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ANALYSIS OF CONJUGATE MIXED CONVECTION FLOW OF A NANOFLUID OVER A VERTICAL FIN IN POROUS MEDIA

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This paper presents a detailed study of the interaction between conduction through a vertical fin and conjugate mixed convection of a nanofluid flowing in a porous medium. The fin model and the primitive partial differential equations governing the nanofluid with boundary conditions are transformed into dimensionless forms. For the nanofluid, fin, and fin-nanofluid interface equations, a second-level nonsimilarity transformation is obtained and solved by the bvp4c solver. A validation of the computational code is ensured by comparing the results to a conventional fluid. It was found that the fin temperature is strongly controllable by the geometrical parameters and thermal conductivity of the fin, while Brownian motion and thermophoresis have a moderate effect on it. In addition, low values of Nr and Ω favor the fin efficiency. An analysis on very different values of the Pr number reveals that the use of nanofluids with a suitable base fluid allows high fin dissipations. A more advantageous thermal design can be achieved by combining a nanofluid in a porous medium with a fin in specific applications. These main results provide valuable information on the necessary optimization of the fin efficiency.

Key words: fin, porous media, nanofluid, mixed convection, conjugate problem.

1. Introduction

In recent decades, the attractiveness of porous media has been steadily increasing in many fields of technology and medicine. Due to their large specific surface area, and offering a good compromise between lightness and mechanical strength, they constitute structures for great thermal dissipation and mass exchanges. The porous media in which solid fin devices are incorporated allow significant intensification of heat transfers. For example, combined porous medium-fins improve the cooling of microprocessors and high-power electronic components by increasing heat dissipation. Similarly, the energy efficiency of car radiator and engine cooling systems can be increased. When fin-porous media tandem is used into hyperthermia treatment devices, the system's heat transfer is enhanced.

The fin is a structure in relation with a surrounding fluid is used to increase the heat transfer by convection and by increasing the contact surface. The theme extended surfaces like fins is well developed follow up by their utilization and improvement [1-3]. They are frequently immersed in a porous medium. A review on characteristics of porous medium and fluid laws through it, is squissed by Mahdi *et al.* [4]. A porous media saturated by a fluid causes the fin-fluid system to become strongly-dependent: the fluid's convection determines the fin's temperature distribution and the fin's temperature determines the fluid's convection until a final heat exchange is established. Thus, this conjugate problem must be solved from an appropriate mathematical fin-fluid models connected with an interface. The iteratively process of calculation is complete when acceptable convergence is reached between the wall fluid and fin temperature as explained by Cebeci and Bradshaw [5]. For more comprehensive conjugate natural convection several different configurations are considered in the review of Kimura *et al.* [6]. Considering a vertical fin under a prescribed uniform heat coefficient, and for free, purely forced and mixed convection, respectively, Sparrow and Acharya [7], Sparrow and Chyu [8], and Sunden [9] concluded that the predictions are not precise compared with those taking into account the variations of the local

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heat transfer rate induced by the fluid-solid interactions. Since then, the conjugate problems of convection, mainly mixed, have received considerable attention for a better accuracy heat transfer exchange resulting between a solid and the adjacent free fluid. Different geometries have been studied simulating in a real or approximate way the fluid-structure configurations for more representation of the physical situation. The mixed convection conjugate problem revealing various mechanisms have extensively studied along a vertical circular pin, Huang *et al.* [10], while Wang analysed the problem with a vertical cylindrical fin interactively with a non-Newtonien fluids, in [11]. The triangular fin-visco-elastic fluid couple has examined by Hsiao and Hsu [12]. In their investigation, Chen and Hsu [13] considered a moving vertical cylinder-micropolar fluid, and focused on the most uniform interface temperature profile with the highest contact conductance attachment technique. Kaya [14] showed the MHD effect on the conjugate mixed convection about a vertical hollow cylinder. To elucidate the interest in applied of certain king of fluids, MHD-micropolar fluid is employed by Alliche *et al*. [15]. An unsteady conjugate mixed convection heat transfer around a solid sphere is presented by Nguyen *et al.* [16]. Hsiao and Hsu [17] found that a second-grade visco-elastic fluid past a horizontal flat-plate fin lead to a better heat transfer with larger values of the significant parameters. About the vertical flat plate and conjugate mixed convection heat transfer, few works can be cited. A heated plate which is moving in an ambient power-law fluid has been examined by Kumari and Nath [18]. Later, the effect of magnetic field is highlighted by Mamun *et al.* [19], Azim *et al.* [20], Ali *et al.* [21] and Kaya and Aydin [22]. Hsiao [23] consider the ohmic dissipation mixed convection on a stagnation point induced by Maxwell fluid flow.

