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THERMOELASTIC BEHAVIOR OF A HOLLOW CYLINDER USING SPACE-TIME FRACTIONAL HEAT CONDUCTION: A QUASI-STATIC APPROACH

S. Warbhe[1*] and V. Gujarkar[2]
[1]Department of Applied Mathematics, Laxminarayan Innovation Technological University, Nagpur, Laxminarayan Innovation Technological University, INDIA
[2]Research Scholar, Department of Mathematics, R. T. M. Nagpur University, Nagpur, INDIA
E-mail: shrikantwarbhe@gmail.com

The present study examines thermoelastic behaviour of a hollow cylinder governed by the space-time fractional heat conduction equation, employing a quasi-static approach. The analysis utilizes the Caputo time fractional derivative and finite Riesz space fractional derivative. An arbitrary temperature is applied to the upper surface of the cylinder while, the other boundaries are maintained at zero temperature. The heat conduction equation is solved using the integral transform technique. A mathematical model is developed specifically for pure copper material. The effects of varying the fractional orders of space and time on thermoelasticity, influenced by changes in thermal conductivity, are investigated and the results are depicted graphically.

Key words: integral transform, Mittag-Leffler function, quasi-static, fractional derivatives and integrals, thermal stresses.

1. Introduction

Recently, various engineering fields have integrated fractional calculus into their methodologies for a wide range of applications, including PID controllers, bio-mathematics, fluid mechanics, signal processing, viscoelasticity, and electro-chemistry. The study of fractional-order calculus, offers a fascinating extension of classical calculus to real or complex orders. However, giving tangible meaning to fractional-order calculus remains a challenge. Podlubny [1] offered a geometric viewpoint by comparing fractional integration, and its physical interpretation. An important strength of fractional-order differential equations lies in their capacity to model nonlocal properties, including long-term memory effects and complex chaotic behaviour. The Riemann-Liouville fractional derivative has been instrumental in the development of fractional calculus and its applications within pure mathematics. However, the advancement of modern technology necessitates a reassessment of the traditional pure mathematical framework.

Many researchers are deeply engaged in developing and analyzing various approaches to define and handle fractional-order derivatives. The foundation of fractional theory arises from its capability to account for delayed responses to physical stimuli, a behaviour frequently encountered in natural systems. This is in contrast to the generalized theory of thermoelasticity, which presupposes an instantaneous reaction.

Space-time fractional-order thermoelasticity, which extends classical thermoelastic theory, finds applications in diverse areas such as heat transfer modeling, stress analysis, and the study of material responses to thermal loads. This approach is especially valuable for systems where heat transfer and stress responses exhibit memory effects and are not immediate, as seen in viscoelastic materials or structures with complex geometries.

A novel thermoelasticity theory is developed through the application of fractional calculus by Sherief *et al.* [2]. Povstenko [3-7] carried out comprehensive research on fractional thermoelasticity using the quasi-static framework. Raslan [8] effectively resolved a problem involving a thick plate characterized by a

[*] To whom correspondence should be addressed

symmetric temperature distribution. Warbhe *et al.* [9-10] explored various issues in fractional-order thermoelasticity employing a quasi-static methodology. Employing the quasi-static theory, Tripathi *et al.* [11] examined the deflection in a thin circular plate under fractional order thermoelastic conditions with a constant temperature distribution. Tripathi *et al.* [12] employed the time fractional-order thermoelasticity theory to analyze a half-space problem involving a periodically varying heat source, aiming to regulate wave propagation speed. Warbhe [13] computed the thermal stresses in a rectangular plate with simple supports by employing stress due to temperature change and a time dependent fractional derivative. Ezzat and El-Karmany [14] examined fractional-order thermoelasticity problem. El-Karamany and Ezzat [15] developed a model for thermoelastic diffusion in both isotropic and anisotropic solids using a novel generalized theory that incorporates a memory-dependent derivative. A comprehensive analysis of the two-temperature theory within the framework of G.N. generalized thermoelasticity, incorporating fractional phase-lag heat transfer, is presented in the work by Ezzat *et al.* [16]. Ezzat and El-Bary [17] developed a new mathematical model for two-temperature electro-thermo-viscoelasticity, incorporating a novel approach to heat conduction based on a memory-dependent derivative. Ezzat and El-Bary [18] examined the influence of variable thermal conductivity and fractional-order heat transfer on a perfectly conducting, infinitely long hollow cylinder. Ezzat *et al.* [19] proposed a new mathematical model of generalized thermoelasticity incorporating memory-dependent derivatives, formulated within the framework of the dual-phase-lag heat conduction law.

