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INFLUENCE OF COUPLE STRESS AND MAGNETIC FLUID ON THE PERFORMANCE OF STEP SLIDER BEARING

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This theoretical investigation delves into the performance of step slider bearings, considering the influence of couple stress and magnetic fluid as the lubricating medium. Employing a non-linear model for magnetic field intensity aims to optimize bearing performance. Modifying the Reynolds equation for step slider bearings incorporates theories from Neuringer and Rosensweig for magnetic fluid flow dynamics and the Stokes micro-continuum model to account for couple stress effects. Solving the Reynolds equation with appropriate boundary conditions determines key parameters such as pressure, load capacity, center of pressure, and frictional force. Analytical formulations for load-bearing capacity and pressure distribution are produced, and MATLAB is used to analyse the integrals that appear in these two expressions. The results of the investigation are then explained via graphical representations. The findings demonstrate a notable improvement in load-carrying capacity ferrofluid based system as compared to non-magnetic fluid. Additionally, increasing coupling stress and magnetization parameters results in a drop in the frictional coefficient and an increase in both load capacity and frictional force. The use of magnetic fluid lubrication significantly increases the beneficial effects of couple stress fluids. It is discovered that the impact of magnetic fluid lubrication increases the load-carrying capacity by at least *22.30 %* in the presence of couple stress. The synergistic effect of couple stress and magnetic fluid lubrication demonstrates the potential for optimizing the bearing performance beyond conventional methods.

Key words: step slider bearing, load capacity, magnetic fluid, couple stress, magnitude of magnetic field.

1. Introduction

In the realm of tribology, step slider bearings play a pivotal role in facilitating the smooth operation and longevity of machinery and mechanical systems. These bearings, known for their ability to support rotating or sliding shafts, offer even load distribution, reduced friction, and tolerance to misalignment, thereby enhancing the efficiency of various mechanical setups including turbines, compressors, pumps, and engines. Naduvinamani and Angadi [1] investigated how surface roughness affects the dynamic behavior of a Rayleigh step-bearing under squeezing action. Their findings showed that using a non-Newtonian couple stress fluid improved the steady load-carrying capacity, dynamic stiffness, and damping coefficients. It has been also noted that, in the case of Newtonian fluids, negatively skewed surface roughness resulted in a reduced volume flow rate. Kashinath and Upadhya [2] examined an inclined porous slider bearing lubricated with couple stress fluid, accounting for both slip and squeeze velocities. It has been demonstrated that the inclusion of these velocities enhanced the load-carrying capacity. In another study, Kashinath and Upadhya [3] analyzed a secant-shaped porous slider bearing under similar lubrication conditions. They reported that an increase in the slip parameter reduced the load capacity, friction, and coefficient of friction. Conversely, while the load capacity and friction

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decreased with an increase in the permeability parameter, the coefficient of friction showed an increasing trend. Naduvinamani and Siddangouda [4] investigated the influence of surface roughness on hydrodynamic lubrication in porous step-slider bearings. Their findings revealed that negatively skewed surface roughness positively altered the characteristics of the porous step bearing, while the presence of positively skewed surface roughness detrimentally affected bearing performance, highlighting the significance of surface texture in tribological behavior. Rahmani *et al.* [5] conducted an analytical analysis of infinite-width Rayleigh step slider bearings using the Reynolds hydrodynamic lubrication equation, accounting for hydrostatic pressure differences between bearing ends. Their study revealed that optimal bearing parameters were closely tied to pressure boundary variations, and highlighted a discrepancy between the optimal points for load capacity and those for friction coefficient/lubricant flow rate, even in the absence of pressure differences between bearing ends, emphasizing the multifaceted nature of optimizing bearing performance.

Furthermore, the magnitude of the magnetic field emerges as a fundamental parameter, indispensable in various scientific and engineering domains. From physics research to electrical engineering and magnetism-based technologies like MRI machines, understanding the strength of magnetic fields is crucial for predicting their effects on materials and charged particles, as well as for optimizing technological applications. Maiti [6] explored the lubricating properties of micropolar fluids through the analysis of composite and step slider bearings employing micropolar fluid as a lubricant. It was revealed greater load-supporting capacity in micropolar fluids compared to Newtonian fluids, with the load capacity of the composite slider bearing surpassing that of the ordinary slider bearing, underscoring the enhanced performance of micropolar lubricants in bearing applications. Dass [7] examined slider bearings with a comprehensive approach, incorporating the lubricant as an isothermal, incompressible, electrically conducting couple stress fluid under a uniform magnetic field. Results indicated that maximum load capacity and inlet-outlet film ratio were contingent upon both couple stress and magnetic parameters, as well as bearing geometry, highlighting the interplay of these factors in determining bearing performance, particularly when compared to Newtonian lubricant scenarios. Barik *et al.* [8] investigated the impact of a sinusoidal magnetic field on a rough porous hyperbolic slider-bearing system. Their findings revealed that the sinusoidal magnetic field augmented the load-bearing capacity by a factor of $\frac{\mu^*}{\pi}$, resulting in a notable *38.3 %* increase in the load-carrying capacity, providing valuable insights into the beneficial effects of magnetic field modulation on bearing performance.

