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IMPACT OF TRIANGULAR IRREGULARITY, MATERIAL HETEROGENEITY AND INITIAL STRESS ON THE PROPAGATION OF SHEAR WAVES IN A TRANSVERSELY ISOTROPIC POROUS LAYER

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In this work, the behaviour of shear waves in a FSPL that is initially stressed exhibits transverse isotropy is investigated. The layer is located on an elastic half-space, characterized by a triangular shaped irregularity at interface of contact. This research derives the dispersion equation of shear waves utilizing elasticity theory by Biot, combined with perturbation techniques and Fourier transformations. Computational simulations of the dispersion equation, performed using MATLAB, highlight important findings, such as when inhomogeneity, anisotropy, and porosity increase, a noticeable decrease in phase velocity is observed. Additionally, phase velocity drops significantly with rising wave numbers. The graphical results reveal that there is a significant influence on the dimensionless phase velocity by the wave number, irregularity depth, initial stress, and anisotropy, highlighting the complex interplay between these factors in wave propagation in such layered media. This research provides deeper insight into the behaviour of shear waves in complex geological formations, with potential applications in geophysical exploration and material science.

Key words: triangular irregularity, material heterogeneity, shear waves, initial stress, porous media.

1. Introduction

The scientific investigation of seismic waves that are the energy waves produced by an instantaneous release of energy within the Earth is termed as seismology. Earth is considered a layered medium made up of layers that have different mechanical properties and differing thicknesses. The seismic wave propagation is greatly influenced by the Earth's heterogeneous composition, which contains a particularly hard layer, the stiff interface, and the medium porosity. With more in-depth research on Earth's structure, seismologists have found that wave propagation is more influenced by the effects of inhomogeneity and boundaries with free surfaces. Many researchers have examined the seismic wave propagation in stratified material with varying degrees of success. This phenomenon has significant applications in seismology and geophysics. Theoretical research on seismic waves and elastic layering mediums is highly valuable due to its numerous potential applications in soil mechanics, structural and earthquake engineering, geophysical science, and seismology. Rich insights into the Earth's interior have been obtained via a variety of research projects and experiments involving the seismic wave behaviour through the layers of Earth. Geological media generally have the property of anisotropy. Transverse isotropy, most fundamental kind of anisotropy that represents media characterized by a single axis of symmetry describes genuine media of geophysical importance such as the inner core, the upper mantle, layered lower crust, and a stack of sediment layers. The study of elastic waves spread out from an earthquake possess the highest level of reliability and conclusions regarding the Earth’s internal constitutions. The first study of elastic waves showing the effect of the initial stress was conducted by Biot [1]. This research involved deriving the wave equations, studying the wave transmission and reflection phenomena and exploring the influence of the different factors on the wave propagation characteristics. A hypothesis for the movement of the stress vibrations in a poroelastic material with a viscous compressible fluid inside was developed by Biot

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[2]. A material that has porous voids within it is said to be porous. Generally speaking, the pores are filled with liquid, or gasses. A fluid saturated medium is a phenomenon in fluid mechanics that consists of an incompressible fluid phase combined with elastic solids. Since the Earth's structure is complicated and differs across its strata, seismologists are primarily interested in studying how waves propagate in monoclinic media. These types of mediums are vastly used in the seismology field to comprehend the seismic waves behaviour generated during earthquakes. Numerous researchers have examined the propagation of seismic waves, but only few have investigated the impact of various types of irregularities on the seismic wave behaviour. However, majority of the research on this topic does not focus on porous fluid-filled media with surface irregularities. Bhattacharya [3] investigated the dispersion curve for the Love waves resulting from the crustal layer that is transversely isotropic with irregularities. Chattopadhyay *et al.* [4] investigated the effect of irregularities in the thickness of crustal layers that is transversely isotropic on the dispersion equation for Love waves, providing rich insights into the seismic wave behaviour inside the Earth. Chattopadhyay *et al.* [5] investigated shear waves behaviour in a monoclinic layered structure placed over an elastic half-space characterized with a rectangular shaped irregularity at point of contact. Gupta *et al.* [6] explored the impact of anisotropy and irregularities on shear waves movement. The perturbation technique was employed to derive the dispersion equation and to generate phase velocity curves for various irregularities by utilizing the porous medium parameters which were proposed by Biot [7] that introduced a theoretical framework describing coupled behaviour of the solid and fluid within porous medium. Crampin [8] studied the shear waves that stress changes before occurrence of earthquake. The Love waves dispersion equation in a FSPL that is transversely isotropic situated atop an elastic half-space was obtained by Konczak [9]. In another study, Konczak [10] examined how shear waves move through multilayered media containing fluid-saturated porous layers. Numerous researchers have since analysed shear wave propagation across diverse heterogeneous media. Because a porous media is intricate in nature and has many geometrical properties, such as porosity and specific internal area, it is challenging to characterize geometrically. Fluid-saturated porous rocks can be found on or beneath the surface of the Earth as stone, groundwater, and oil sentiments. Thus, research on shear wave behaviour in fluid saturated porous medium has a significant influence. Biot worked on the single-phase elastic wave propagation for liquid-filled porous materials. Using the perturbation technique, the frequency relation was calculated for various irregularity sizes, and Biot examined the impact of these irregularities on phase velocity curves. The significance of irregularity and rigidity in FSPL, anisotropic elastic layers for both single and multi-layered was evaluated by Kumar *et al.* [11]. Saha *et al.* [12] studied how irregularities affect shear wave movement in layered systems. Lastly, Saini and Kumar [13] examined the influence of a parabolic shaped irregularity and initial stress on the behaviour of Love waves in a FSPL overlying an elastic half-space. The frequency equation was formulated by Pal *et al.* [14] after the examination of the surface waves behaviour in a FSPL between a homogeneous liquid layer and an orthotropic half-space. The behaviour of Love waves in a transversely isotropic layer situated atop an elastic half-space was investigated by Alam and Kundu [15] under the conditions of initial stress. Kakar and Kakar [16] examined the influence of initial stress, irregularity, and porosity on the behaviour of Love waves inside an inhomogeneous crustal layer lying above an inhomogeneous half-space. The irregularity has been represented as a parabola in half-space. Love waves are seen to propagate through this presumptive medium and it is possible to derive the phase velocity dispersion equation. Mahanty *et al.* [17] studied the behaviour of shear waves in both inhomogeneous and homogeneous fiber-reinforced media using the model of cylindrical Earth. Sahu *et al.* [18] investigated the behaviour of shear waves in a FSPL situated between elastic half-spaces. The study's main objectives were to analyse the behaviour of shear waves as they propagate through this complex medium, taking into account the effects of saturation and the half-space's heterogeneity. It also involved deriving the dispersion equations, studying how different parameters the influence the wave propagation, and studying wave transmission as well as reflection phenomena. The impact of rectangular shaped irregularity on the behaviour of Love waves in a fiber-reinforced media situated above a half space was examined by Prasad and Kundu [19]. The impact of several types of material inhomogeneity on a complex structure comprising two superficial layers situated above a homogeneous isotropic initially stressed half space was examined by Saha *et al.* [20]. The Love waves behaviour in compressible elastic media under uniform initial stress was studied by Ejaz and Shams [21]. Lastly, Kumar and Saini [22], examined behaviour of Love waves in a FSPL exhibits anisotropy resting on a

