises as the wavenumber increases.

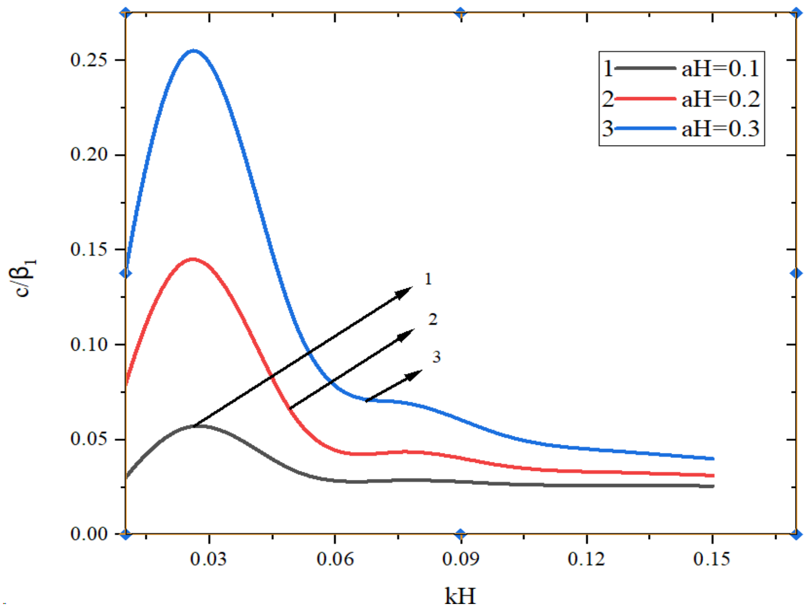

Fig.2. Discrepancy of phase velocity $\left(\dfrac{c}{\beta_1}\right)$ with wavenumber (kH) for various values of heterogeneity (aH).

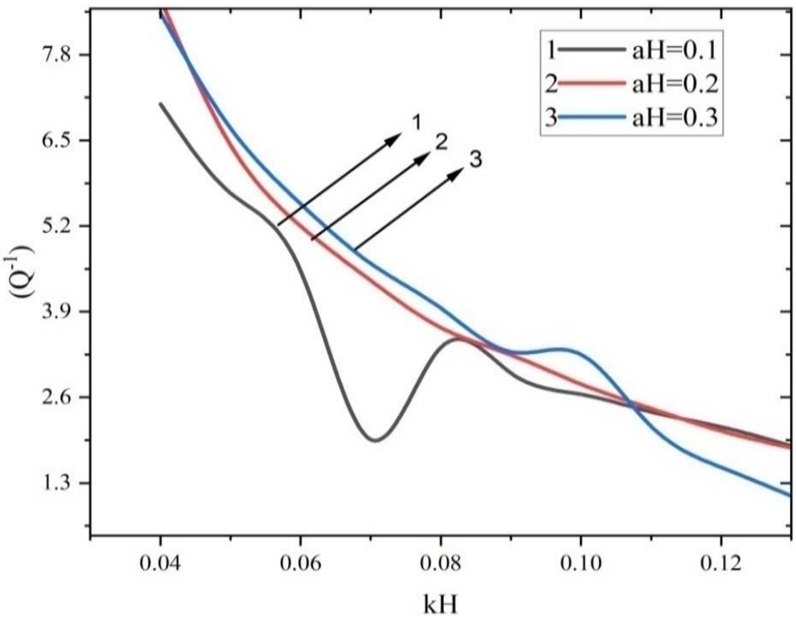

Fig.3. Discrepancy of attenuation coefficient with wavenumber (kH) and various values of heterogeneity (aH).