In the study of mechanical behavior, the concept of couple stress emerges as a critical consideration, especially in structures with small-scale features or complex geometries. Couple stress encapsulates the internal stresses within a material arising from microscopic moments or couples, influencing its mechanical properties and deformation behavior. This framework provides a nuanced understanding of material response, particularly relevant in the analysis of microstructures, biological tissues, and small-scale mechanical systems. Vinutha *et al.* [9] studied the combined effect of magnetohydrodynamic (MHD) and slip velocity on the squeeze film lubrication of long cylinders and infinite plates with couple stress fluids. It has been found that the pressure, load-carrying capacity, and squeeze film time increased with the combined effect of MHD, couple stress fluids, and slip velocity. Devani *et al.* [10] explored the novel behavior of curved circular and porous flat plates under the influence of MHD and slip velocity. They concluded that fluids with couple stresses performed better than Newtonian fluids. Patil *et al.* [11] examined the effect of MHD on curved circular and porous flat plates lubricated with couple stress. They revealed that the applied magnetic field and couple stress lubricants increased the squeeze film pressure, load-carrying capacity, and squeezing time. Naduvinamani *et al.* [12] investigated the synergistic impact of couple stresses and surface roughness on the hydrodynamic lubrication of slider bearings with diverse film shapes. It was revealed that negatively skewed surface roughness consistently enhanced load-carrying capacity, frictional force, and temperature rise across all lubricant film shapes while reducing the coefficient of friction. Conversely, positively skewed surface roughness exhibited the opposite trend, indicating the nuanced effects of surface texture on bearing performance. Lin *et al.* [13] investigated the impact of non-Newtonian couple stresses on the dynamic behavior of Rayleigh step slider bearings under fluid-film lubrication. It was indicated that non-Newtonian couple stresses contributed to enhanced bearing characteristics, including increased steady load-carrying capacity, dynamic stiffness coefficient, and dynamic damping coefficient, along with a reduced steady volume flow rate requirement, showcasing the advantageous effects of non-Newtonian behavior on bearing performance compared to traditional Newtonian lubricants. Kasinath and Hanumagowda [14] investigated the impact of a transverse magnetic field on a wide composite slider bearing utilizing a couple-stress fluid. The analysis has been revealed that an increase in the strength of the magnetic field correlated with elevated fluid film pressure, load carrying capacity, frictional force, and coefficient of friction, shedding light on the significant influence of magnetic field strength on bearing tribological properties. Naduvinamani and Angadi [15] explored the effects of surface roughness on Rayleigh step bearings lubricated with couple stress fluid, observing enhancements in load carrying capacity, dynamic stiffness coefficient, and dynamic damping coefficient when compared to smooth surfaces, elucidating the beneficial impact of surface roughness on bearing performance. In their study, Naduvinamani and Ganachari [16] analyzed the efficacy of double-layered porous Rayleigh step slider bearings lubricated with couple stress fluid, juxtaposed with single-layered counterparts. It was observed that the incorporation of a double-layer porous facing led to heightened load carrying capacity and frictional force, while concurrently reducing the coefficient of friction, indicative of enhanced lubrication system efficiency. Anthony and Elamparithi [17] studied the rough-porous Rayleigh step slider bearing lubricated with couple stress fluid, emphasizing MHD effects. The findings observed that increased workload and frictional force compared to smooth plates, while the coefficient of friction decreased, highlighting the intricate interplay of surface roughness and fluid properties in tribological systems.

Alongside the advancements in bearing technology, the integration of ferrofluids, has introduced novel possibilities for sealing, lubrication, cooling, and vibration damping in mechanical systems. Ferrofluids, composed of stable colloidal suspensions containing ferromagnetic particles dispersed within a liquid medium, exhibit unique responses to magnetic fields, enabling their application in diverse fields such as biomedical engineering and industrial processes. Gomathi and Poulomi [18] investigated the steady electro-magnetohydrodynamic (EMHD) flow of Casson-Williamson nanofluid over a porous, exponentially-shaped vertical cone. The findings had broad industrial, environmental, and biomedical implications, including enhanced energy efficiency and targeted drug delivery. Amudhini and Poulomi [19] investigated the influence of Soret-Dufour effects on the MHD unsteady flow of a tetra-hybrid nanofluid (composed of Al_2O_3, Cu, SiO_2 and TiO_2 with water as the base fluid) within a non-Darcy porous stretching cylinder. These findings highlighted the potential of tetra-hybrid nanofluid in enhancing performance across various industrial and environmental processes. Sangeetha and Poulomi [20] examined the stagnation point flow of MHD nanofluid with a focus on bioconvection in a non-Darcy porous environment. It was revealed that the velocity profiles decreased with increasing porous parameter and inertia coefficient, while the density profile of microorganisms weakened with increasing bioconvection Lewis number, Peclet number, and microorganism difference parameter. Shah and Bhat [21] examined the impact of slip velocity and material parameter of a magnetic fluid lubricant on a parallel plate porous slider bearing. Results indicated that increasing slip parameter decreased slider frictional force and coefficient of friction, while maintaining load capacity and center of pressure position. Conversely, an increase in material parameter decreased frictional force and coefficient of friction, shifting the center of pressure towards the bearing inlet without altering load capacity. Patel and Deheri [22] examined the performance of a transversely rough porous parallel plate slider bearing under slip velocity conditions, employing magnetic fluid as the lubricant. Their findings revealed that while a parallel plate slider bearing failed to support a load with conventional lubricant, adoption of a magnetic fluid lubricant facilitated load support with a constant frictional force on the slider, underscoring the superior performance of magnetic fluid lubrication in sustaining bearing functionality under varying operational conditions. Shukla and Deheri [23] evaluated the performance of a magnetic-fluid-based porous rough step bearing, taking into account slip velocity effects. The analysis revealed that transverse surface roughness negatively impacted bearing performance; however, the introduction of magnetization exhibited a mitigating effect. Notably, minimizing the slip parameter, particularly in the presence of negatively skewed roughness, contributed to enhancing the bearing system's performance to some extent. Shah and Patel [24] investigated the influence of diverse porous structures on the performance of step bearings lubricated with magnetic fluid. The study indicated that the permeability and width of the porous matrix exhibited minimal impact on the dimensionless load-carrying capacity, highlighting the robustness of magnetic fluid lubrication in mitigating the effects of porous structure variations on bearing performance. Deheri and Patel [25] scrutinized the performance of a magnetic fluid-based squeeze film within a rough porous parallel plate slider bearing. It has been revealed that the bearing exhibited load support capabilities even in the absence of flow, underscoring the resilience and effectiveness of magnetic fluid-based lubrication systems in maintaining bearing functionality under varying operational conditions. Patel and Deheri [26] explored the performance characteristics of an infinitely long rough porous slider bearing lubricated with ferrofluid and subjected to thin film lubrication at the nanoscale. The results observed that the utilization of nanoscale thin film lubrication yielded overall enhanced performance, even under lower magnetic intensity, with the additional contribution of couple stress further improving the system's efficiency, highlighting the synergistic effects of thin film lubrication and couple stress in optimizing bearing behaviour. In their study, Patel and Deheri [27] scrutinized the performance of a parallel plate rough slider bearing lubricated with magnetic fluid. Their analysis suggested the utilization of the Shliomis model for higher loads to ensure prolonged bearing lifespan, while the Neuringer-Rosensweig model was recommended for lower to moderate loads, offering strategic insights into model selection for optimizing bearing longevity across varying load conditions. Andharia and Pandya [28] investigated the impact of longitudinal surface roughness on Rayleigh step bearing performance, finding that bearing load capacity rises with increasing standard deviation while decreasing with aspect ratio, variance, and height ratio, offering insights into the nuanced influence of surface roughness parameters on bearing behavior. Patel *et al.* [29] conducted an inquiry into the operational efficacy of porous step bearings lubricated with ferrofluid. The investigation revealed a notable increase in load capacity with the adoption of appropriate step size, underscoring the critical role of geometric parameters in optimizing the performance of ferrofluid-based lubrication systems. Patel *et al.* [30] dealt with porous squeeze film performance in curved annular plates considering slip velocity, Kozeny-Carman's porous structure, and Rosensweig's viscosity in Shliomis model-based magnetic fluid lubrication and revealed that the Shliomis model goes ahead of the other magnetic fluid flow models in overall improvement of bearing performance characteristics. Patel *et al.* [31] investigated the performance of a magnetic fluid-based squeeze film between infinitely long porous rough parallel plates with a porous matrix of non-uniform thickness. This study revealed that besides providing an additional degree of freedom from the design point of view, the thickness ratio parameter may play an important role in the case of negatively skewed roughness especially, when negative variance is involved. Patel *et al.* [32] discussed the combined effect of longitudinal surface roughness and deformation on the behavior of a ferrofluid-based squeeze film in canonical plates. It was found that the effect of surface roughness and deformation was relatively adverse. Deheri *et al.* [33] analyzed the performance characteristics of a Shliomis model based ferrofluid lubrication of a rough porous convex pad slider bearing. The investigation revealed that the adverse effect of surface roughness could be reduced to a certain extent by the positive effect of Shliomis model based ferrofluid lubrication.