The spatial non-locality is characterised by kernels, leading to fractional differential operators in spatial coordinates. Fil'Shtinskii *et al.* [20] successfully solved the space-time fractional heat conduction equation and explored its thermoelatic behaviour in a one dimensional half-space. Povstenko [21] explores issues related to the space-time fractional diffusion equation. Sherief and Abd El-Latief [22] employed the fractional-order thermoelastic theory in addressing a 2D problem for a half-space. Salama *et al.* [23] addresses the problem of thermoelasticity with fractional-order in a half space by employing a time-dependent thermal shock.

This study presents a mathematical framework for modelling heat conduction in materials exhibiting both spatial and temporal variability. To capture memory effects, the model incorporates a time-fractional differential operator, while spatial nonlocal interactions are represented through a space-fractional differential operator. Motivated by the practical relevance of fractional calculus, a comprehensive model integrating space-time fractional differential operators is developed. A quasi-static formulation is employed to investigate the resulting thermoelastic behaviour.

This study investigates a hollow cylindrical structure with space and time fractional derivatives and arbitrary temperature using the quasi-static approach. The problem is solved through the application of the integral transform technique. We analyse the thermoelastic behaviour of the hollow cylinder with respect to both time and space fractional order parameters. A modelling framework was established to represent the behaviour of pure copper and visually presented the graphical results depicting temperature distribution, displacement potential, and stresses within the hollow cylinder. The validation of this model was performed using Mathcad Prime 1.0 version.

2. Formulation of the problem

Consider a two-dimensional space-time fractional heat conduction equation defined within a hollow cylinder, incorporating fractional-order effects in both spatial and time domains, with dimensions $b \le r \le c;\ 0 \le z \le h$. Maintained temperature in inner boundary $(r = b)$, outer boundary $(r = c)$ and the lower surface $(z = 0)$ of the cylinder are at zero temperature and at the upper surface $(z = h)$, the temperature is defined in terms of $\dfrac{Q(t)\delta(r)}{2\pi r}$. The integral transform technique is employed to derive the solution to the problem.

The Caputo fractional derivative is defined according to the equation presented by Povstenko [3] as:

$$\frac{\partial^\alpha f(t)}{\partial t^\alpha} = \frac{1}{\Gamma(n-\alpha)} \int_0^t (t-\tau)^{n-\alpha-1} \frac{d^n f(\tau)}{d\tau^n} d\tau, \quad n-1 < \alpha < n. \tag{2.1}$$

According to Warbhe *et al.* [10], the Laplace transform of the Caputo derivative is given by the equation as:

$$L\left\{\frac{\partial^{\alpha} f(t)}{\partial t^{\alpha}}\right\}=s^{\alpha} f^{*}(s)-\sum_{k=0}^{n-1} f^{(k)}(0^{+}) s^{\alpha-1-k}, \quad n-1<\alpha<n. \tag{2.2}$$

The expression for the finite Riesz fractional derivative is introduced in El-Sayed, [24] as:

$$\frac{\partial^{\beta}\phi(z)}{\partial z^{\beta}}=\frac{I_{0+}^{2-\beta}\dfrac{d^2\phi}{dz^2}+I_{-}^{2-\beta}\dfrac{d^2\phi}{dz^2}}{2\cos\dfrac{(2-\beta)\pi}{2}}, \quad \text{for} \quad \beta=2 \tag{2.3}$$

where

$$I_{0+}^{\omega}\phi(z)=\frac{1}{\Gamma(\omega)}\int_{0}^{z}(z-\zeta)^{\omega-1}\varphi(\zeta)d\zeta, \qquad I_{-}^{\omega}\phi(z)=\frac{1}{\Gamma(\omega)}\int_{z}^{\infty}(\zeta-z)^{\omega-1}\varphi(\zeta)d\zeta,$$

are the Riemann-Liouville fractional integrals, $\omega>0$.