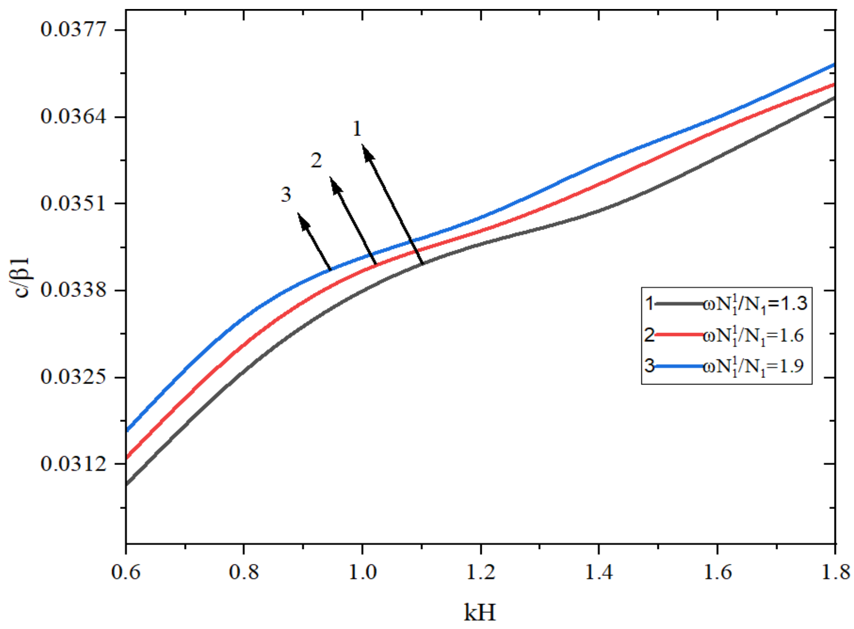

Fig.4. Discrepancy of phase velocity $\left(\dfrac{c}{\beta_1}\right)$ against wavenumber (kH) for various values of $\left(\dfrac{\omega N_1'}{N_1}\right)$.

From this Fig.4, it is concluded that the behavior remains constant when the ratio varies. A similar conclusion could be drawn from Fig.5, which shows that phase velocity increases as wavenumber rises under the specified values of $\left(\dfrac{\omega N_2'}{N_2}\right)$.Throughout the data in both figures, the trend remains constant. Attenuation $\left(Q^{-1}\right)$ varies with wavenumber (kH) for different levels of heterogeneity (aH), and fixed ratio values of $\left(\dfrac{\omega N_1'}{N_1}\right), \left(\dfrac{\omega N_2'}{N_2}\right)$ as illustrated in Fig.6. Attenuation increases for all values of heterogeneity as the wavenumber increases. This illustrates how waves with higher wavenumber dissipate more energy, regardless of the level of heterogeneity, the effect remains same.

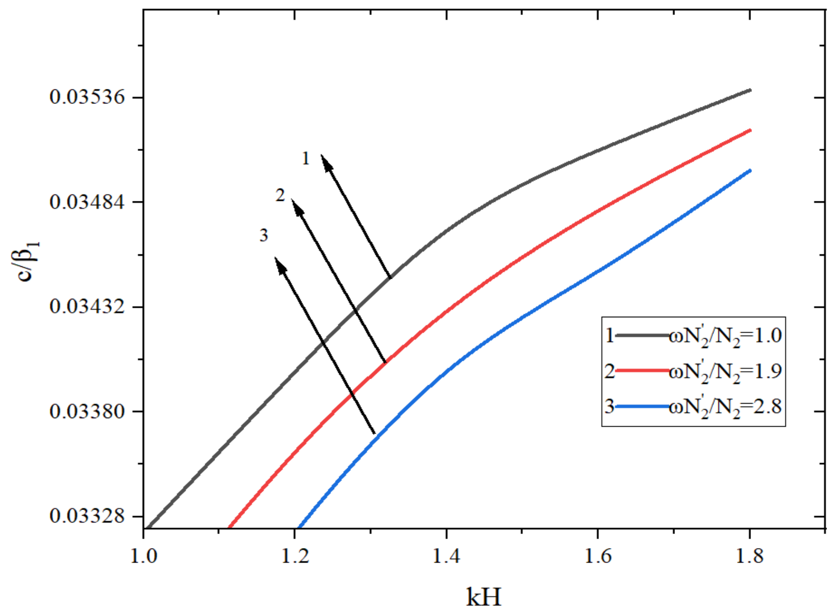

Fig.5. Discrepancy of phase velocity $\left(\frac{c}{\beta_1}\right)$ with wavenumber (kH) for various values of $\left(\frac{\omega N_2'}{N_2}\right)$.