half-space. It has been found that pre-stressing in the media, along with the anisotropy and porosity of the porous layer significantly influences the phase velocity of seismic waves. These results advance our understanding on wave propagation in complicated and varied mediums. Many researchers have conducted research in this field such as Poonia *et al.* [23], Saini and Poonia [24], Saini and Kumar [25] and Kumar *et al.* [26] etc.

The main idea of this paper is to study the behaviour of shear waves in a FSPL which is transversely isotropic overlying an elastic half-space characterized with triangular interfacial irregularities under initial stress conditions. In this work, the perturbation technique combined with Fourier transform method was employed to derive the dispersion relation for shear waves. The resulting dispersion equation demonstrates the influence of several parameters (depth of irregularity, wave number and layer's width) on shear wave behaviour for the specified problem. The phase velocity versus the wave number have been plotted for various ratios of irregularity depth to the width of the layer and various parameter values of inhomogeneity using MATLAB.

2. Mathematical formulation of the problem

Consider a model having a FSPL that is transversely isotropic of thickness H (termed as medium M_1) resting above an elastic half-space that is non-homogeneous in nature under the initial stress conditions (termed as medium M_2). Here, we consider the triangular shaped irregularity with width $2s$ and depth H_1. The location of origin of xz plane is on the centre point of irregularity's interface. Along the positive z axis direction, the disturbance source is positioned at d distance from the origin (where $d > H_1$). The problem's geometry is illustrated in the figure.

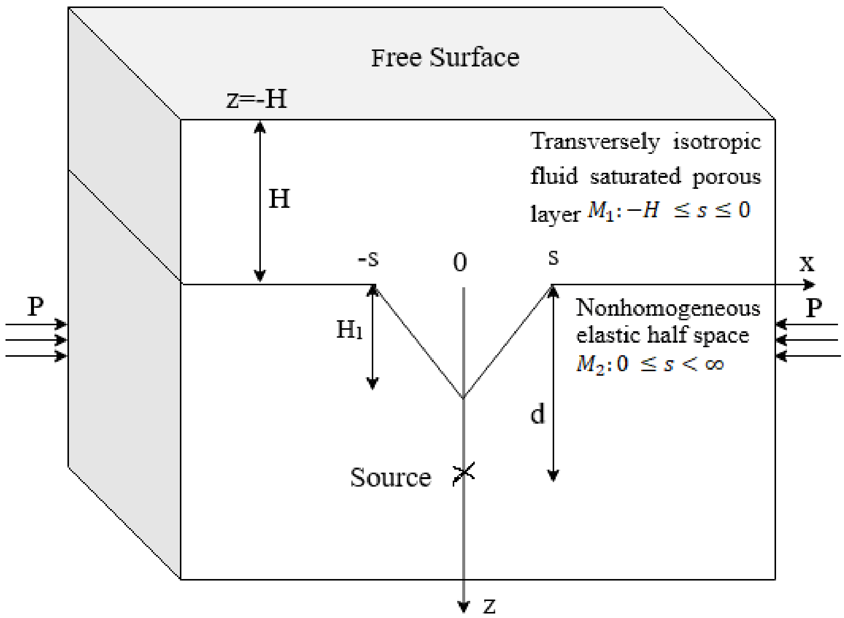

Fig.1. Mathematical modeling of the problem.

The equation of the considered triangular shaped irregularity has been taken using Kumar *et al.* [26] as

$$z = \epsilon h(x),$$

$$h(x)=\begin{cases} 0; & x>|s|, \\ H_1\left(1+\dfrac{x}{s}\right); & -s<x<0, \\ H_1\left(1-\dfrac{x}{s}\right); & 0<x<s \end{cases} \tag{2.1}$$

where $\epsilon=\dfrac{H_1}{2s}<<1$, $s\neq 0$, H_1 is the height of irregularity and $2s$ is the width of the irregularity.

3. Governing equations

To study the propagation of Love waves through the structure, it is important to first set up the governing field equations and material relations for both mediums.