The space-fractional derivative of order β is defined by Saichev [25] as a pseudo-differential operator, characterized by the following rule for the finite Fourier transform:

$$F\left\{\frac{d^{\beta}\phi(z)}{d|z|^{\beta}}\right\}=-|\xi|^{\beta} F\{\phi(z)\}. \tag{2.4}$$

The equations for the displacement potential $\phi(r,z,t)$, as well as the stress functions σ_{rr} and $\sigma_{\theta\theta}$ are given in Warbhe *et al.* [9] as:

$$\frac{\partial^2\varphi}{\partial r^2}+\frac{1}{r}\frac{\partial\varphi}{\partial r}=(1+v)a_t T \tag{2.5}$$

with $\varphi=0$ at $r=b$ and $r=c$ for $t>0$,

$$\sigma_{rr}=-\frac{2\mu}{r}\frac{\partial\varphi}{\partial r}, \tag{2.6}$$

$$\sigma_{\theta\theta}=-2\mu\frac{\partial^2\varphi}{\partial r^2}. \tag{2.7}$$

Under the condition of plane stress within the hollow cylinder is

$$\sigma_{rz}=\sigma_{zz}=\sigma_{\theta z}=0. \tag{2.8}$$

The heat conduction equation with space-time fractional-order and arbitrary temperature for a hollow cylinder within the specified domain $b\le r\le c$, $0\le z\le h$ is defined as in Warbhe [13] as:

$$\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}+\frac{\partial^\beta T}{\partial z^\beta}=\frac{1}{a}\frac{\partial^\alpha T}{\partial t^\alpha}, \quad 0<\alpha<2, \quad 0<\beta<2, \tag{2.9}$$

boundary conditions

$$T=0 \quad \text{at} \quad r=b,\ 0\le z\le h, \tag{2.10}$$

$$T=0 \quad \text{at} \quad r=c,\ 0\le z\le h, \tag{2.11}$$

$$T=0 \quad \text{at} \quad z=0,\ b\le r\le c, \tag{2.12}$$

$$T=\frac{Q(t)\delta(r)}{2\pi r} \quad \text{at} \quad z=h,\ b\le r\le c, \tag{2.13}$$

initial conditions

$$T=0 \quad \text{when} \quad t=0,\ 0<\alpha<1, \tag{2.14}$$

$$\frac{\partial T}{\partial t}=0 \quad \text{when} \quad t=0,\ 1<\alpha<2. \tag{2.15}$$

The operator $\frac{\partial^\alpha}{\partial t^\alpha}$ represent Caputo fractional derivative predict memory effect whereas $\frac{\partial^\beta}{\partial z^\beta}$ finite Riesz fractional derivative predicted the long range interaction.

3. Solution of the problem

The finite Hankel transform and its inverse with respect to the spatial variable r over the range $b\le r\le c$ are defined here, as detailed in [26] as follows:

$$\overline{T}(\eta_p,z,t)=\int_{r=b}^{c} rK_0(\eta_p,r)T(r,z,t)\,dr, \tag{3.1}$$

$$T(r,z,t)=\sum_{p=1}^{\infty}K_0(\eta_p,r)\,\overline{T}(\eta_p,z,t). \tag{3.2}$$

Upon applying the Hankel transform to the system of Eqs (2.9) through Eq.(2.15), we obtain as:

$$-\eta_p^2\overline{T}+\frac{\partial^\beta \overline{T}}{\partial z^\beta}=\frac{1}{a}\frac{\partial^\alpha \overline{T}}{\partial t^\alpha} \tag{3.3}$$

with

$$\overline{T}=0 \quad \text{at} \quad z=0,\ b\le r\le c, \tag{3.4}$$

$$\overline{T}=\frac{Q(t)}{2\pi} \quad \text{at} \quad z=h,\, b\le r\le c\,, \tag{3.5}$$