In the present investigation, a comprehensive analysis has been developed to understand the interplay between couple stress and ferrofluid on the step slider bearing. Till today in most of the studies the effect of couple stress and magnetic fluid have been treated separately. However, the present investigation goes into combined influence. Through a synthesis of existing literature and empirical studies, our aim is to elucidate the underlying mechanisms governing the behavior of these intricate systems, offering insights into the design and optimization of efficient mechanical setups of step slider bearings.

2. Analysis

Various theories have been developed to elucidate the anomalous behavior of fluids with substructure, like polymeric fluids (Ariman *et al.* [34-35]). The basic micro continuum theory extends the classical theory to accommodate polar phenomena, including the influence of couple stress, body couples, and asymmetric stress tensors, as proposed by Stokes [36-38].

The analytical method is used to solve the mathematical equations for pressure, Load carrying capacity, the center of pressure, and frictional force to observe the performance of step slider bearing.
The fundamental equations governing the motion of fluids with couple stress, expressed in Cartesian tensor notation, are delineated as follows:

$$\dot{\rho} + \rho v_{k,k} = 0 \tag{2.1}$$

$$\rho\dot{v}_i = \rho b_i + T_{ji,j} \ , \tag{2.2}$$

$$\rho g_i + e_{ijk}T_{jk} + M_{ji,j} = 0 \, . \tag{2.3}$$

The superscript dot denotes a material time derivative, and a subscript followed by a comma indicates partial differentiation.
The constitutive equations governing the stress tensor and the couple stress tensor are provided as follows:

$$T_{ij} = \left(-p + \lambda v_{k,k}\right)\delta_{ij} + \mu\left(v_{i,j} + v_{j,i}\right) - \frac{\rho}{2}e_{ijk}g_k + \eta\nabla^2\left(v_{i,j} - v_{j,i}\right), \tag{2.4}$$

$$M_{ij} = 2\eta e_{j\alpha\beta}v_{\beta,\alpha i} + 2\eta' e_{i\alpha\beta}v_{\beta,\alpha j} \, . \tag{2.5}$$

The substitution of T_{ij} and M_{ij} from Eqs (2.4) and (2.5) into Eq.(2.3) is found to satisfy Eq.(2.3) identically. Equation (2.2) yields the field equation for velocity, represented as:

$$\rho\dot{v}_i = -p_{,i} + \left(\lambda + \mu + \eta\nabla^2\right)v_{k,ki} + \left(\mu - \eta\nabla^2\right)v_{i,jj} + \rho b_i + \frac{1}{2}e_{ijk}\left(\rho g_k\right)_{,j} \, . \tag{2.6}$$

In the study of Ramanaiah and Sarkar [39], the couple stress theory of fluids was applied to analyze squeeze films and thrust bearings. This study focuses on evaluating the performance of slider bearings lubricated by fluids exhibiting couple stress properties, extending the investigation into the tribological behavior of such systems.