A fin embedded in porous medium is an attractive topic and there were some studies are related to conjugate of free or mixed convection problems, as detailed analysis of Pop *et al.* [24]. Assuming a vapor satured non Darcy porous medium, Char *et al.* [25] investigate numerically the effect of the wall conduction on the filmwise condensation heat transfer. Pop *et al*. [26] conducted their analysis of a vertical fin heated by a plate heat source in free convection condition at high Rayleigh numbers. For conjugate mixed convection in porous medium, Liu *et al.* [27] note that the trend results in porous media are similar to those of the classical fluid in non-porous media. The effect of lateral surface mass transfer on the same geometric case is studied by Liu and Minkowycz [28]. Both Gill and Minkowycz [29], and Hung [30], treated the vertical plate fin in a high porosity medium. Specific cases of non-uniform porosity and non-Darcian effects are examined by Cha *et al.* [31] and in non-Darcian porous media by Chen and Chiou [32]. Vaszi et *al*. [33] have analysed the interactive system with singularity at the tip fin. The coupled vertical fin-non Newtonian fluid in a porous medium are discussed by Pop *et al*. [34], showing the effects of the fin shape and power-law index parameters. Liu *et al*. [35] extended their previous analysis to a cylindrical fin in porous medium. With the lateral mass flux, a revisited paper of conjugate mixed convection of cylindrical fin in porous medium has adjusted by Liu *et al.* [36]. For a vertical circular pin, Kuang and Chien-Hsin [37] found that the non-Darcian effect significantly change the heat transfer compared to a Darcian porous media

In past three decades, the search for unconventional fluids characterized by intense heat transfer has become a reality. Choi [38] proposed metal nanoparticles dispersed in a conventional base fluid. Until today, this discovery appears as a major breakthrough. Due to increased thermal conductivity, these nanofluids have been extended to various branches of technology. The Brownian motion and the thermophoresis are found the most mechanisms that explains this improvement. To model the behavior of nanofluids, Buongiorno [39] introduce the above mechanisms adequately in the boundary layer equations. A comprehensive review about the developments of nanofluids in porous media is given by Kasaeian *et al*. [40]. For a vertical plate in contact with a nanofluid, some developed works explain the interaction of nanoparticles added to the base fluid. Kusnetsov and Nield [41] have applied the above findings to the natural boundary layers. They also studied double diffusion heat and mass transfer activated by a nanofluid [42]. Khan and Aziz focused on natural convectively heated vertical plate [43]. Khrisna *et al*. [44] are presented a free convection micropolar nanofluid flow through a porous surface and more recently Chamkha *et al*. [45] analyse the thermophoresis effect in free convection from vertical plate embedded in porous medium. Khademi *et al.* [46] adressed a numerical analysis for conjugate mixed convection flow of a nanofluid over an inclined plate in non-Darcian porous medium. Some works are dedicated to the mixed convection of nanofluid in porous media, over a vetical wedge, Gorla *et al*. [47], on a stretching sheet, Hsiao [48]. Entropy generation analysis is conducted in porous medium by

Hussain *et al*. [49], in double lid driven cavity by Mehmood *et al*. [50] and Hussain *et al*. [51]. Micropolar nanofluid towards stretching sheet is studied by Hsiao [52].

Nowadays, there is a great deal of attention being paid to the use of nanofluids in porous media and fins, which can enhance heat transfer rates. On a recent article-review, Khalaf *et al*. [53] note the importance of adding porous materials, nanomaterials and fins which contribute significantly and clearly to improving heat transfer. Several and specific configurations are exposed with reporting the numerical and/or experimental works in details.

The present work is motivated by these particularities:

- the porous structure, offering a very large expanded contact surface for circulating nanofluids, enhances convection and thermal diffusion.
- thermal conductivity of base-fluid is highly improved by the nature, concentration, size, and shape of the nanoparticles.
- the fin's heat exchange surface promotes heat removal from hot zones.

These interesting characteristics, combined in an integrated fin-nanofluid-porous medium assembly, reduce the size of thermal systems through accelerated heat dissipation without significantly impacting performance. For examples, such technology applied to power converters and processors improves significantly cooling for safe operation. Similarly, the absorber of a solar collector equipped with this combination provides improved efficiency.

Based on the above considerations and the reviewed literature, a conjugate mixed convection heat transfer, based on the interaction between a vertical fin filled and saturated by a nanofluid in porous medium, is here investigated. An analysis of fin-nanofluid in porous media is a contribution filling a gap in the literature, elucidating the fundamental effects of Browian motion, thermophoresis as well as the buoyancy ratio on the fin temperature distribution predicted. The impact of the Prandtl number on the fin efficiency is also investigated in accordance with the diversity and constant extension of the base fluids.

2. Mathematical analysis

2.1. Problem description and assumptions

A vertical fin is immersed in porous media saturated by a nanofluid. So, the heat flux coming from duct temperature T_f is transmitted by conduction to the attached fin which dissipated it to a nanofluid flowing in a porous medium.

The physical model is represented in Fig.1 as a schematic diagram in which all necessary geometric variables are reported. This system is under an uniform free stream u_∞ along the coordinate x and subject to a great interaction fin-nanofluid in porous medium. Dynamic and thermal boundary layers take place from the interface fin-nanofluid. So, a conjugate problem is here assumed because the convection along the nanofluid-fin interface is not primarily quantified. The fin temperature and the nanofluid temperature calculated by different models at the interface remain unknown until equality is established between the two temperatures.