$$\overline{T}=0 \quad \text{when} \quad t=0,\;\; 0<\alpha<1\,, \tag{3.6}$$

$$\frac{\partial\overline{T}}{\partial t}=0 \quad \text{when} \quad t=0,\;\; 1<\alpha<2 \tag{3.7}$$

where,

$$K_0(\eta_p,r)=\frac{\pi}{\sqrt{2}}\,\frac{\eta_p.J_0(\eta_p c).Y_0(\eta_p c)}{\left[1-\dfrac{J_0{}^2(\eta_p c)}{J_0{}^2(\eta_p b)}\right]^{\frac{1}{2}}}\left[\frac{J_0(\eta_p r)}{J_0(\eta_p c)}-\frac{Y_0(\eta_p r)}{Y_0(\eta_p c)}\right].$$

The transcendental equation has positive roots given by $\eta_1,\eta_2,\eta_3...$

$$\frac{J_0(\eta b)}{J_0(\eta c)}-\frac{Y_0(\eta b)}{Y_0(\eta c)}=0\,.$$

To solve the Eq.(3.3), we present Fourier Sine transform along with its inverse, defined in [27] as:

$$F\left\{\overline{T}(\eta_p,z,t)\right\}=\overline{\overline{T}}(\eta_p,\xi_m,t)=\int_0^h\overline{T}(\eta_p,z,t)\sin(\xi_m\,z)\,dz\,, \tag{3.8}$$

$$F^{-1}\left\{\overline{\overline{T}}(\eta_p,\xi_m,t)\right\}=\overline{T}(\eta_p,z,t)=\frac{2}{h}\sum_{m=1}^{\infty}\overline{\overline{T}}(\eta_p,\xi_m,t)\sin(\xi_m\,z)\,. \tag{3.9}$$

Upon utilizing the Fourier Sine transform as specified in Eq.(3.8), on the set of Eqs (3.3) through Eq.(3.5), we obtain;

$$\frac{d^\alpha\overline{\overline{T}}}{dt^\alpha}+a\left(\eta_p^2+\xi_m^\beta\right)\overline{\overline{T}}=a\,\xi_m^{\beta-1}(-1)^{m+1}\,\overline{Q}(t) \tag{3.10}$$

with

$$\overline{\overline{T}}=0 \quad \text{when} \quad t=0,\;\; 0<\alpha<1\,, \tag{3.11}$$

$$\frac{\partial\overline{\overline{T}}}{\partial t}=0 \quad \text{when} \quad t=0,\;\; 1<\alpha<2 \tag{3.12}$$

where $\xi_m=\dfrac{m\pi}{2},\; m=1,2,3,.......$

By applying Laplace transform to the aforementioned equations, we obtains;

$$\overline{\overline{T}}^{*}=\frac{a\,\xi_m^{\beta-1}(-1)^{m+1}\,\overline{Q}^{*}(s)}{[s^{\alpha}+a\left(\eta_p^2+\xi_m^{\beta}\right)]} \tag{3.13}$$

where, * denotes the Laplace transform.

Finally, upon performing the inversions of Laplace, Fourier Sine and Hankel transform on Eq.(3.13), the thermal distribution function is derived as:

$$T(r,z,t)=\frac{2a}{h}\sum_{p=1}^{\infty}\sum_{m=1}^{\infty}K_0(\eta_p,r)\sin(\xi_m z)\frac{1}{\xi_m}\left[1-\cos(\xi_m h)\right]\left(\frac{m\pi}{2}\right)^{\beta-1}(-1)^{m+1}\times$$
$$\times\left(\int_0^t t^{\alpha-1}E_{\alpha,\alpha}\left[-a\left(\xi_m^{\beta}+\eta_p^2\right)t^{\alpha}\right]Q(t-\tau)\,d\tau\right) \tag{3.14}$$

where $L^{-1}\left[\frac{1}{s^{\alpha}+a\left(\eta_p^2+\xi_m^{\beta}\right)}\right]=t^{\alpha-1}E_{\alpha,\alpha}\left[-a\left(\xi_m^{\beta}+\eta_p^2\right)t^{\alpha}\right]$,

$E_{\alpha,\alpha}(.)$ is the Mittag-Leffler function.