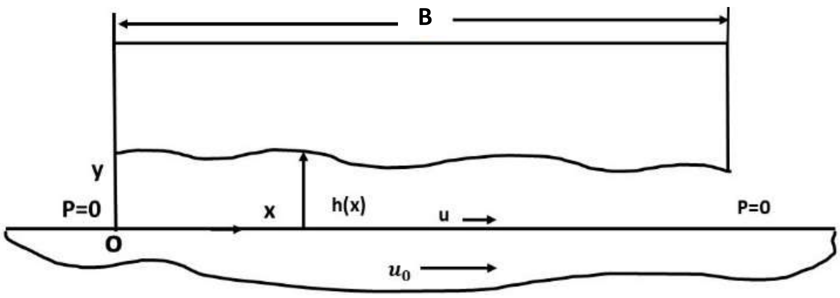

Fig.1. The geometry of the slider bearing.

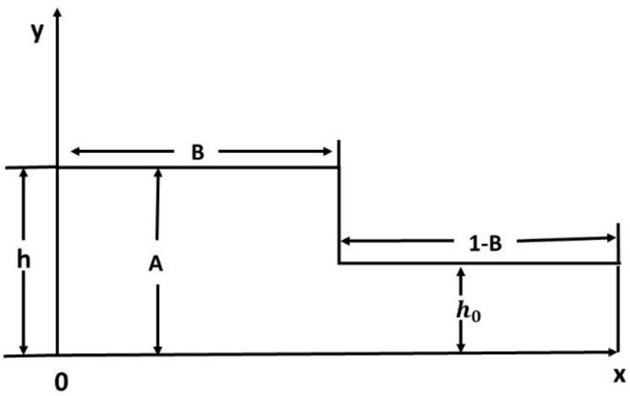

Fig.2. Configuration of step slider bearing.

The geometry of the two-dimensional slider bearings depicted in Fig.1 comprises two closely spaced rigid surfaces undergoing relative motion. The lubricant filling the gap between the surfaces is modeled as an incompressible fluid with couple stress, with the absence of body forces and body couples. Assuming a velocity vector $(u,v,0)$ and the applicability of fluid film lubrication for thin (Cameron [40]), Eqs (2.1) and (2.6) are simplified to:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{2.7}$$

$$\mu\frac{\partial^2 u}{\partial y^2}-\eta\frac{\partial^4 u}{\partial y^4}=\frac{dp}{dx} \tag{2.8}$$

In the current context, the pertinent components of the stress tensor and the couple stress tensor are derived as:

$$T_{21}=\mu\frac{\partial u}{\partial y}-\eta\frac{\partial^3 u}{\partial y^3}, \tag{2.9}$$

$$M_{23}=-2\eta\frac{\partial^2 u}{\partial y^2}, \tag{2.10}$$

which are derived from Eqs (2.4) and (2.5).

The boundary conditions for Eqs (2.7) and (2.8) consist of the standard no-slip conditions:

$$u=u_0 \text{ at } y=0; \quad u=0 \text{ at } y=h. \tag{2.11}$$

Additionally, the no-couple stress conditions, derived from Eq.(2.10), are imposed as:

$$\frac{\partial^2 u}{\partial y^2}=0;\text{ at } y=0 \quad \text{and} \quad y=h. \tag{2.12}$$

By employing Eqs (2.11) and (2.12) to solve Eq.(2.8), the solution for u is determined as:

$$u=\frac{1}{2\mu}\frac{dp}{dx}\left(y^2-hy+2l^2\left[1-\frac{\cosh\left\{\frac{2y-h}{2l}\right\}}{\cosh\left(\frac{h}{2l}\right)}\right]\right)+u_0\left(1-\frac{y}{h}\right) \tag{2.13}$$

where, $l=(\eta/\mu)^{1/2}$.

The volume flow rate q is determined as:

$$q=\int_0^h u\,dy=\int_0^h\frac{1}{2\mu}\frac{dp}{dx}\left(y^2-hy+2l^2\left[1-\frac{\cosh\left\{\frac{2y-h}{2l}\right\}}{\cosh\left(\frac{h}{2l}\right)}\right]\right)+u_0\left(1-\frac{y}{h}\right)dy= \tag{2.14}$$

$$=\frac{1}{2\mu}\frac{dp}{dx}\left[\int_0^h (y^2-hy)dy+\int_0^h 2l^2\left[1-\frac{\cosh\left\{\frac{2y-h}{2l}\right\}}{\cosh\left(\frac{h}{2l}\right)}\right]dy\right]+u_0\int_0^h\left(1-\frac{y}{h}\right)dy= \tag{2.14 cont.}$$

$$=\frac{u_0 h}{2}-\frac{1}{12\mu}\frac{dp}{dx}\frac{1}{s(l,h)}$$

where $s(l,h)=\left[h^3-12l^2h+24l^3\tanh\left(\frac{h}{2l}\right)\right]^{-1}$.

Integrating Eqs (2.7) across the fluid film with Eqs (2.14) and boundary conditions

$$v=0 \text{ at } y=0 \quad \text{and} \quad y=h, \tag{2.15}$$

one can obtain the Reynolds equation for the bearings

$$\frac{d}{dx}\left\{\frac{1}{s(l,h)}\frac{dp}{dx}\right\}=6\mu u_0\frac{dh}{dx}. \tag{2.16}$$

Combining Eqs (2.9) and (2.13) provides

$$T_{21}=\left(y-\frac{h}{2}\right)\frac{dp}{dx}-\frac{\mu u_0}{h}. \tag{2.17}$$

The friction force on the bearing surface $(y=0)$ is expressed as:

$$f=-\int_0^b T_{21}dx=\int_0^b\left(\frac{h}{2}\frac{dp}{dx}+\frac{\mu u_0}{h}\right)dx. \tag{2.18}$$