3.1. For M_1 medium

Biot [2] gives the basic equations of motion, assuming no body forces are present.

$$\begin{aligned} \sigma_{ij,j} &= \rho_{11}\ddot{u}_i+\rho_{12}\ddot{U}_i-b_{ij}\left(\dot{U}_j-\dot{u}_j\right), \\ \sigma_{,i} &= \rho_{12}\ddot{u}_i+\rho_{22}\ddot{U}_i+b_{ij}\left(\dot{U}_j-\dot{u}_j\right), \end{aligned} \tag{3.1}$$

where the terms σ_{ij}; $i,j=1,2,3$ represents the tensor components of stress per unit area for the solid. The fluid pressure is denoted as $\rho=-fp$, where p represents fluid pressure and f represents medium's porosity. The components of the solid and fluid are u_i and U_i respectively. The effects of inertia for the moving fluid are taken into consideration by mass coefficients $\rho_{11},\rho_{12},\rho_{22}$.
The stress strain relation for medium M_1 using Kończak [10]:

$$\begin{bmatrix}\sigma_{11}\\ \sigma_{22}\\ \sigma_{33}\\ \sigma_{23}\\ \sigma_{31}\\ \sigma_{12}\\ \sigma\end{bmatrix}=\begin{bmatrix} 2C_1+C_2 & C_2 & C_3 & 0 & 0 & 0 & C_6\\ C_2 & 2C_1+C_2 & C_3 & 0 & 0 & 0 & C_6\\ C_3 & C_3 & 2C_4 & 0 & 0 & 0 & C_7\\ 0 & 0 & 0 & 2C_5 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 2C_5 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 2C_1 & 0\\ C_6 & C_6 & C_7 & 0 & 0 & 0 & C_8\end{bmatrix}\begin{bmatrix}e_{11}\\ e_{22}\\ e_{33}\\ e_{23}\\ e_{31}\\ e_{12}\\ E\end{bmatrix}, \tag{3.2}$$

where $$E=divU\equiv U_{j,j},\quad e=divu\equiv u_{k,k},\quad e_{ij}=\frac{1}{2}\left(u_{i,j}+u_{j,i}\right), \tag{3.3}$$

and $C_1, C_2, ..., C_8$ are materials constant.
The fundamental equation of shear-waves moving in the xz plane while particles displacement takes place in the y-axis,

$$U_1 \equiv 0, \quad U_3 \equiv 0, \quad U_2 \equiv U(x,z,t),$$
$$u_1 \equiv 0, \quad u_3 \equiv 0, \quad u_2 \equiv u(x,z,t). \tag{3.4}$$

By using (3.2) and (3.3) in (3.1), the system of equations is formulated as:

$$C_1 \frac{\partial^2 u_2}{\partial x^2} + C_5 \frac{\partial^2 u_2}{\partial z^2} = \frac{\partial^2}{\partial t^2}(\rho_{11} u_2 + \rho_{12} U_2) - b_{11} \frac{\partial}{\partial t}(U_2 - u_2). \tag{3.5}$$

After the elimination of u_2 and U_2 from the equation (3.5), yields to

$$\left\{ C_1 \frac{\partial^2 u_2}{\partial x^2} + C_5 \frac{\partial^2 u_2}{\partial z^2} - \left[\rho_{11}\partial_t^2 + b_{11}\partial_t - \frac{\left(\rho_{12}\partial_t^2 - b_{11}\partial_t\right)^2}{\rho_{22}\partial_t^2 + b_{11}\partial_t} \right] \right\} (u_2, U_2) = 0. \tag{3.6}$$

where ρ_{11}, ρ_{12}, and ρ_{22} are the mass coefficients, and the displacement vector components for the fluid and the solid represented as U_2 and u_2 respectively and C_1, C_5 are the constant for the material.

3.2. For M_2 medium

Ewing *et al.* [27] provides the basic motion equations for the lower half-space, assuming no body forces are present,

$$\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} + \frac{\partial \sigma_{xz}}{\partial z} - P\left[\frac{\partial \Omega_3}{\partial y} - \frac{\partial \Omega_2}{\partial z}\right] = \rho \ddot{u}_0,$$

$$\frac{\partial \sigma_{yx}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \sigma_{yz}}{\partial z} - P\left[\frac{\partial \Omega_3}{\partial x}\right] = \rho \ddot{u}_2, \tag{3.7}$$

$$\frac{\partial \sigma_{zx}}{\partial x} + \frac{\partial \sigma_{zy}}{\partial y} + \frac{\partial \sigma_{zz}}{\partial z} - P\left[\frac{\partial \Omega_2}{\partial y}\right] = \rho \ddot{u}_3,$$

$$\Omega_1 = \frac{1}{2}\left[\frac{\partial u_3}{\partial y} - \frac{\partial u_2}{\partial z}\right], \quad \Omega_2 = \frac{1}{2}\left[\frac{\partial u_1}{\partial z} - \frac{\partial u_3}{\partial x}\right], \quad \Omega_3 = \frac{1}{2}\left[\frac{\partial u_2}{\partial x} - \frac{\partial u_1}{\partial y}\right],$$

where $\sigma_{ij,j}$ represents stress tensor components, u_i represents the vector displacement components, P is the initial stress when there is no body forces and ρ is the density. The term Ω represents vector component for rotation in the lower half-space.
The constitutive relations using Kończak [10] are

$$\sigma_{ij} = \lambda e_{kk}\delta_{ij} + 2\mu e_{ij},$$

$$2e_{ij} = (u_{i,j} + u_{j,i}), \tag{3.8}$$

$$e_{kk} = u_{k,k} = e.$$

In this context, λ and μ represents the elastic coefficients given by Lame varying with x, y, z and δ_{ij} represents the Kronecker delta. This paper focuses on shear waves propagating in the xz plane with displacements aligned along the y direction and represents independently of the y co-ordinate. Hence,

$$U^{(2)} \equiv W^{(2)} \equiv 0, \qquad V^{(2)} \equiv V^{(2)}(x,z,t),$$

$$u^{(1)} \equiv w^{(1)} \equiv 0, \qquad v^{(1)} \equiv v^{(1)}(x,z,t), \tag{3.9}$$

$$u^{(2)} \equiv w^{(2)} \equiv 0, \qquad v^{(2)} \equiv v^{(2)}(x,z,t),$$

$$\left\{\left[1 - \frac{P}{2\mu}\right]\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}\right\}v_2 = \frac{1}{\beta^2}\frac{\partial^2 v_2}{\partial t^2}, \tag{3.10}$$

where $\beta = \sqrt{\frac{u}{\rho}}$, μ represents the Lame's elastic coefficient, P represents the initial stress when there is no body forces and ρ is the density.