Determination of thermal stresses

By substituting Eq.(3.14), into Eq.(2.5), we determine the displacement function as:

$$\varphi(r,z,t)=\frac{-2a}{h}(1+v)a_t\sum_{p=1}^{\infty}\sum_{m=1}^{\infty}\frac{1}{\eta_p^2}K_0(\eta_p,r)\sin(\xi_m z)\frac{1}{\xi_m}\left[1-\cos(\xi_m h)\right]\left(\frac{m\pi}{2}\right)^{\beta-1}\times$$
$$\times(-1)^{m+1}\left(\int_0^t t^{\alpha-1}E_{\alpha,\alpha}\left[-a\left(\xi_m^{\beta}+\eta_p^2\right)t^{\alpha}\right]Q(t-\tau)\,d\tau\right). \tag{3.15}$$

Now, by incorporating Eq.(3.15) into Eqs (2.6) and (2.7), we obtain the expressions for the radial and angular stress functions, respectively, as:

$$\sigma_{rr}=\frac{4a}{h}(1+v)a_t\mu\sum_{p=1}^{\infty}\sum_{m=1}^{\infty}\frac{1}{r}\frac{1}{\eta_p^2}K_1(\eta_p,r)\sin(\xi_m z)\frac{1}{\xi_m}\left[1-\cos(\xi_m h)\right]\left(\frac{m\pi}{2}\right)^{\beta-1}\times$$
$$\times(-1)^{m+1}\left(\int_0^t t^{\alpha-1}E_{\alpha,\alpha}\left[-a\left(\xi_m^{\beta}+\eta_p^2\right)t^{\alpha}\right]Q(t-\tau)\,d\tau\right) \tag{3.16}$$

where $K_1(\eta_p,r)=\frac{-\pi}{\sqrt{2}}\frac{\eta_p^2.J_0(\eta_p c).Y_0(\eta_p c)}{\left[1-\frac{J_0^{\,2}(\eta_p c)}{J_0^{\,2}(\eta_p b)}\right]^{\frac{1}{2}}}\left[\frac{J_1(\eta_p r)}{J_0(\eta_p c)}-\frac{Y_1(\eta_p r)}{Y_0(\eta_p c)}\right]$

$$\sigma_{\theta\theta}=\frac{4a}{h}(1+v)a_t\mu\sum_{p=1}^{\infty}\sum_{m=1}^{\infty}\frac{1}{\eta_p^2}K_2(\eta_p,r)\sin(\xi_m z)\frac{1}{\xi_m}\left[1-\cos(\xi_m h)\right]\left(\frac{m\pi}{2}\right)^{\beta-1}(-1)^{m+1}\times$$
$$\times\left(\int_0^t t^{\alpha-1}E_{\alpha,\alpha}\left[-a\left(\xi_m^{\beta}+\eta_p^2\right)t^{\alpha}\right]Q(t-\tau)\,d\tau\right) \tag{3.17}$$

where

$$K_2(\eta_p,r)=\frac{-\pi}{\sqrt{2}}\frac{\eta_p^3.J_0(\eta_p c).Y_0(\eta_p c)}{\left[1-\frac{J_0^2(\eta_p c)}{J_0^2(\eta_p b)}\right]^{\frac{1}{2}}}\times$$
$$\times\left\{\frac{1}{J_0(\eta_p c)}\left[J_0(\eta_p r)-\frac{J_1(\eta_p r)}{\eta_p r}\right]-\frac{1}{Y_0(\eta_p c)}\left[Y_0(\eta_p r)-\frac{Y_1(\eta_p r)}{\eta_p r}\right]\right\}.$$

4. Numerical computation

To formulate the mathematical model with various parameters and functions for pure copper, aimed at analyzing the fractional-order thermal effects, we adopt the following values as specified in Warbhe [10]:

$$b=1\ m,\quad c=2\ m,\quad z=0.4\ m,\quad h=0.4\ m,\quad w=5,\quad t=5\text{ sec.},\quad \nu=0.35,$$

$$a=112.34\times10^{-6}\ m^2s^{-1},\quad \mu=26.67\ GPa,\quad \tau=4.5\text{ sec.},\quad a_t=16.5\times10^{-6}/K.$$

To account for the discontinuity in the function at time $t>0$, we define the function $Q(t)$ as follows:

$$Q(t)=e^{-wt},\quad t>0,\ w>0.$$

Numerical calculations were performed using Mathcad Prime 1.0 and the graphs were generated using Microsoft Excel 2007.