Several important assumptions are then considered in this analysis:

1. It is assumed that the length of the fin is longer than its thickness so heat conduction along the fin is approximated by one-dimensional model.
2. Heat loss from the tip fin is neglected.
3. The temperature at the base of the fin is constant.
4. Except the density in the buoyancy term, all physical properties are assumed to be constant.
5. The Boussinesq approximation is adopted to the buoyancy mixed term.
6. Darcy flow is applied because laminar flow occurs.
7. Inertia term is neglected considering small pores and small velocities in the porous media.
8. Mixed convection and steady-state flow are considered.
9. The convective nanofluid and the porous media are everywhere in local thermal equilibrium, due to laminar flow in small pores.
10. No thermal resistance is between the duct and the base of the fin.

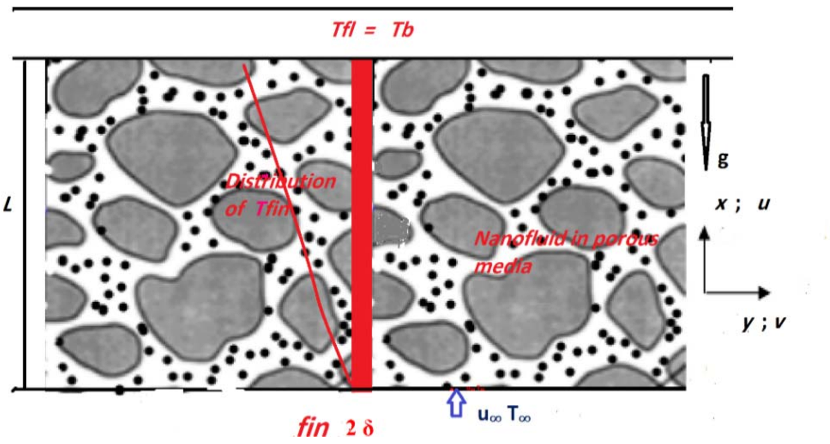

Fig.1. Schematic geometrical configuration of physical problem.

Based on the aforementioned assumptions, coupled equations for the fin conduction and combination of the Boussinesq and boundary layer approximations for the nanofluid flow in porous media are written.

2.2. Heat conduction in fin and B.C

$$k_f \delta \frac{d^2 T_f}{dx^2} = h(x)\left(T_f - T_\infty\right) \quad \text{and} \quad x = 0: \frac{d\, T_f}{dx} = 0;\; x = L:\; T_f = T_b. \tag{2.1}$$

Where δ, L are the half-tickness and the fin length; k_f is the thermal conductivity; $h(x)$ is the local heat transfer coefficient. T_f or T_b and T_∞ are the temperatures at the base of the fin and out the boundary layer.

Let $\theta_f = \frac{\left(T_f - T_\infty\right)}{\left(T_b - T_\infty\right)}$, and $\xi = \frac{x}{L}$. Then the above fin equation with B.C, Eq.(2.1), is in dimensionless form:

$$\frac{d^2\theta_f}{d\xi^2} = Ncc \cdot \lambda(\xi) \cdot \theta_f \quad \text{and} \quad \xi = 0: \frac{d\,\theta_f}{d\xi} = 0;\; \xi = 1:\; \theta_f = 1. \tag{2.2}$$

Where $Ncc = kL\sqrt{Re}/k_f$ is the convection-conduction parameter. The appeared parameter $\lambda(\xi) = hL/k\sqrt{Re}$ is the dimensionless heat transfer coefficient. The Reynolds number expressed here is $Re = u_\infty L/\nu$, where ν is the kinematic viscosity of the base-nanofluid.

2.3. Nanofluid in porous media and B.C

In primitive form and according to the Buongiorno's model [39], the convective boundary layers for the nanofluid in the porous media are:

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0, \tag{2.3}$$

$$\frac{\partial p}{\partial x} = -\frac{\mu}{K}u + (1 - \phi_\infty)\rho g \beta (T - T_\infty) - \left(\rho_p - \rho_{f\infty}\right) g (\phi - \phi_\infty), \tag{2.4}$$

$$\frac{\partial p}{\partial y}=0, \tag{2.5}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha_{nf}\frac{\partial^2 T}{\partial y^2}+\varepsilon\frac{(\rho c)_p}{(\rho c)_f}D_B\frac{\partial\phi}{\partial y}\frac{\partial T}{\partial y}+\frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial y}\right)^2, \tag{2.6}$$

$$\frac{1}{\varepsilon}\left(u\frac{\partial\phi}{\partial x}+v\frac{\partial\phi}{\partial y}\right)=D_B\frac{\partial^2\phi}{\partial y^2}+\frac{D_T}{T_\infty}\frac{\partial^2 T}{\partial y^2}. \tag{2.7}$$

The appropriate boundary conditions are:

$$y=0:\ v=0,\ \ T=T(x,0),\ \ \phi=\phi_w;\ \ y\to\infty,\ u=u_\infty,\ T=T_\infty,\ \phi=\phi_\infty. \tag{2.8}$$

u, v are the velocity components along with the x and y directions respectively. ϕ, ϕ_w and ϕ_∞ are the local volume fraction of the nanoparticles at current location, fin wall and out the boundary layer. ρ_f, ρ_p and ρ are the density of the base-fluid, the nanoparticle and nanofluid. μ is the absolute viscosity of the base fluid. $(\rho c)_p/(\rho c)_f$ are the heat capacity ratio of nanoparticle to the base fluid. α_{nf} is the thermal diffusivity of the nanofluid. β is the volumetric thermal expansion coefficient of the base fluid, D_B and D_T are the Brownian diffusion coefficient and the thermophoretic diffusion coefficient. ε and K are the porosity and permeability of the porous medium. T and T_∞ are the temperature of the nanofluid and out the boundary layer. p is the pressure and g the gravitational acceleration.