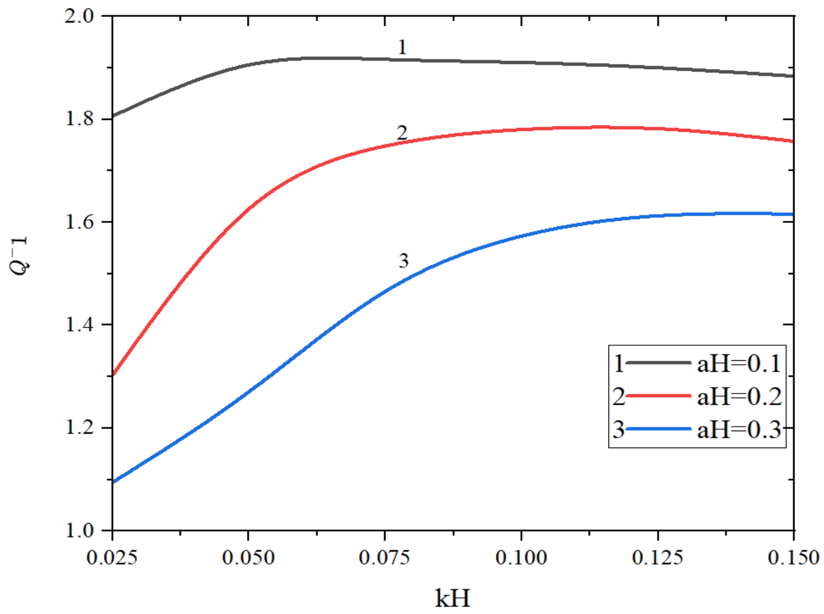

Fig.6. Discrepancy of attenuation coefficient with wavenumber (kH) and various ratio values of $\left(\frac{\omega N_1'}{N_1}\right), \left(\frac{\omega N_2'}{N_2}\right)$.

It is observed that higher heterogeneity increases attenuation, especially at lower wavenumbers.The curves show that as aH increases, wave energy dissipates more rapidly due to enhanced scattering and internal friction. Fluctuations in the curves indicate wave-mode interactions and resonance effects within the heterogeneous layer. These results highlight the significant role of heterogeneity in controlling seismic wave attenuation in viscoporoelastic media. This behavior shows that the relationship between heterogeneity and attenuation is not always linear, it depends on the interplay between wavelength, layer depth, and material properties. In the absence of the fluid parameter, Fig.7 shows how the phase velocity varies with wavenumber for particular case (ii). As the wavenumber grows, the phase velocity generally decreases. The current findings align with those published in previous investigations [5]. However, while the fluid parameter is present, Fig.5 demonstrates that the phase velocity rises with increasing wavenumber; when the fluid absent, Fig.7 reverses this tendency.

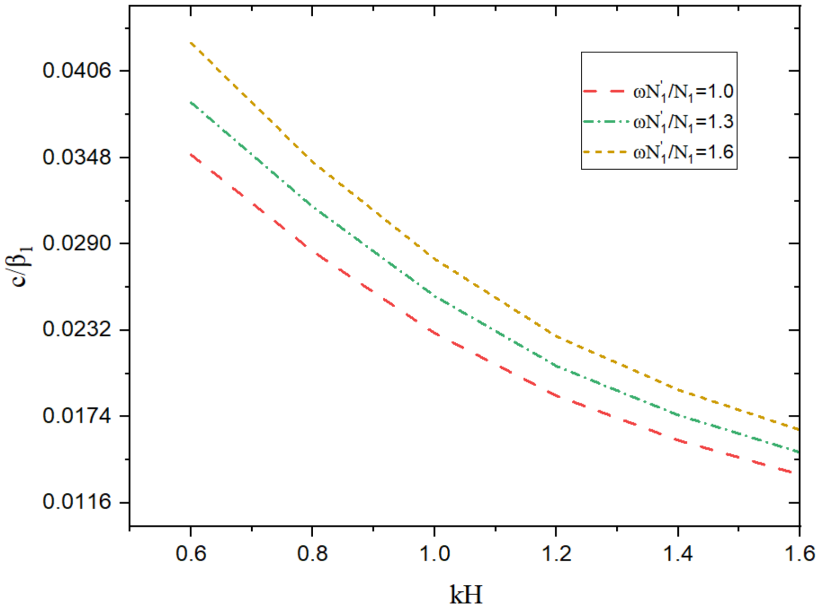

Fig.7.Variation of phase velocity with wavenumber in particular case (ii).