The graphical representations are shown as under:

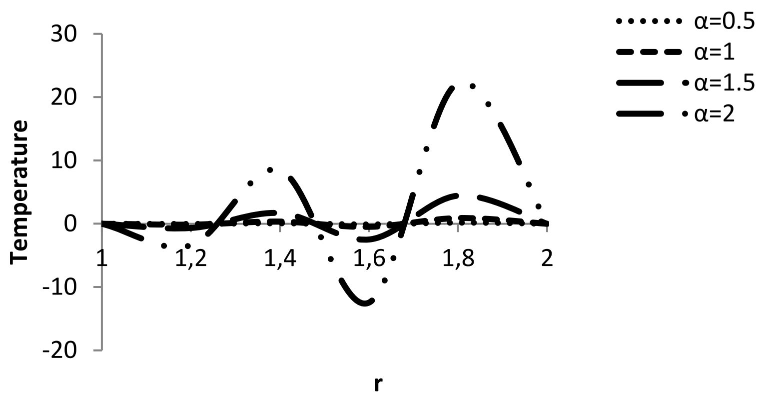

Fig.1. Dependence of temperature on r for $\beta=1.75$ and different values of α.

Temperature-time dependence for $\beta = 1.75$ along with different values of α illustrated in Fig.1. When the parameter α increases, the heat disturbance becomes oscillatory throughout the $1 \le r \le 2$ region, resulting in wave like structure.

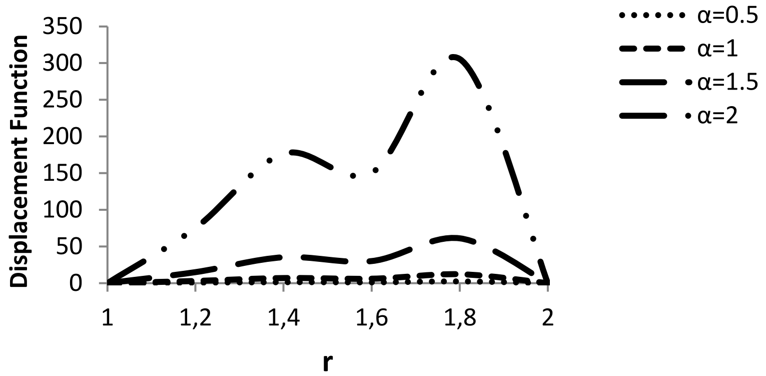

Fig.2. Dependence of displacement on r for $\beta = 1.75$ and different values of α.

Displacement potential function time dependence for $\beta = 1.75$ along with different values of α illustrated in Fig.2. It is observed that the peaks of displacement increases with the increase of parameter α and the graph illustrate that the displacement is zero at both ends of the cylinder.

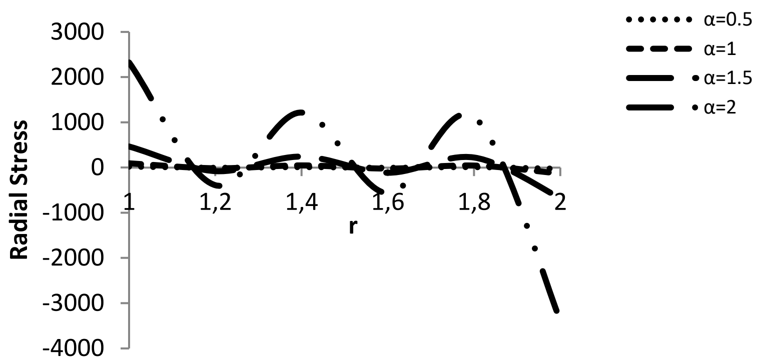

Fig.3. Dependence of radial stress on r for $\beta = 1.75$ and different values of α.