Introducing the stream function Ψ such $u=\frac{\partial\Psi}{\partial y}$, $v=-\frac{\partial\Psi}{\partial x}$. Then, by cross differentiation of Eq (2.4) /y and Eq.(2.5)/x and combination give a commode system:

$$\frac{\partial^2\Psi}{\partial y^2}=\frac{K}{\mu}(1-\phi_\infty)\rho g\beta\frac{\partial T}{\partial y}-\frac{K}{\mu}(\rho_p-\rho_{f\infty})g\frac{\partial\phi}{\partial y}, \tag{2.9}$$

$$\frac{\partial^2 T}{\partial y^2}=\frac{1}{\alpha_{nf}}\left[\frac{\partial\Psi}{\partial y}\frac{\partial T}{\partial x}-\frac{\partial\Psi}{\partial x}\frac{\partial T}{\partial y}-\varepsilon\frac{(\rho c)_p}{(\rho c)_f}\left(D_B\frac{\partial\phi}{\partial y}\frac{\partial T}{\partial y}+\frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial y}\right)^2\right)\right], \tag{2.10}$$

$$\varepsilon D_B\frac{\partial^2\phi}{\partial y^2}=\left(\frac{\partial\Psi}{\partial y}\frac{\partial\phi}{\partial x}-\frac{\partial\Psi}{\partial x}\frac{\partial\phi}{\partial y}\right)-\varepsilon\frac{D_T}{T_\infty}\frac{\partial^2 T}{\partial y^2}. \tag{2.11}$$

The new appropriate similarity variables are defined as:

$$\eta=y\sqrt{\frac{u_\infty}{\alpha x}}\ ;\ \xi=\frac{x}{L}\quad \text{and}\quad \Psi=\sqrt{\alpha u_\infty x}\,f(\eta,\xi)\ ;\ \theta=\frac{(T-T_\infty)}{(T_b-T_\infty)}\ ;\ s=\frac{(\phi-\phi_\infty)}{(\phi_w-\varphi_\infty)}.$$

Where η and ξ are the pseudo-similarity variable and the dimensionless variable in the x-direction; f, θ and s are the dimensionless velocity, temperature and the volumetric concentration of nanoparticles. First of all, introducing these new functions in Eqs (2.9-2.11), we get the first and second derivatives of them using the composite derivation rule. The primes ' and " are the derivatives with respect to η. Likewise operators $\frac{\partial}{\partial x}$ are transformed to $\frac{\partial}{\partial \xi}$ The above system Eqs (2.9-2.11) with Eqs (2.7-2.8) becomes:

$$f'' = (Gr/Re)\theta' - N_r\left(\frac{Gr}{Re}\right)s', \tag{2.12}$$

$$\theta'' + \frac{1}{2}f\theta' + N_b s'\theta' - N_t(\theta')^2 = \xi\left[\frac{\partial f}{\partial \eta}\frac{\partial \theta}{\partial \xi} - \frac{\partial f}{\partial \xi}\frac{\partial \theta}{\partial \eta}\right], \tag{2.13}$$

$$s'' + \frac{1}{2}L_n fs' + \frac{N_t}{N_b}\theta'' = L_n\,\xi\left[\frac{\partial f}{\partial \eta}\frac{\partial s}{\partial \xi} - \frac{\partial f}{\partial \xi}\frac{\partial s}{\partial \eta}\right], \tag{2.14}$$

$$\eta = 0: f = -2\xi\frac{\partial f}{\partial \xi},\ \theta = \theta(0,\xi),\ s = 1;\ \eta \to \infty,\ f' = 1,\quad \theta = 0,\ s = 0. \tag{2.15}$$

As seen, the system is not similar. To bypass a complete numerical solution by solving the above partial differential equations system, a subsequent approximate solutions are obtained using the local nonsimilarity method.

In the first level of truncation, the right-hand sides of Eqs (2.13-2.14), are ignored in this step. From Eq.(2.15), f is equal to zero. This corresponds to local similarity system.

For better precision, a second level of truncation is developed, with new unknowns functions G, H and J, so that $\frac{\partial f}{\partial \xi} = G$; $\frac{\partial \theta}{\partial \xi} = H$ and $\frac{\partial s}{\partial \xi} = J$. The next procedure consists in obtaining new equations by a derivation of the previous ones with respect to ξ.