In the absence of the fluid parameter, Fig.7 shows how the phase velocity varies with wavenumber. As the wavenumber grows, the phase velocity generally decreases, as shown in Fig.7. The current findings align with those published in previous investigations [5]. However, while the fluid parameter is present, Fig.5 demonstrates that the phase velocity rises with increasing wavenumber; when the fluid absent, Fig.7 reverses this tendency.

6. Conclusion

The behavior of Love wave propagation with a heterogeneous viscoporoelastic layer with a viscoporoelastic half-space is being studied in the described research. The investigation assesses the real and imaginary quantities from the dispersion relation. The phase velocity is given by the real portion. The wave's attenuation, which explains how the wave's amplitude diminishes with time or distance, as a result of variables like viscosity and heterogeneity is provided by the imaginary component. According to the study, Love wave's phase velocity is increased by medium heterogeneity. The way several layers and their characteristics interact with one another in complex media can be linked to this effect. The results presented here, however, imply that viscosity actually accelerates the wave in the context of the heterogeneous viscoporoelastic half-space. Variations in the medium's characteristics can cause scattering and absorption of wave energy, contributing to attenuation. This research demonstrates that both the heterogeneity of the medium and the viscosity of the layer have significant effects on wave characteristics.

Nomenclature

A, Q, R – poroelastic constants

aH – heterogeneity coefficient

$\frac{c}{\beta_1}$ – phase velocity
e , ε – dilatations of solid and fluid
$e_{ij}\ (i, j = 1,2)$ – strain components
H – width of a layer
kH – wavenumber
$N_i\ (i = 1,2)$ – shear modulus
$N_i^{'}\ (i = 1,2)$ – viscosity of the fluid
Q^{-1} – attenuation
s – fluid pressure
$\vec{u}_i, \vec{U}_i\ (i, j = 1,2)$ – displacements of solid and fluid
$\delta_{ij}\ (i, j = 1,2)$ – Kronecker delta function
$\rho_{ij}, \rho_{ij0}\ (i, j = 1,2)$ – mass coefficients
$\left(\sigma_{yz}\right)_i (i = 1,2)$ – stress components
ω – frequency
∇^2 – Laplacian operator

References

[1] Chakraborty S.K. and Dey S. (1982): *The propagation of Love waves in water-saturated soil underlain by a heterogeneous elastic medium.*– J. Acta Mechanica, vol.44, pp.169-176. https://doi.org/10.1007/BF01303335.

[2] Romeo M. (2003): *Interfacial viscoelastic SH waves.*–J. International Journal of Solids and Structures, vol.40, pp.2057-2068.

[3] Kundu S., Kumari A., Gupta S. and Pandit D.K. (2016): *Effect of periodic corrugation, reinforcement, heterogeneity and initial stress on Love wave propagation.*– J. Waves in Random and Complex Media, vol.26, No.4, pp.485-515.

[4] Singh A.K., Mistri K.C. and Das A. (2016): *Propagation of Love type wave in a corrugated fibre-reinforced layer.*– Journal of Mechanics, vol.32, No.6, pp.693-708.

[5] Dewangan N. and Sahu S.A. (2017): *On phase velocity of Love type waves in heterogeneous visco-elastic medium.*– J. Procedia Engineering, vol.173, pp.1034-1041.

[6] Kakar R., Kakar S. (2017): *Love wave in a Voigt-type viscoelastic heterogeneous layer overlying heterogeneous viscoelastic half-space.*– International Journal of Geomechanics, vol.17, No.1,https://doi.org/10.1061/(ASCE)GM.1943-5622.0000675.

[7] Billon F. and El Baroudi A. (2021): *Mathematical modelling of Love wave propagation in viscoelastic waveguide loaded with complex fluids.*– J.Applied Mathematical Modelling, vol.96, pp.559-569.

[8] Chattopadhyay A., Chakraborty M. and Kushwaha V. (1986): *On the dispersion equation of Love waves in a porous layer.*– J.Acta Mechanica, vol.58, pp.125-136, https://doi.org/10.1007/BF01176595.

[9] Ke L.L., Wang Y.S. and Zhang Z.M. (2006): *Love waves in an inhomogeneous fluid-saturated porous layered half-space with linearly varying properties.*– J. Soil Dynamics and Earthquake Engineering, vol.26, pp.574-581.