Step bearings have garnered attention in tribology due to their remarkable load capacity compared to other feasible bearing geometries. Prior research has established the superior load-bearing capabilities of step bearings, attributing their performance to an optimized geometric configuration. In this study, it has been investigated step bearings employing the theoretical framework proposed by Ramanaiah and Sarkar [39]. The non-dimensional film thickness of step bearing as displayed in Fig.2 is defined as:

$$H(X)=H=\frac{h}{h_0}=\begin{cases}A>1; & \text{when } 0<X<B,\\ 1; & \text{when } B<X<1.\end{cases}$$

A simplistic steady flow model for ferrofluid subjected to gradually alternating external magnetic fields was proposed by (Neuringer and Rosensweig [41]). The model comprises the following equations (Prajapati [42], Bhat [43]):

$$\rho(\bar{q}\nabla)\bar{q}=-\nabla p+\eta\nabla^2\bar{q}+\mu_0(\bar{M}\nabla)\bar{H}, \tag{2.19}$$

$$\nabla\bar{q}=0, \tag{2.20}$$

$$\nabla \times \bar{H} = 0, \tag{2.21}$$

$$\bar{M} = \bar{\mu}\bar{H}, \tag{2.22}$$

$$\nabla\left(\bar{H} + \bar{M}\right) = 0. \tag{2.23}$$

Utilizing Eqs (2.20)-(2.23), Eq.(2.19) transforms into:

$$\rho\left(\bar{q}\nabla\right)\bar{q} = -\nabla\left(p - \frac{\mu_0\bar{\mu}}{2}M^2\right) + \eta\nabla^2\bar{q}.$$

The lubricant of the film is characterized as isoviscous and incompressible, exhibiting laminar flow. The corresponding modified Reynolds-type equation governing the film pressure for the step slider bearing is as follows:

$$\frac{d}{dx}\left\{\frac{1}{s\left(l,h\right)}\frac{d}{dx}\left(p - \frac{\mu_0\hat{\mu}}{2}M^2\right)\right\} = 6\mu u_0\frac{dh}{dx}$$

where $M^2 = kb^2\frac{x}{b}\left(1 - \frac{x}{b}\right)$ which is taken in the algebraic form and k is adjusted to harmonize with the dimensions of both sides.

Introducing dimensionless quantities

$$X = \frac{x}{b},\; L = \frac{2l}{h_0},\; Q = \frac{2q}{u_0 h_0},\; P = \frac{ph_0^2}{6\mu u_0 b},\; F = \frac{fh_0}{6\mu u_0 b},\; \mu^* = \frac{\mu_0\hat{\mu}kbh_0^2}{6\mu u_0},\; C = \frac{F}{W},$$

$$S\left(L,H\right) = \left[H^3 - 3L^2 H + 3L^3\tanh\frac{H}{L}\right]^{-1}. \tag{2.24}$$

The boundary conditions governing the pressure distribution within the lubrication film of the step bearing, expressed in dimensionless terms, are stipulated as follows:

$$\left(P\right)_{X=0} = 0;\quad \left(P\right)_{X=1} = 0. \tag{2.25}$$

With the help of Eq.(2.24) and the boundary conditions (2.25), the dimensionless pressure P, load capacity W, the center of pressure $\bar{X}$ and dimensionless frictional force F and frictional coefficient C is determined as:

$$P = \frac{\mu^* X\left(1 - X\right)}{2} + \int_0^X\left[\left(H - Q\right)S\left(L,H\right)\right]dX \tag{2.26}$$

where $$Q = \frac{\int_0^1 HS\left(L,H\right)dX}{\int_0^1 S\left(L,H\right)dX},$$

$$W = \int_0^1 P dX = \frac{\mu^*}{12} + \int_0^1 \left[X\left(Q - H\right) S\left(L, H\right) \right] dX \,, \tag{2.27}$$

$$\bar{X} = \frac{1}{W}\int_0^1 XPdX = \frac{\mu^*}{24W} + \frac{1}{2W}\int_0^1 \left[X^2\left(Q - H\right) S\left(L, H\right) \right] dX \tag{2.28}$$

and

$$F = \frac{1}{6}\int_0^1 \left[3H\left(H - Q\right) S\left(L, H\right) + \frac{1}{H} \right] dX \,. \tag{2.29}$$

The basic characteristics of the slider bearing, W, $\bar{X}$ and F are obtained by evaluating the integrals (2.27)-(2.29) provided the function $H(X)$ is given.

$$\begin{gathered} W = \frac{\mu^*}{12} + \frac{1}{2}B\left(1 - B\right)\left(A - 1\right) \times \\ \times \left[B\left\{ 1 - 3L^3\left(\frac{1}{L} - \tanh\frac{1}{L} \right) \right\} + \left(1 - B\right)\left\{ A^3 - 3L^3\left(\frac{A}{L} - \tanh\frac{A}{L} \right) \right\} \right]^{-1} , \end{gathered} \tag{2.30}$$

$$\bar{X} = \frac{1 + B}{3} + \frac{\mu^*}{12W}\left(\frac{1}{2} - \frac{1 + B}{3} \right), \tag{2.31}$$

$$F = \left(A - 1\right)\left(W - \frac{\mu^*}{12} \right) + \frac{1}{6}\left(\frac{B}{A} + 1 - B \right). \tag{2.32}$$

3. Results and discussion

In this investigation, the study scrutinizes the impact of both couple stress and magnetic fluid on the operational characteristics of the step slider bearing. The analysis reveals that parameters such as the couple stress parameter L, magnetic fluid parameter μ^* and step height ratio A significantly influence the behaviour of the step slider bearing. Furthermore, it has been explored the collective influence of couple stress and ferrofluid, incorporating an algebraic representation of the magnetic field magnitude, to improve the bearing's performance. Expression for pressure distribution and load carrying capacity are obtained analytically while the integrals occurring in these two expressions are evaluated using MATLAB.