Similarly, at this step, the system is then reduced by considering the terms $\frac{\partial G}{\partial \xi} = \frac{\partial H}{\partial \xi} = \frac{\partial J}{\partial \xi} = 0$, so the result system is:

$$f'' = (Gr/Re)\theta' - N_r\left(\frac{Gr}{Re}\right)s', \tag{2.18}$$

$$\theta'' + \frac{1}{2}f\theta' + N_b s'\theta' - N_t(\theta')^2 = \xi[f'H - G\theta'], \tag{2.19}$$

$$s'' + \frac{1}{2}L_n fs' + \frac{N_t}{N_b}\theta'' = L_n\,\xi[f'J - s'G], \tag{2.20}$$

$$G'' = (Gr/Re)H' - N_r\left(\frac{Gr}{Re}\right)J', \tag{2.21}$$

$$H'' + (0.5f + N_b s' - 2N_t\theta' + \xi G)H' - (f' - \xi G')H = -\frac{3}{2}G\theta' - N_b J'\theta', \tag{2.22}$$

$$J'' + \frac{1}{2} L_n \left(f + \xi G \right) J' - L_n \left(f' + \xi G' \right) J + \frac{N_t}{N_b} H'' = -0.5 L_n G s'. \tag{2.23}$$

The boundary conditions becomes:

$$\eta = 0 : f\left(\xi, 0\right) = 0\ ;\ \theta\left(\xi, 0\right) = \theta_f\left(\xi\right);\ s = 1;\ G\left(\xi, 0\right) = 0\ ;\ H\left(\xi, 0\right) = \frac{\partial \theta_f\left(\xi\right)}{\partial \xi},\ J = 0, \tag{2.24}$$

$$\eta \to \infty\ : f'\left(\xi, \infty\right) = 1;\ \theta\left(\xi, \infty\right) = 0;\ s = 0;\ G'\left(\xi, \infty\right) = 0;\ H\left(\xi, \infty\right) = 0,\ J\left(\xi, \infty\right) = 0. \tag{2.25}$$

The above dimensionless parameters are defined as:

- $L_n = \alpha_{nf} / \varepsilon D_B$, the nanofluid Lewis number,
- $Nb = \varepsilon\left(\rho c\right)_p D_B \left(\phi_w - \phi_\infty\right) / \left(\rho c\right)_f \alpha_{nf}$, the Brownian motion parameter,
- $Nt = \varepsilon\left(\rho c\right)_p D_T \left(T_w - T_\infty\right) / \left(\rho c\right)_f T_\infty \alpha_{nf}$, the thermophoresis parameter,
- $Nr = \left(\rho_w - \rho_{f\infty}\right)\left(\phi_w - \phi_\infty\right) / \left\{\left(1 - \phi_\infty\right)\rho_{f\infty}\beta\left(T_f - T_\infty\right)\right\}$, the buoyancy ratio parameter,
- $Pr = \nu / \alpha_{nf}$, the Prandlt number; $Gr = g\beta KL\left(T_b - T_\infty\right) / \upsilon^2$.

The Grashof number and *Gr/Re* is the mixed convection parameter denoted Ω.

2.4. Fin-nanofluid interface

Physically, the temperatures on the fin side and the fluid side as well as for the heat flows must be equal.

$$y = 0 : T_f\left(x\right) = T\left(x, 0\right);\ -k\frac{\partial T}{\partial y} = h\left(x\right)\left(T_f - T_\infty\right). \tag{2.26}$$

Or in dimensionless form:

$$\eta = 0 :\ \theta = \theta_f;\ \lambda\left(\xi\right) = -\left(\frac{Pr}{\xi}\right)^{\frac{1}{2}} \frac{\theta'\left(0, \xi\right)}{\theta_f\left(\xi\right)}. \tag{2.27}$$

$\lambda\left(\xi\right)$ is the dimensionless local heat transfer coefficient expressed by Eq.(2.27) following the similarity variables. It depends on the thermal boundary layer slope and fin temperature in dimensionless variables. Likewise, this coefficient is linked to the Prandtl number *Pr* defined as ratio of the viscous forces on the thermal diffusion of the nanofluid. Through this expression The fin-fluid interface shows the high conjugate problem in temperature.

3. Results and discussion

3.1. Discretisation, accuracy and process

Both the first and second level of truncation equations can be easily solved using the software Matlab built in bvp4c techniques. The implicit Lobatto finite difference discretisation process is rather accurate than the numerical Runge-Kutta methods. Local truncation error for individual step is estimates and solutions are calculated for each integration subinterval, using the midpoint quadrature rule at both the beginning and end.

The computation were performed by a choice of initial solution vector, a constant mesh $\Delta h = 0.001$ and a fixed 10^{-6} convergence criterion was selected.

Starting with analytically solution of Eq.(2.2):

$$\theta_f = cosh\left(\xi\sqrt{Ncc \cdot \lambda(\xi)}\right) / cosh\left(\sqrt{Ncc \cdot \lambda(\xi)}\right). \tag{2.28}$$

Thus, the developed and implemented code requires also an iterative computational loop used to take into account the conjugate fin-nanofluid interface problem, including the Eqs (2.18-2.25). So, for each ξ, the convective transfer coefficient is first estimated, then $\theta'(0,\xi)$ is extracted by the above described code based on bvp4c of Matlab, and finally this dimensionless coefficient $\lambda(\xi)$ is replaced by the new value from Eq.(2.28). The iterative process is continued until a precision of 10^{-6} is reached between the final and equal temperatures fin-nanofluid at $\eta = 0$. For ξ in Eq. (2.27), its expression has replaced by $\xi + 0.001$ in the code to avoid division by 0 corresponding to $x = 0$.