4. Boundary conditions

To account the behaviour of the displacement components and stress component at the boundaries, certain conditions are included when applying the basic motion equations to a bounded medium. The problem under consideration has the following boundary conditions:

- At free surface, where $z = -H$, there is no shear stress component, i.e.,

$$\sigma_{32}(x, z = -H, t) = 0, \tag{4.1}$$

- At interface of contact of the half-space and layer, where $z = \varepsilon h(x)$, the stresses and displacement are continuous, i.e.,

$$u_2\left(x, z = \epsilon h(x), t\right) = v_2\left(x, z = \epsilon h(x), t\right), \tag{4.2}$$

and

$$C_5\frac{\partial u_2}{\partial z} - C_1\epsilon h'(x)\frac{\partial u_2}{\partial x} = \mu\left(\frac{\partial v_2}{\partial z} - \epsilon h'(x)\frac{\partial v_2}{\partial x}\right). \tag{4.3}$$

5. Solution of the problem

The following equation is given for the waves that moves along the x-axis and vary harmonically with time t,

$$\{u_2,U_2,v_2\}(z,x,t)=\{u_2^0,U_2^0,v_2^0\}(z,x)\exp(i\omega t), \tag{5.1}$$

where ω represents the angular velocity.
Now, using Eq.(5.1) in Eq.(3.6) and Eq.(3.10), it reduces to:

$$\begin{aligned}&\left\{C_1\frac{\partial^2}{\partial x^2}+C_5\frac{\partial^2}{\partial z^2}+\xi_1^2\right\}\left(u_2^0,U_2^0\right)=0,\\ &\left[1-\frac{P}{2\mu}\right]\frac{\partial^2 v_2^0}{\partial x^2}+\frac{\partial^2 v_2^0}{\partial z^2}+\frac{\omega^2}{\beta_1^2}v_2^0=0,\end{aligned} \tag{5.2}$$

where $\xi_1^2=\alpha_1+i\alpha_2,\quad \alpha_1=\dfrac{F\omega^2}{c_G^2},\quad \alpha_2=\dfrac{R\omega^2}{c_G^2},\quad F=F(\omega)=\dfrac{1+\Omega^2\gamma_{22}C'}{1+(\Omega\gamma_{22})^2}\dfrac{\gamma_{22}}{C'},$

$$R=R(\omega)=\frac{(C'-\gamma_{22})\Omega}{1+(\Omega\gamma_{22})^2}\frac{\gamma_{22}}{C'},\qquad C'=\gamma_{11}\gamma_{22}-\gamma_{12}^2,\qquad \gamma_{kl}=\frac{\rho_{kl}}{\rho}(k,\, l=1,\, 2),$$

$$c_G^2=(\rho_{11}-\rho_{12}^2/\rho_{22})^{-1},\qquad \Omega=\frac{\rho\omega}{b_{11}}.$$

Ω represents the frequency(dimensionless), and C_G denotes the shear wave phase velocity in medium M_1.
Define the Fourier transform $\overline{u}_2^0(z,\eta)$ of $u_2^0(z,\eta)$ as

$$\overline{u}_2^0(z,\eta)=\int_{-\infty}^{\infty}u_2^0(z,x)e^{i\eta x}dx, \tag{5.3}$$

and the Fourier inverse transform $u_2^0(z,\eta)$ of $\overline{u}_2^0(z,\eta)$ is given by

$$u_2^0(z,x)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\overline{u}_2^0(z,\eta)e^{-i\eta x}d\eta. \tag{5.4}$$

By applying Fourier transformations, following by Kumar *et al.* [11], Eq.(5.2) changes to

$$\begin{aligned}&\frac{d^2\overline{u}_2^0}{dz^2}+\Psi_1^2\overline{u}_2^0=0,\\ &\frac{d^2\overline{U}_2^0}{dz^2}+\Psi_1^2\overline{U}_2^0=0,\end{aligned} \tag{5.5}$$

$$\frac{d^2\bar{v}_2^0}{dz^2}+q\frac{d\bar{v}_2^0}{dz}-\Psi_2^2\bar{v}_2^0=0, \tag{5.5 cont.}$$

$$\Psi_1^2=\frac{C_1}{C_5}\left(\frac{\xi_1^2}{C_1}-\eta^2\right),$$

$$\Psi_2^2=\left(\eta^2\left(1-\frac{P}{2\mu}\right)-\frac{\omega^2}{\beta^2}\right). \tag{5.6}$$

The solution of Eq.(5.5),

$$\bar{u}_2^0=A\cos\Psi_1 z+B\sin\Psi_1 z,$$

$$\bar{U}_2^0=\bar{A}\cos\Psi_1 z+\bar{B}\sin\Psi_1 z, \tag{5.7}$$

$$\bar{v}_2^0=D\exp(-\chi z),\ 0\le z<\infty,$$

where

$$\chi=\frac{1}{2}\left(q+\sqrt{q^2+4\Psi_2^2}\right), \tag{5.8}$$

here, $\bar{A},\bar{B},A,B,D$ all depend on η.
Thus, by applying inverse Fourier transformations, proceeding as by Kumar *et al.* [11], results in

$$U_2^0(z,x)=\frac{1}{2\pi}\int_{-\infty}^{\infty}(\bar{B}sin\Psi_1 z+\bar{A}cos\Psi_1 z)e^{-i\eta x}d\eta,$$

$$u_2^0(z,x)=\frac{1}{2\pi}\int_{-\infty}^{\infty}(B\sin\Psi_1 z+A\cos\Psi_1 z)e^{-i\eta x}d\eta, \tag{5.9}$$

$$v_2^0(z,x)=\frac{1}{2\pi}\int_{-\infty}^{\infty}(De^{-\chi z}+\frac{2}{\chi}e^{\chi z}e^{-\chi d})e^{-i\eta x}d\eta.$$

the last term introduced in the integrand of $v_2^0(z,x)$ is due to the source of the disturbance in the lower elastic half-space.