Figure 3 illustrates the Load Carrying Capacity (LCC), denoted as W, plotted against L for various values of μ^* with $A = 2$. The graph indicates a noticeable enhancement in load carrying capacity as the couple stress parameter and magnetic fluid values increase. This observed improvement can be attributed to the additional influence of the algebraic magnitude of the magnetic field.

The elaboration of W with different values of L is depicted in Fig.4 for various μ^* with $A = 3$. It is observed that the W increases with an increase in the couple stress parameter for different magnetic parameter values. Additionally, a higher W is noted for $A = 2$ compared to $A = 3$ from Figs 3 and 4.

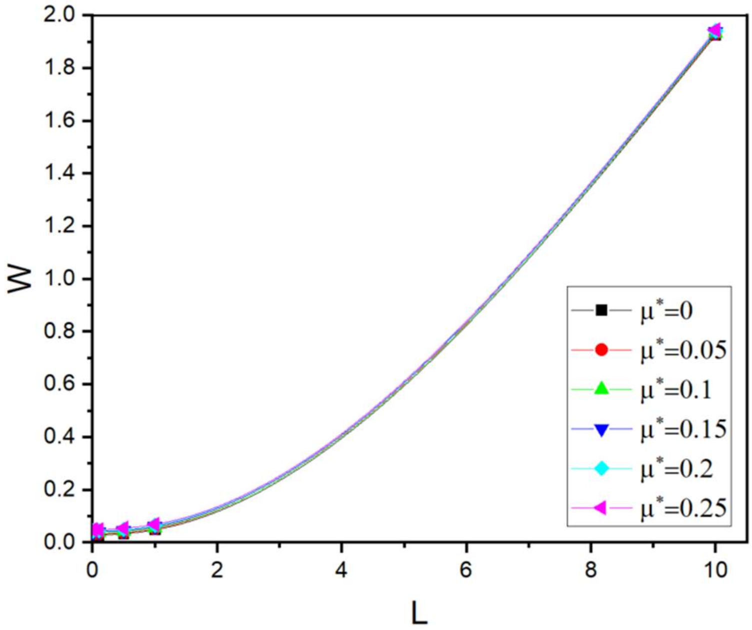

Fig.3. W as a function of L for different μ^* and $A = 2$.

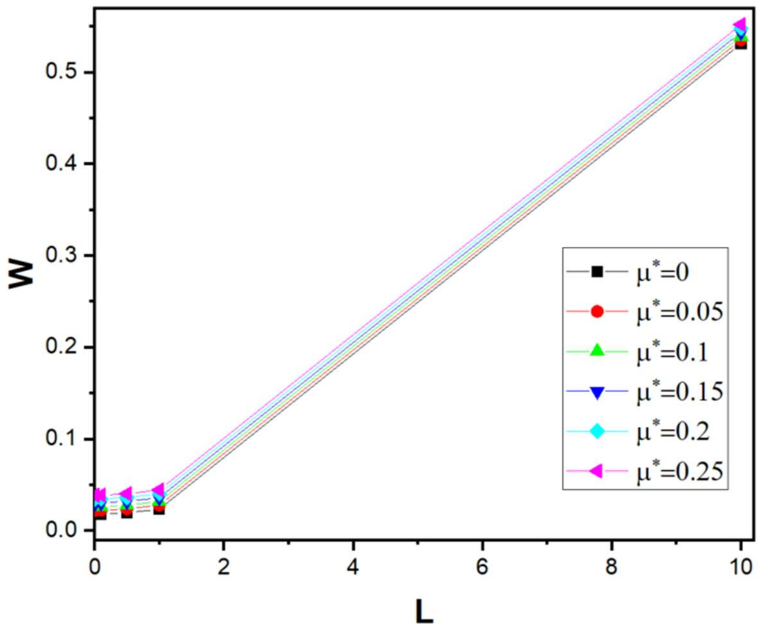

Fig.4. W as a function of L for different μ^* and $A = 3$.

The variation of the non-dimensional friction coefficient C for different values of L and μ^* is shown in Figs 5 and 6 for $A = 2$ and $A = 3$, respectively. It is observed that the friction coefficient gradually decreases with increasing L and μ^* in both cases. Particularly, the friction coefficient tends to a certain value when $L = 10$ and sharply decreases for initial values of L , for both $A = 2$ and $A = 3$.

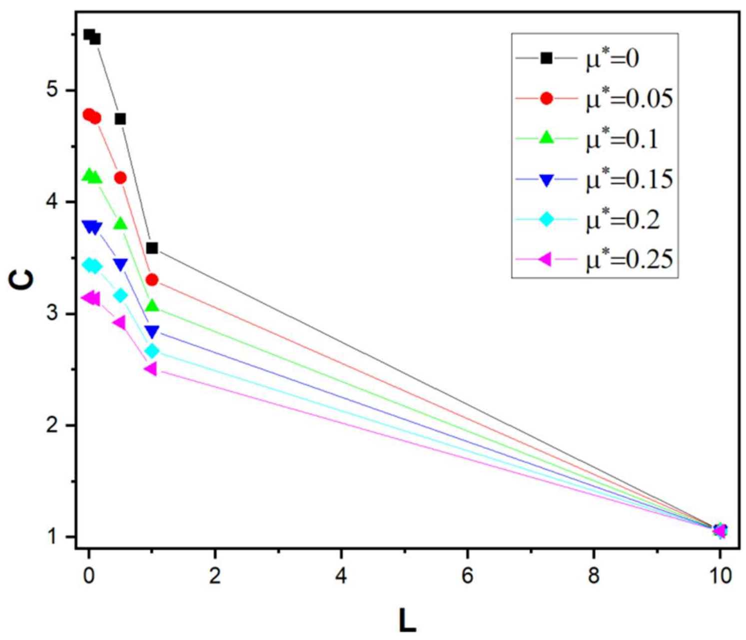

Fig.5. C as a function of L for different μ^* and $A = 2$.