3.2. Validation of the code implemented

In order to verify the code and the numerical method developed with Eq.(2.28) and its derivative with respect to ξ associated to present second level of truncation Eqs (2.18-2.25), linked by Eq.(2.27), numerous computations are conducted. The results plotted in Fig.2, are obtained with $Pr = 5.5$; $Nb = Nt = Ln = 0$ and for $Ncc = 0.1, 2$ and $\Omega = 0.1, 1, 5,$ are compared with fin temperature distributions of Liu *et al.* [27], presented for a classical fluid. Although made by these authors using a third level, it should be noted that present high accuracy solution (at the second level) is showed, and validate the computational code, one can see in Fig.2.

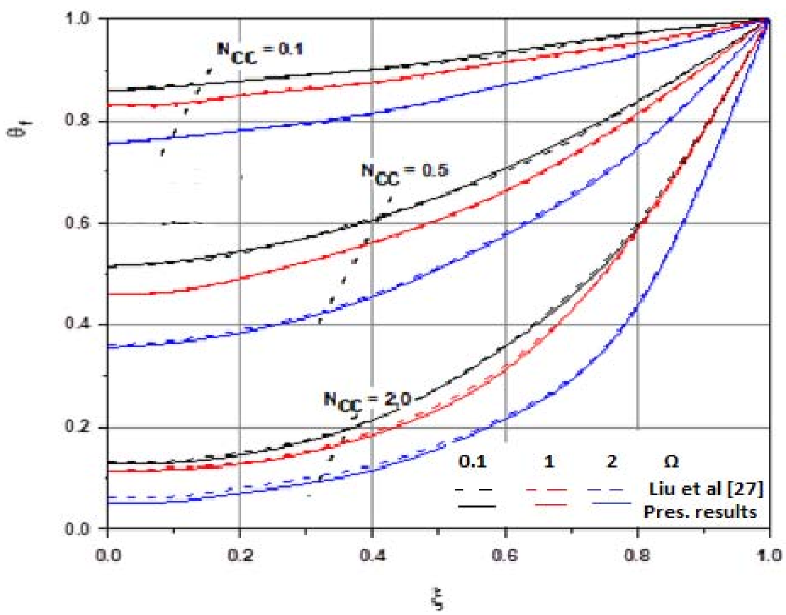

Fig.2. Validation of the code by comparison between the present and Liu *et al.* results [27] of fin temperature distributions for different values of Ncc and Ω $(Pr = 5.5;\ Nb = Nt = Ln = 0)$.

The temperature distributions along the fin are strongly dependent on *Ncc* and less dependent on Ω which represents the relative importance of free and forced convection. Physically, it is obvious that a more conductive fin with lower *Ncc* is more efficient than one with higher *Ncc*. It is explains that more is *Ncc* parameter, more is the convection effectiveness to fin conduction. As expected, the lower Ω is, meaning the fin is subjected to important forced convection, more is the fin' efficiency.

3.3. Discussion of results

For a non-classical fluid as nanofluid in which nanoparticles with higher thermal conductivity are added, Fig.3 shows the incidence of the buoyancy ratio parameter of the nanofluid surrounding the fin. The parameter *Nr* characterizes a difference in density due to temperature variations in the nanofluid and is dependent on the expansion coefficient and the volume fraction of nanoparticles. Some values of *Nr* are chosen corresponding to those encountered in the open literature. As shown in this figure, and for low *Nr*, the fin temperatures are high, due to limited convection and this result is compatible with a low sedimentation of nanoparticles. Physically, it is explained by the high fin temperature consistent with low buoyancy ratio. On the contrary and for higher *Nr* values, the temperatures are reduced, reflecting strong concentration of nanoparticles. The fin efficiencies are better when the thermal regime is purely forced in the porous medium, as well known. In the present presentation, this condition corresponds to weak values of Ω.

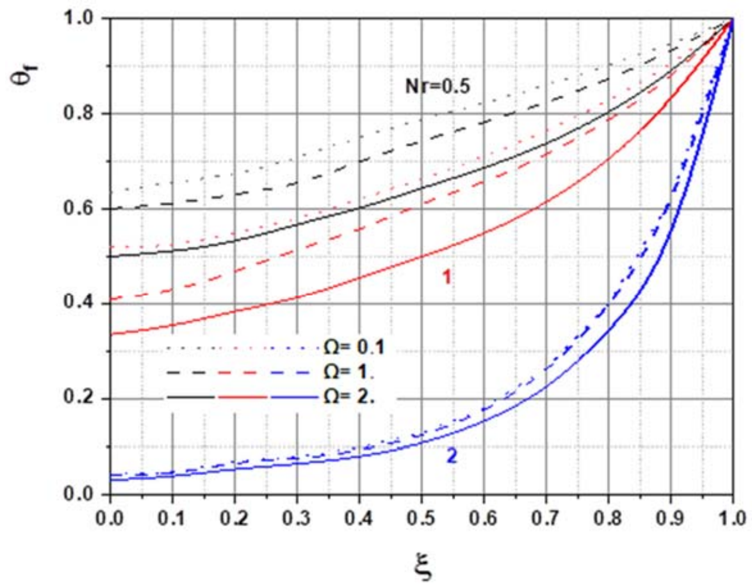

Fig.3. Interaction between the nanofluid and the fin. Effect of *Nr* and Ω on the fin temperature $(Pr = 7;$ $Ncc = 0.5;$ $Nb = Nt = 0.1;$ $Ln = 10)$.