6. Methodology

The following approximations given by Eringens and Samuels [28] is used. As ε is very small and α can be arbitrary.

$$A\cong A_0+A_1\varepsilon,\quad B\cong B_0+B_1\varepsilon,\quad D\cong D_0+D_1\varepsilon, \tag{6.1}$$

$$e^{\pm\alpha\varepsilon h} \cong 1 \pm \alpha\varepsilon h, \quad \sin(\Psi_1 \varepsilon h) \cong \Psi_1 \varepsilon h, \quad \cos(\Psi_1 \varepsilon h) \cong 1. \tag{6.2}$$

Now using the boundary conditions (4.1)-(4.3) in Eq.(5.7) and approximation method from Eqs (6.1)-(6.2), we have

$$\begin{aligned}
&(A_0 + A_1\varepsilon)\sin\Psi_1(-H) + (B_0 + B_1\varepsilon)\cos\Psi_1(-H) = 0, \\
&(A_0 + A_1\varepsilon)\cos(\Psi_1\varepsilon h) + (B_0 + B_1\varepsilon)\sin(\Psi_1\varepsilon h) = (D_0 + D_1\varepsilon)e^{-\chi\varepsilon h} + \frac{2}{\chi}e^{-\chi d}e^{\chi\varepsilon h}, \\
&C_5\left[(B_0 + B_1\varepsilon)\Psi_1\cos\Psi_1\varepsilon h - (A_0 + A_1\varepsilon)\Psi_1\varepsilon h\right) + i\eta C_1\varepsilon h'(x)(A_0 + A_1\varepsilon) + (B_0 + B_1\varepsilon)\Psi_1\varepsilon h = \\
&= \mu\left[-\chi(D_0 + D_1\varepsilon)e^{-\chi\varepsilon h} + 2e^{-\chi d}e^{\chi\varepsilon h} + i\eta\varepsilon h'(x)\left(-\chi(D_0 + D_1\varepsilon)e^{-\chi\varepsilon h} + 2e^{-\chi d}e^{\chi\varepsilon h}\right)\right].
\end{aligned} \tag{6.3}$$

Equating the term which is not containing ε and coefficients of ε from the Eqs (6.3) yields,

$$\begin{aligned}
&A_1 \sin\Psi_1 H + B_1 \cos\Psi_1 H = 0, \\
&A_0 \sin\Psi_1 H + B_0 \cos\Psi_1 H = 0, \\
&A_0 - D_0 = \frac{2}{\chi}e^{-\chi d}, \\
&D_1 - A_1 = W_1(k), \\
&\Psi_1 C_5 B_0 + \mu\chi D_0 = 2\mu e^{\chi d}, \\
&\Psi_1 C_5 B_1 + \mu\chi D_1 = W_2(k).
\end{aligned} \tag{6.4}$$

Solving the above system of Eqs (6.4) yields,

$$\begin{aligned}
&A_0 = \frac{4\mu e^{-\chi d}}{S(k)}, && A_1 = \frac{[W_2(k) - \mu\chi W_1(k)]}{S(k)}, \\
&B_0 = -\frac{4\mu e^{-\chi d}\tan\Psi_1 H}{S(k)}, && B_1 = -\frac{[W_2(k) - \mu\chi W_1(k)]\tan\Psi_1 H}{S(k)}, \\
&D_0 = \frac{2e^{-\chi d}[\mu\chi + C_5\Psi_1\tan\Psi_1 H]}{\chi S(k)}, && D_1 = \frac{W_2(k) - \Psi_1 C_5 W_1(k)\tan\Psi_1 H}{S(k)},
\end{aligned} \tag{6.5}$$

where

$$S(k)=\mu\chi-C_5\Psi_1\tan\Psi_1 H,$$

$$W_1(k)=B_0\Psi_1 h+D_0\chi h-2he^{-\chi d}, \tag{6.6}$$

$$W_2(k)=\mu(2\chi he^{\chi d}-D_0\chi^2 h)+C_5\Psi_1^2 A_0 h+i\eta h'(x)\left[\mu(D_0+\frac{2}{\chi}e^{-\chi d})-C_1A_0\right].$$

Thus, the displacement vector in isotropic layer using Eq.(6.5) becomes

$$u_2^0(z,x)=\frac{1}{2\pi}\times\int_{-\infty}^{\infty}\frac{4\mu e^{-\chi d}}{S(k)}\left[1+\frac{\varepsilon\left(W_2(k)-\mu\chi W_1(k)\right)e^{\chi d}}{4\mu}\right](\cos\Psi_1 z-\tan\Psi_1 H\sin\Psi_1 z)e^{-ikx}dk\,. \tag{6.7}$$

Now, using the Fourier and inverse Fourier transformations of $h(x)$, results in

$$\overline{h}(\lambda)=\frac{2s}{\lambda^2}\sin(\lambda s), \tag{6.8}$$

$$W_2-\mu\chi W_1=\frac{2s\mu}{\pi}\int_{-\infty}^{\infty}\left[\phi(k-\lambda)+\phi(k+\lambda)\right]\frac{1}{\lambda^2}\sin\lambda s d\lambda, \tag{6.9}$$

where

$$\phi(k-\lambda)=\left[(A_2+A_3)\frac{e^{\chi d}}{S(k)}\right],$$

$$A_2=A_0\Psi_1^2C_5-\mu\chi\left(B_0\Psi_1-4e^{-\chi d}\right),$$

$$A_3=-\lambda k\left(\mu D_0\chi-2e^{\chi d}-C_1A_0\right). \tag{6.10}$$

Using Willis's asymptotic formula [29] and ignoring terms with $2/\mathrm{s}$ and higher coefficients powers of $2/\mathrm{s}$ when s is large, yields

$$\int_{-\infty}^{\infty}\frac{1}{\lambda^2}\sin\frac{\lambda s}{2}\left[\phi(k-\lambda)+\phi(k+\lambda)\right]d\lambda\equiv\frac{\pi}{2}\,2\phi(k)=\pi\phi(k)\,. \tag{6.11}$$