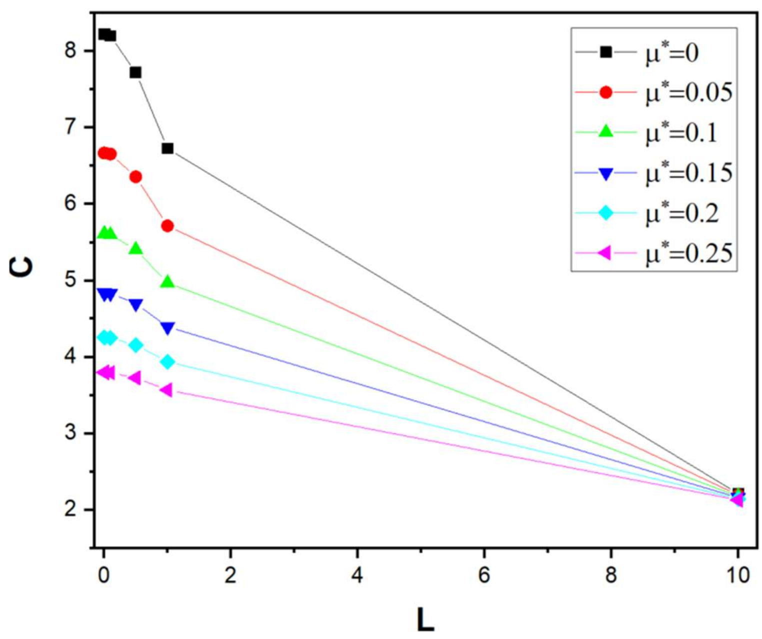

Fig.6. C as a function of L for different μ^* and $A = 3$.

Figure 7 describes the deviation in W concerning A across different scenarios: (a) $L = 0$; $\mu^* = 0$ represents the conventional fluid-based step bearing, (b) $L = 0.1$; $\mu^* = 0$ signifies the presence of only couple stress, (c) $L = 0$; $\mu^* = 0.1$ indicates the existence of only magnetic fluid, and (d) $L = 0.1$; $\mu^* = 0.1$ denotes the combined effect of couple stress and magnetic fluid. Upon closer examination of Fig.7, it is evident that W follows similar trends across all cases. However, cases (c) and (d) notably enhance the load-carrying capacity of the step-bearing system. Furthermore, minimal variation in W is observed for higher values of A across all scenarios, although Case (d) exhibits a higher load due to the presence of couple stress and ferrofluid. A comparison of the present study with the investigation of Ramanahia and Sarksar [39] indicated that the Load-carrying capacity is enhanced by *22.30* % in the presence of couple stress.

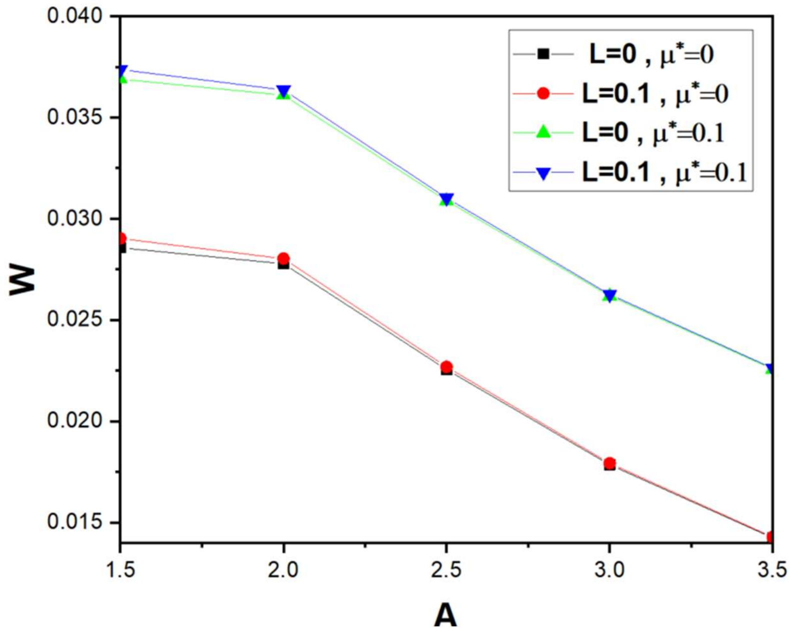

Fig.7. W as a function of A for different Couple stress (L) and magnetic fluid parameter μ^*.

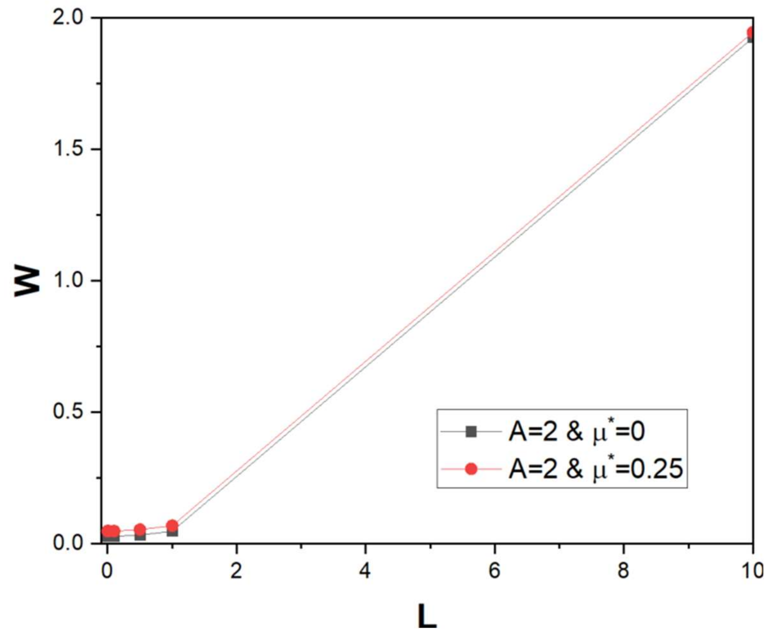

Fig.8. W as a function of L for conventional fluid and ferrofluid.