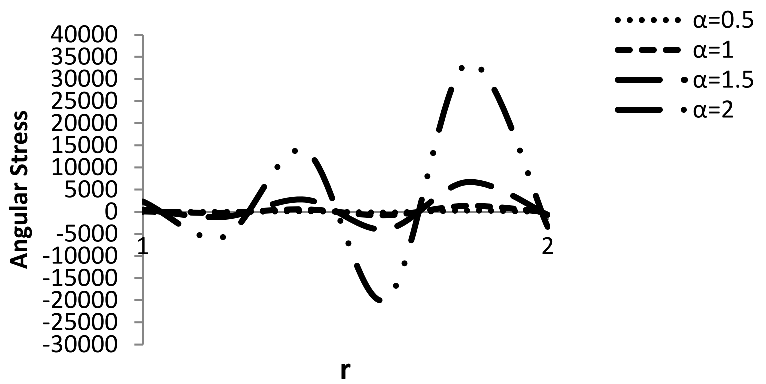

Fig.4. Dependence of angular stress on r for $\beta = 1.75$ and different values of α.

Figure 3 represents the dependence of radial stress on r for $\beta = 1.75$ and different values of α. It is observed that the radial stresses are compressive and exhibit a wave-like pattern till $r = 1.9$ and then it decreases towards outer radii gradually.

Figure 4 shows the dependence of angular stress on distance r for $\beta = 1.75$ and different values of α. The angular stresses remain tensile throughout the region, and their behaviour becomes increasingly wave-like with the growth of the fractional-order parameter.

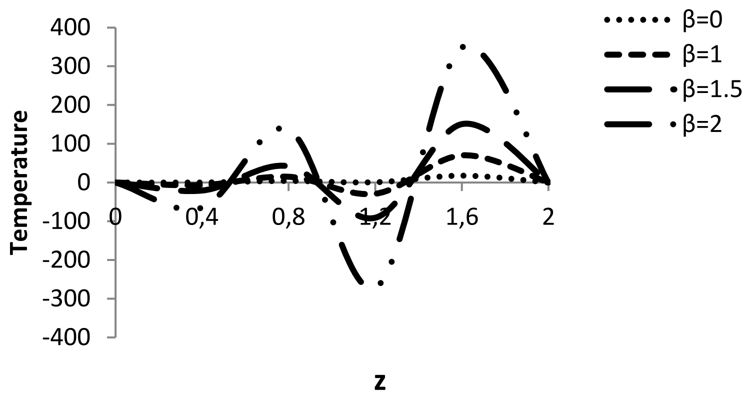

Fig.5. Dependence of temperature on z for $\alpha = 1.9$ and different values of β.

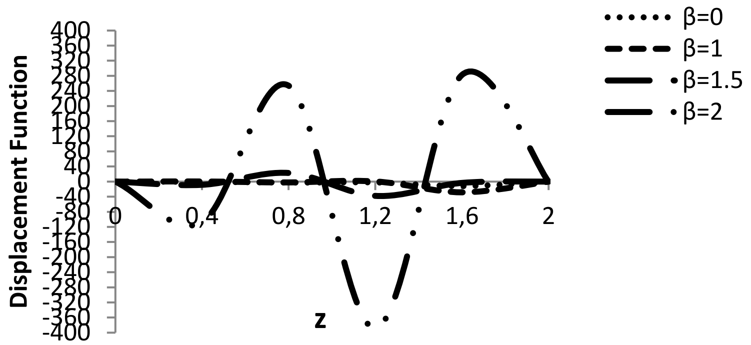

Fig.6. Dependence of displacement on z for $\alpha = 1.9$ and different values of β.

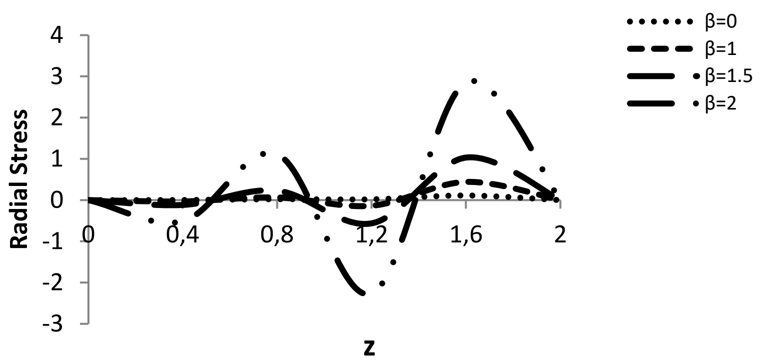

Fig.7. Dependence of radial stress on z for $\alpha = 1.9$ and different values of β.