Using Eq.(6.11) in (6.9), we obtain

$$W_2-\mu\chi W_1=2s\mu\phi(k)=2\mu\frac{H_1}{\varepsilon}\phi(k). \tag{6.12}$$

Thus, the component of displacement vector in considered layer is given as

$$u_2^0(z,x)=\frac{1}{2\pi}\times\int_{-\infty}^{\infty}\frac{4\mu e^{-\chi d}}{S(k)}\left[1+\frac{\varepsilon\left(W_2(k)-\mu\chi W_1(k)\right)e^{\chi d}}{4\mu}\right]\left(\cos\Psi_1 z-\tan\Psi_1 H\sin\Psi_1 z\right)e^{-ikx}dk\,. \tag{6.13}$$

This integral's value depends simply on the 'poles of the integrand', which located at the roots of the equation.

$$\left[1-H_1\phi(k)e^{\chi d}\right]S(k)=0. \tag{6.14}$$

If the wave velocity along the surface is c, then by substituting $\omega = ck$ (where ω represents circular frequency and wave number is denoted by k) in Eq.(6.14), yields to

$$\Psi_1 = P_1 k,\ q = Qk,\ \chi = P_2 k,$$

where
$$P_1 = \sqrt{\left(\frac{1}{C_5}\left(\frac{c^2}{c_G^2}F(\omega)-C_1\right)+i\frac{1}{C_5}\frac{c^2}{c_G^2}R(\omega)\right)}.$$

and solving Eq.(6.14), provides

$$\tan P_1 kH = \frac{\mu P_2 - 4H_1\mu k\left(C_5P_1^2+\mu P_2^2\right)}{P_1\left[C_5\left(1-H_1P_2\right)+H_1\mu P_2\right]}. \tag{6.15}$$

As the value of P_1^2 is complex, it can be represented as

$$P_1 = k_1 + ik_2, \tag{6.16}$$

where
$$k_{1,2} = \left\{\frac{1}{2}\left(\left[\left(\frac{1}{C_5}\left(\frac{c^2}{c_G^2}F(\omega)-C_1\right)^2+\left(\frac{1}{C_5}\frac{c^2}{c_G^2}R(\omega)\right)^2\right]^{\frac{1}{2}}\pm\frac{1}{C_5}\frac{c^2}{c_G^2}F(\omega)-C_1\right)\right\}^{1/2} \tag{6.17}$$

Thus, by Eq.(6.16), the dispersion Eq.(6.17) for shear waves, changes to

$$\tan\left(k_1+ik_2\right)kH = A_r + iA_i, \tag{6.18}$$

where
$$A_r = \frac{k_1\left[\mu P_2(1-P_2)-H_1k\left\{C_5\left(k_1^2-k_2^2\right)-2k_2^2\right\}\right]}{\left(k_1^2+k_2^2\right)\left[C_5\left(1+H_1P_2k\right)+H_1\mu P_2k\right]},$$

$$A_i = \frac{-k_2\left[\mu P_2(1-P_2)-H_1k\{C_5(k_1^2+k_2)+2k_1^2\}\right]}{(k_1^2+k_2^2)\left[C_5(1+H_1P_2k)+H_1\mu P_2k\right]}. \tag{6.19}$$

As k_2 is small, so we have

$$\tan\left(k_1+ik_2\right)kH \approx \frac{\tan k_1kH + ik_2kh}{1-ik_2kh\tan k_1kH} = A_r + iA_i. \tag{6.20}$$

So, by making use of equations (6.19), (6.20) and separating the imaginary and real components of Eq.(6.18), yields to two real solutions

$$\tan k_1 kH = \frac{A_r}{1 - A_i k_2 kH}, \qquad k_2 kH\left(A_r k_2 kH \tan k_1 kH + 1\right) = A_i . \tag{6.21}$$

Since, the real component of Eq.(6.15) provides the shear wave dispersion equation for shear waves which is given by

$$\tan k_1 kH = \frac{A_r}{1 - A_i k_2 kH} . \tag{6.22}$$

7. Particular case

By substituting the value of $H_1 = 0$ in Eq.(6.14), the classical dispersion relation is derived for shear waves in a FSPL which is transversely isotropic lying above a non-homogeneous substrate which is consistent with the results previously derived by Konczak [10].

$$\tan(k_1 + ik_2)H = \frac{\mu\chi}{C_5 \Psi_1} = \frac{\mu}{2C_5} \frac{\left(q + \sqrt{q^2 + 4\Psi_2^2}\right)}{\Psi_1} . \tag{7.1}$$

8. Numerical results and discussions

This work aims to examine the impact of irregularity in the FSPL which is transversely isotropic and to conduct a numerical comparison of the different values of wave number and the phase velocity.