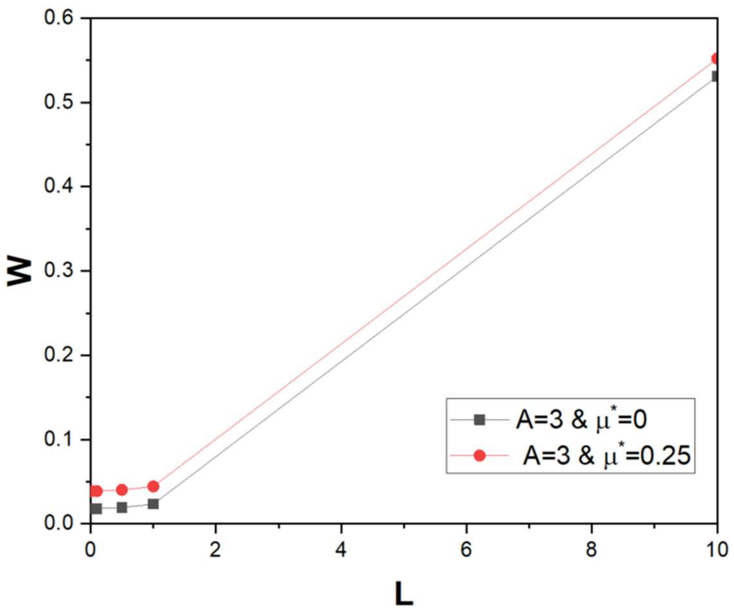

Fig.9. W as a function of L for conventional fluid and ferrofluid.

The elaboration of W with L for A and μ^* is depicted in Figs 8 and 9. It is observed that the LCC increases by incorporating ferrofluid as a lubricant compared to conventional fluid. Additionally, a higher W is noted for $A = 2$ compared to $A = 3$ from Figs 8 and 9.

The article compares conventional fluid with ferrofluid and aligns the current research with the findings of (Ramanaiah and Sarkar [39]) in Figs 8 and 9, employing ferrofluid. It primarily investigates the impact of couple stress and magnetic fluid on the step slider bearing. Naturally, the step slider bearing boasts a greater load capacity compared to other bearings. Introducing ferrofluid as a lubricant marginally boosts the load-carrying capacity, especially in the presence of couple stress.

Generally, the W decreases with an increase in the value of A, suggesting that the performance of the step-bearing system can be enhanced by selecting appropriate values for the magnetic fluid and couple stress parameters, especially for lower step height ratios. This has implications for the design of step bearings and can impact industrial efficiency and productivity.
The present study registers higher LCC as compared to the investigations of (Ramanaiah and Sarkar [39]).

4. Conclusion

An investigation has studied the performance of step slider bearings based on the Neuringer-Rosensweig model for magnetic fluid flow dynamics, along with the Stokes micro-continuum model to consider couple stress effects.

This article conducts a comparative analysis between distinct systems: one is a step slider integrated with both couple stress and ferrofluid, while the other is a conventional fluid step slider operating with couple stress, and also step bearing with conventional fluid only. The results for key factors affecting the bearing's behavior, including the couple stress parameter, magnetic fluid parameter, magnitude of magnetic field, and step height ratio, along with the results, have been graphed. The following conclusions are drawn:

- In the absence of magnetism this study reduces to the investigation of Ramanaiah and Sarkar [39].
- In the absence of couple stress this type of bearing system can support a good amount of load.
- It is interesting to know that in the absence of both couple stress and magnetic fluid certain amount of load is sustained by the bearing system which is unlikely in the case of a conventional fluid-based bearing system.

- The increase in LCC is attributed to the presence of the ferrofluid used as a lubricant.
- It is observed that an increase in the step height ratio leads to a decrease in LCC and an increase in the friction coefficient, validating the presented results.
- The overall performance of the step slider bearing is enhanced in terms of load-bearing capacity and reduced friction coefficient by the combined effect of couple stress and magnetic fluid as compared to traditional lubrication.
- If properly designed, this investigation could be very useful in many industrial applications owing to their high Load carrying capacity and low manufacturing. (for example, compressors, turbines & hard disk drives).
- This observation underscores the importance of considering the bearing's performance not only under normal operational conditions but also under transient or non-ideal situations.
- In our opinion tells that careful consideration of the step size ratio is crucial when designing step bearings, to the usual in turbine compressors.
- A future direction of research could be to analyze the impact of couple stress and magnetic fluid on the performance of step porous rough slider bearing.

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Nomenclature

A – step height ratio
b, B – breadth of bearing
b_i – body force per unit mass
C – frictional coefficient
e_{ijk} – the permutation tensor
f, F – frictional force
g_i – the body couple per unit mass
$\bar{H}$ – external magnetic field
h – film thickness
h_0 – minimum film thickness
l, L – couple stress parameter
M_{ij} – couple stress tensor
p, P – pressure
$\bar{q}$ – fluid velocity
T_{ij} – the stress tensor
v_i – velocity vector
w, W – load capacity
$\bar{X}$ – center of pressure
δ_{ij} – the Kronecker delta
η – fluid viscosity
η' – material constants
λ, μ – classical viscosity coefficients

$\bar{\mu}$ – magnetic susceptibility

μ^* – magnetic fluid parameter

μ_0 – free space permeability

ρ – density

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