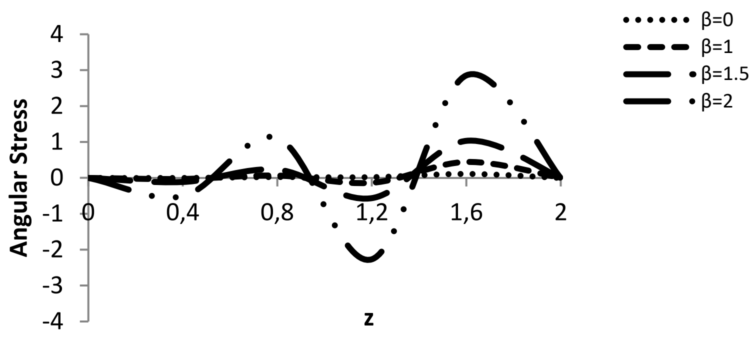

Fig.8.Dependence of radial stress on z for $\alpha = 1.9$ and distinct values of β.

Temperature-space dependence for $\alpha = 1.9$ and various values of β is presented in Fig.5. Here we observed that the variations are obtained due the impact of conductivity in the axial direction for the distinct spatial fractional order parameter β.

Displacement function-space dependence for $\alpha = 1.9$ along with different values of β is illustrated in Fig.6. It is observed that displacement is increases for larger value of fractional order parameter β and forms sinusoidal form in the axial direction.

From Figs 7 and 8 the stress function σ_{rr} and $\sigma_{\theta\theta}$ are shown tensile as well as compressive in nature due to the impact of displacement for distinct values of β in the axial direction.

5. Conclusion

This study is founded on the space-time fractional heat conduction equation incorporating Caputo and Riesz fractional derivatives for a hollow cylinder and discussed the thermoelastic behaviour by quasi-static approach.

Figures 1-8 illustrate the dependence of temperature, displacement, and thermal stresses in both radial and axial directions for varying values of the space-time fractional order. These graphical representations highlight the differences between classical and fractional-order thermoelasticity. Due to the impact of conductivity the weak conductivity, moderate conductivity, super conductivity occurs in the heated region with distinct space-time fractional order parameter α and β for a fixed time $t = 5$ sec., which predicted the infinite wave propagation.

The fractional-order theory foresees a delayed response to physical stimuli, while the space fractional differential operator effectively accounts for long-range interactions, aligning with observations in the natural world. To sum up, the outcomes detailed in this article are expected to be of value to researchers in the field of material sciences, as well as to material designers and those dedicated to advancing the theory of thermoelasticity through a quasi-static approach that incorporates fractional calculus.

The purpose of applying fractional-order thermoelasticity to space-time non-local heat conduction is to regulate wave propagation speeds across weak, moderate, and superconducting materials, as well as to analyze the nature of the problem, which predicts a delayed response. This approach is applied in electronic devices to forecast the retarded response.

The main objective of this study on fractional-order thermoelasticity in a hollow cylinder is to generalize classical uncoupled thermoelastic problems. As a result, this type of problem has not been explored previously for cylindrical bodies with thickness.

Nomenclature

a – thermal diffusivity

a_t – coefficient of linear thermal expansion

b – inner radius of the disk
c – outer radius of the disk
E – Young's modulus
h – thickness of the hollow cylinder
r – radius, m
s – parameter of Laplace transform
T – temperature distribution function
z – thickness, m
α – fractional order parameter for time
β – fractional order parameter for space
δ – Dirac-delta function
μ – Lamé constant
v – Poisson ratio
ξ – Fourier transform variable
σ_{rr} – radial stress function
$\sigma_{\theta\theta}$ – angular stress function
φ – displacement potential function

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