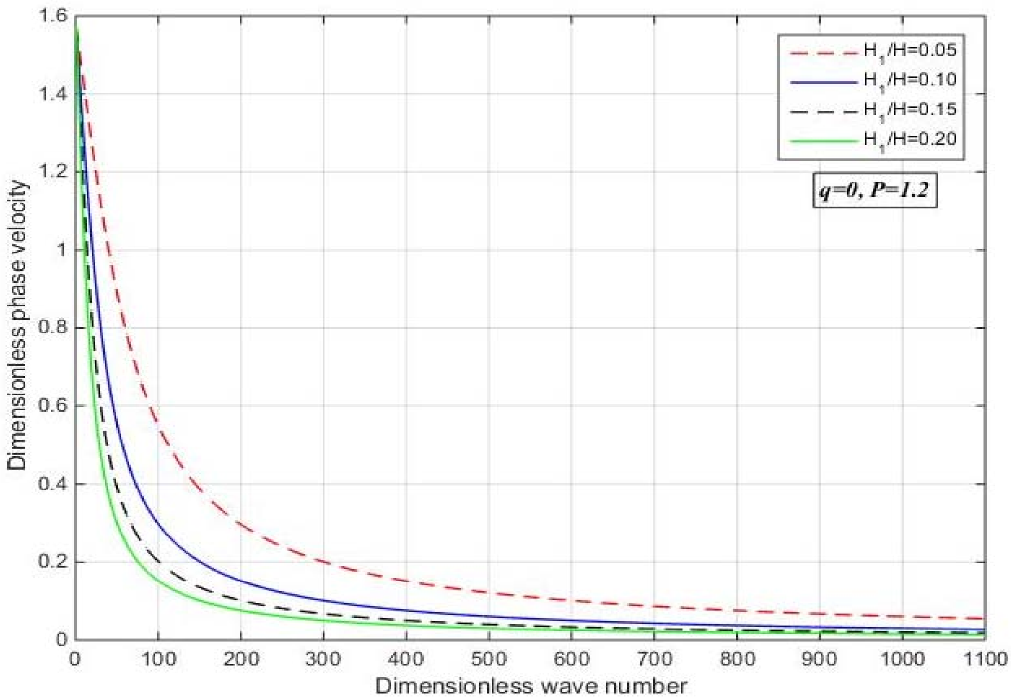

Fig.2. Dispersion curves $q=0$ and for the ratio $H_1 / H = 0.05,\ 0.10,\ 0.15,\ 0.20$ with initial stress $P = 1.2$.

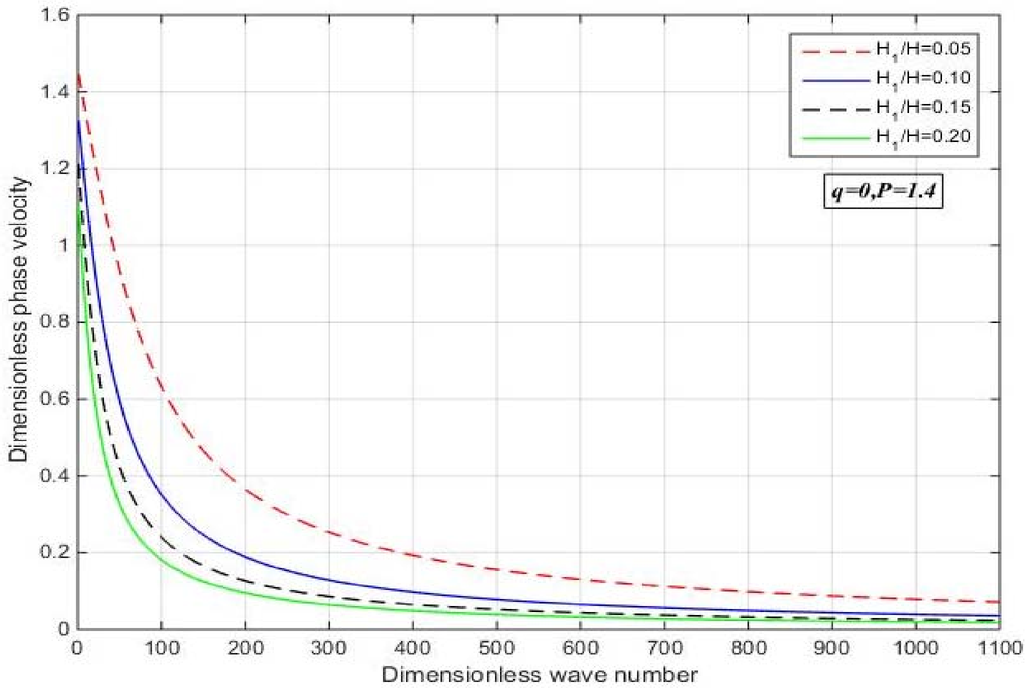

Fig.3. Dispersion curves for $q=0$ and the ratio $H_1 / H=0.05, 0.10, 0.15, 0.20$ with initial stress $P = 1.4$.

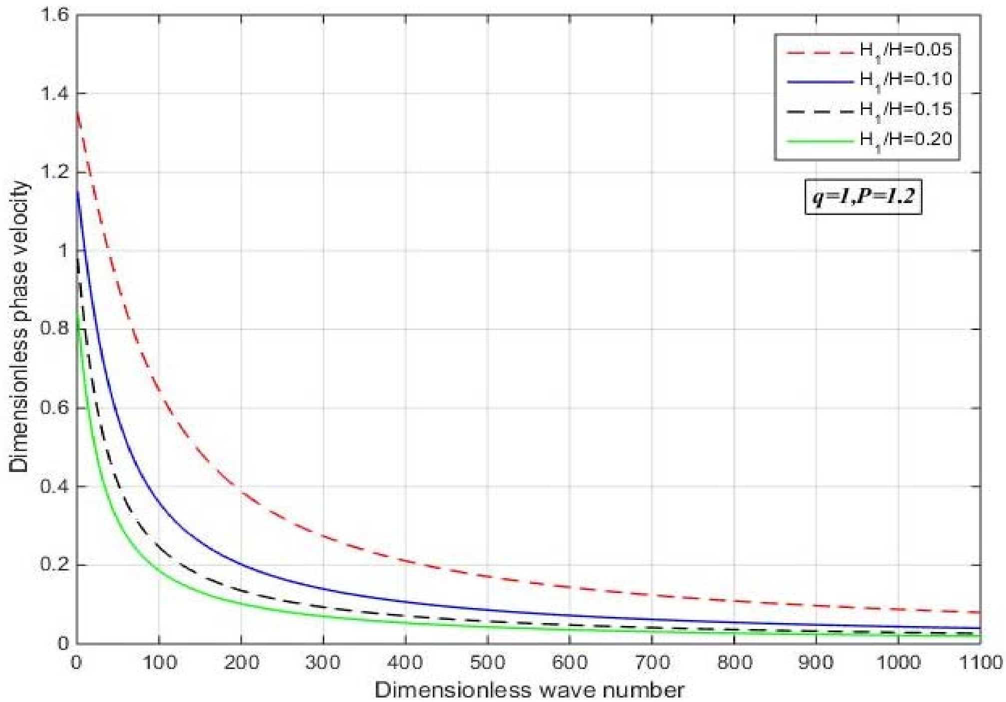

Fig.4. Dispersion curves for $q=1$ and the ratio $H_1 / H=0.05, 0.10, 0.15, 0.20$ with initial stress $P = 1.2$.