Figure 4 is drawn up to best answer the opportunity of using a highly viscous nanofluid in this context. The choice of Prandtl number should reflect a large difference to cover the range of hybrid and highly diverse nanofluids representing the wide variety of applications. *Pr* highlighted the balance between the viscosity and the heat diffusivity of the nanofluid. Here, its influence on the dimensionless convective heat transfer coefficent $\lambda(\xi)$, in Eq.(2.17), is clear. For *Pr* three values were chosen, that correspond to large nanofluids based on light organic liquids, water, and oils, respectively. The results reveal that the behavior of a nanofluid with a high Prandtl number becomes of point of view interesting in terms of fin efficiency. With a low thermal diffusivity relative to the viscosity, i.e high *Pr*, the heat remains more localized leading to a reduction of heat by conduction in the nanofluid. At the interface and therefore at the level of the exchange surface of the fin, the heat becomes significant compared to strong conduction within the nanofluid. This explains that very high fin temperatures are observed in this case leading to high fin efficiency. In the same sense, a high concentration of nanoparticles is competitive towards the same objective. In addition, the interest is to work in mixed convection rather than in a strong free convective regime. This is explained by a less convection, induced by a slow heat diffusion as opposed to the momentum that diffuses in the nanofluid's mechanical boundary. However, if cooling of the fin is enforced, low *Pr* is recommended regardless of purely forced or strongly mixed convection. However, if *Pr* becomes too high, the fin may heat up locally, because the heat is not transported well enough in the fluid.

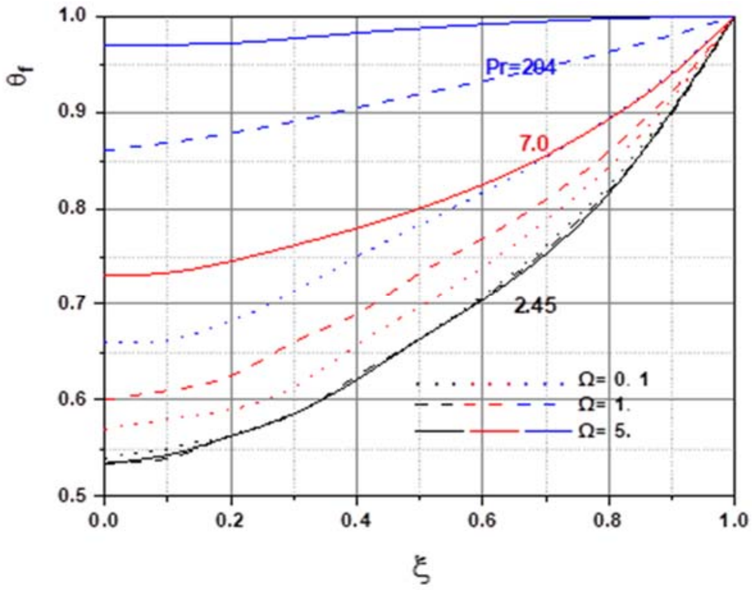

Fig.4. Interaction between the nanofluid and the fin. Effect of *Pr* and Ω on the fin temperature $(Ncc = 0.5$; $Nr = 0.2$; $Nb = Nt = 0.1$; $Ln = 10)$.

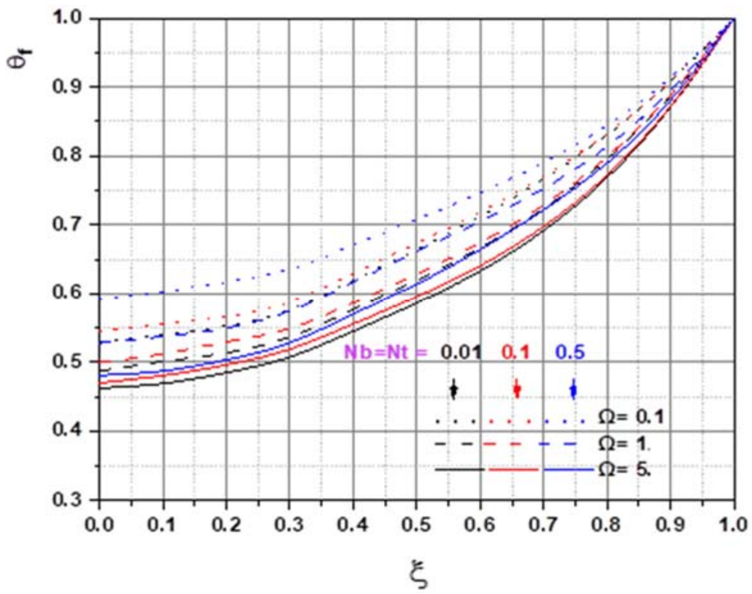

Fig.5. Interaction between the nanofluid and the fin. Effect of *Nb*, *Nt* and *Ω* on the fin temperature $(Pr = 7$; $Ncc = 0.5$; $Nr = 0.2$; $Ln = 10)$.