[10] Kundu S., Manna S. and Gupta S. (2014): *Love wave dispersion in pre-stressed homogeneous medium over a porous half-space with irregular boundary surfaces.*– International Journal of Solids and Structures, vol.51, pp.3689-3697.

[11] Kumhar R., Kundu S. and Gupta S. (2020): *Modelling of Love waves in fluid saturated porous viscoelastic medium resting over an exponentially graded inhomogeneous half-space influenced by gravity.*– Journal of Applied and Computational Mechanics, vol.6,https://doi.org/10.22055/JACM.2019.29246.1575.

[12] Alam P., Satya Singh K., Anjum Badruddin I., Yunus Khan T.M. and Kamangar S. (2022): *Attenuation and dispersion phenomena of torsional waves in self-weighted, inhomogeneous, pre-stressed poro-elastic and poro-viscoelastic stratified structure.*– Waves in Random and Complex Media, vol.32, No.6, pp.2729-2750.

[13] Biot M.A. (1956): *Theory of elastic waves in a fluid-saturated porous solid. I. Low frequency range.*– Journal of the Acoustical Society of America, vol.28, pp.168-178.

[14] Venugopal M., Rajitha G. and Malla Reddy P. (2022):*G-type wave propagation in an initially stressed fluid saturated Viscoporoelastic layer lying over heterogeneous poroelastic half space.*– J.Mechanics of Solids, vol.57,No.1, pp.193-204.

[15] Poonem L.M., Gurijala R., Ala S. and Perati M.R. (2022): *Initial stress and heterogeneity effects on torsional waves in dry sand half-space between reinforced half-space and poroelastic medium.*– J.Engineering Computations, vol.39, No.5, pp.1752-1767.

[16] Pramanik D., Manna S. and Şahin O. (2024): *Love type wave propagation in a coated fluid-saturated fractured poro-viscoelastic layer with sliding contacts and point source effect*.– J.Applied Mathematical Modelling, vol.125, pp.424-444.

[17] Saini A. and Poonia R.K. (2023): *Propagation of Love waves under the effect of parabolic irregularity in isotropic fluid-saturated porous medium, Materials Today: Proceedings.*– [In press or volume pending]

[18] Kumar R. and Saini A. (2024):*Effect of anisotropy, inhomogeneity and porosity on Love wave propagation through fluid-saturated porous layers in irregular layered media.*– The European Physical Journal Plus, vol.139, No.12, pp.1-20.

[19] Ala S., Gurijala R. and Perati M.R. (2020):*Shear wave propagation in magneto poroelastic medium sandwiched between self-reinforced poroelastic medium and poroelastic half space.*– J.Engineering Computations, vol.37, No.9, pp.3345-3359.

[20] Gurijala R. and Perati M.R. (2020):*Study of radial vibrations in thick walled hollow dissipative poroelastic spherical shell on elastic foundation.*– J. Materials Physics and Mechanics,vol.44,No.3, pp.411-422.

[21] Bibi F., Ali H., Azhar E., Jamal M., Ahmed I. and Ragab A.E. (2023): *Propagation and reflection of thermoelastic wave in a rotating nonlocal fractional order porous medium under Hall current influence*.– Scientific Reports, vol.13, No.1, p.17703.

[22] Ali H., Bibi F., Azhar E. andJamal M. (2023): *Analyzing Hall current effects and fractional order nonlocal theory on plane wave reflection in a rotating isotropic medium*.– International Journal of Computational Materials Science and Engineering, p.2350050.

[23] Bibi F. and Ali, H. (2024): *Reflection and transmission phenomenon of plane waves at the interface of diffusive viscoelastic porous rotating isotropic medium under hall current and nonlocal thermoelastic porous solid.*– The Journal of Strain Analysis for Engineering Design, vol.59, No.7, pp.491-516.

[24] Ali H., Bibi F., Azhar E. andJamal M. (2024): *Study of fractional-order heat and Hall current on reflection of waves in viscoelastic rotating diffusive porous isotropic medium.*– Multiscale and Multidisciplinary Modeling, Experiments and Design, vol.8, No.7, pp.1-23.

[25] Azhar E., Bibi F., Ali H. and Jamal M. (2024): *Influence of hall current on thermoelastic wave behavior in viscoelastic fractional-order rotating porous solids.*– Arabian Journal for Science and Engineering, vol.49, No.7, 9947-9965.

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