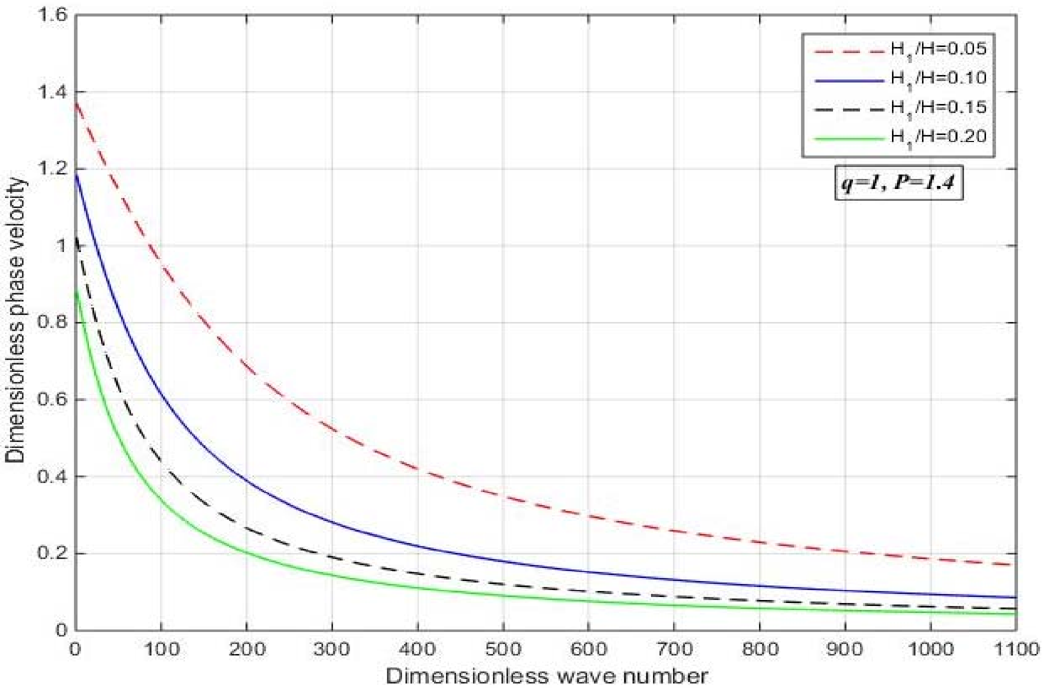

Fig.5. Dispersion curves for $q = 1$ and the ratio $H_1 / H = 0.05, 0.10, 0.15, 0.20$ with initial stress $P = 1.4$.

With the elastic constant values provided by Gubbins [30] and Kończak [10], MATLAB is used to generate the graphs for various inhomogeneity parameter q values demonstrating how phase velocity (dimensionless) drops with respect to raise in the value of wave number.

9. Conclusions

In this work, the shear wave behaviour in a FSPL exhibits transverse isotropy lying over a non-homogeneous isotropic half space characterized with triangular irregularity at the layer-half space interface is investigated. Using Eringen and Samuels [28] perturbation method, the displacement vector components within the layer were derived, leading to the formulation of the dispersion relation for this complex medium. The dispersion equations, considering both the presence and absence of the triangular irregularity, were then analysed graphically using MATLAB simulations. The influence of material's inhomogeneity and initial stress on the dispersion curve is evaluated and shown graphically, focusing on dimensionless wave number and dimensionless phase velocity for both cases of homogeneous half-spaces and non-homogeneous half-spaces. Based on the above discussions, it can be concluded that:

- the phase velocity of the shear waves in FSPL exhibits transverse isotropy falls as the wave number raises when the FSPL is situated over an elastic half-space characterized with triangular shaped irregularity.
- phase velocity depends on depth of irregularity, wave number, and width of the layer.
- figures 2-5 demonstrate that the phase velocity drops as the ratio of the layer's height to the depth of the irregularity raises.

Overall, the study demonstrates that a triangular irregularity has a substantial impact on shear wave behaviour in a FSPL exhibits transverse isotropy. In addition to the irregularity's shape, factors such as wave number, the layer's height to irregularity's depth ratio, and material properties play a crucial role in determining phase velocity. These findings provide valuable insights into wave behaviour in complex layered geological structures, with potential applications in fields like geophysics, civil engineering, and material sciences.

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Nomenclature

c	– common wave velocity
C_G	– velocity of shear waves
C_m	– material constants used in porous layer for $m = 1, 2, ...,8$
d	– distance of source creating irregularity
FSPL	– fluid-saturated porous layer
H_1	– irregularity's maximum depth
k	– wave number
M_1	– initially stressed transversely isotropic FSPL
M_2	– non-homogeneous elastic half-space
P	– initial stress
$2s$	– irregularity's maximum width
U_j	– displacement vector component for fluid in medium M_1
u_j	– displacement vector component for solid in medium M_1
δ_{ij}	– Kronecker delta
λ,μ	– Lame's constants
ρ	– density of medium M_1
$\rho_{11},\rho_{12},\rho_{22}$	– mass coefficients
σ_{ij}	– stress tensor components per unit area in medium M_1
ω	– angular velocity

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