This study is based on the known single-phase Buongiorno approach, where Brownian diffusion and thermophoresis play an essential role on the dimensionless temperature profile. The effects of these parameters on dimensionless fin temperature are shown in Fig.5. Three set of curves with practical *Nb* and *Nt* are shown, and each curve is prepared for three values of the mixed convection parameter. Globally, it can be seen that the fin temperature is better as the Brownian motion and thermophoresis effects increase. This result can be explained by the competitive actions between more dispersion of nanoparticles that promotes the heat transfer, while reduce it caused by low thermal conductivity near the fin. So, it is evident that an optimized couple (*Nb*, *Nt*) should be determined to improve the fin'efficiency. Again, the conjugate fin problem is complex due to the fin-fluid system's convective conductivity, on the one hand. The conflicting impacts of the nanofluid's diffusive phenomena caused by the migrations of the nanoparticles based on temperature changes and their random agitations, constitute an additional complexity, on the other hand. The porosity ε of the medium appears in the *Nb* and *Nt* dimensionless parameters. Thus, The influence of ε on dimensionless fin temperature remains identical to that observed in Fig.15. The variations in porosity are in linear dependence on *Nb* and *Nt* and the trends are the same.

4. Conclusions

A conjugate mixed convection of nanofluid flow along vertical fin in porous medium is modeled. The governing equations for this complex fin-nanfluid interaction are developed, and the partial differential equations are transformed into ordinary equations using the non-similarity method. It was found that a mathematical development at the second truncation level is necessary and adequate to validate the results obtained for a classical fluid. Using from bvp4c and Lobatto III discretisation technique, numerical simulations were conducted to highlight mostly the effects of the dominant characteristics of the fin and the nanofluid on the fin temperature, the natural-force flow ratio, the Prandtl number, the Brownian motion and the thermophoresis .

The main conclusions within this paper are summarized below:

- The desired fin temperature distribution, or level of heat dissipation, can be controlled by a the geometric fin parameters as its thickness and/or thermal conductivity than the nanofluid flow regime.
- More beneficial thermal design of vertical fin in porous medium can be reached by this tandem.
- Low *Nr* is, high are fin temperatures i.e better fin efficiency with a low sedimentation of nanoparticles.
- The fin efficiency is enhanced using nanofluid with high Prandtl number of the base-nanofluid.
- The Brownian motion and the thermophoresis mechanisms should be optimized for a specific application.

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Nomenclature

b – base

Cp – specific heat at constant pressure $\left[J / Kg \cdot K\right]$

D_B – Brownian diffusion coefficient $\left[m^2 / s\right]$

D_T – thermophoretic diffusion coefficient $\left[m^2 / s\right]$

f – dimensionless velocity or fin
fl – fluide of duct
g – gravitational acceleration $\left[m^2/s\right]$
Gr – Grashof number
G, H, J – dimensionless first derivatives vs. ξ functions
$h(x)$ – local heat transfer coefficient $\left[W/m^2 \cdot K\right]$
k_f – fin -thermal conductivity $[W/m \cdot K]$
k – nanofluid thermal conductivity $[W/m \cdot K]$
K – permeability of porous medium $\left[m^2\right]$
L – fin-length $[m]$
Ln – nanofluid Lewis number
Nb – Brownian diffusion parameter
Ncc – conduction-convection parameter
Nr – buoyancy ratio parameter
Nt – thermophoretic diffusion parameter
nf – nanofluid
p – pressure of nanofluid $\left[N/m^2\right]$
Pr – Prandtl number
Re – Reynolds number
s – dimensionless nanoparticle volume fraction
T – temperature of nanofluid $[K]$
T_b – temperature at fin base $[K]$
T_f – fin-temperature $[K]$
u, v – velocity in x, y directions $[m/s]$
x, y – coordinates $[m]$
α_{nf} – thermal diffusivity of the nanofluid $\left[m^2/s\right]$
β – volumetric thermal expansion coefficient $[1/K]$
δ – half-fin thickness $[m]$
ε – porosity
η – pseudo-similarity variable
θ – dimensionless temperature
λ – dimensionless local heat transfer coefficient
ν – kinematic viscosity $\left[m^2/s\right]$
μ – dynamic viscosity $\left[N \cdot s/m^2\right]$
ξ – dimensionless stream wise coordinate
ρ – density of nanofluid $\left[kg/m^3\right]$
$(\rho c)_f$ – heat capacity of the nanofluid $\left[J/m^3 \cdot K\right]$
$(\rho c)_p$ – effective heat capacity of the nanoparticule materiel $\left[J/m^3 \cdot K\right]$
φ – nanoparticle volume fraction

Ψ – stream function $\left[m^2/s\right]$

Ω – mixed convection parameter

∞ – out the boundary layer

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