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KINEMATIC ANALYSIS AND SYNTHESIS OF A COMPOUND EPICYCLIC GEAR MECHANISM WITH INTERMESHING SATELLITES

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In the present work, a study has been conducted on the possible kinematic configurations of a given gear mechanism with intermeshing satellites to compare the values of the transmitted torques. As a result of this analysis, principal schemes of compound epicyclic gear mechanisms achieving high values of transmitted torques at the output have been proposed. Kinematic synthesis of the mechanisms with optimal gear ratios achieving the highest values of output torques has been performed.

Key words: epicyclic gear mechanism, differential mechanism, planetary mechanism, satellites, kinematic scheme.

1. Introduction

With the advancement of the industry, the requirements for products and technical items are increasing. The continuous rise in raw material prices, along with the drive to drastically reduce harmful emissions, places high demands on mechanical engineering products. Reducing the size, weight, and material used, while aiming for high load capacity and efficiency, is a primary challenge for designers in many areas of mechanical engineering.

Gear mechanisms find wide application in technology. They are used in most types of machines and devices that have a mechanical part. Although modern digital controls and the latest generations of electric machines simplify the requirements for power transmission, there is still a need for gear and other types of transmissions. The goal is maximum specific load capacity, compact dimensions, and efficiency. Regarding gear transmissions, it is known that power-splitting, and particularly epicyclic gears, offer potential for high specific load capacity. This is due to the principle of multipath, which allows the load to be divided between several branching elements (Chen and Chen, [1]; Wang, [2]; Yang *et al.* [3]; Guo and Parker, [4]; Xue and Li, [5]).

Epicyclic gears are gear mechanisms where the axes of some gears are movable, or in other words, one or a group of gears perform complex motion-they rotate around their own axis and perform additional rotation around the axis of another element called a carrier. Gears with complex motion are called satellites. A configuration that allows for more satellites to be placed between which the load is distributed creates potential for high specific load capacity. Such are planetary gears with intermeshing satellites. It should be noted that the conditions for the realization of the structure are strictly specific, justified by the meshing of the satellites with each other.

In (Hariharan *et al.* [6]; Ding and Cai, [7]; Mathis and Remond, [8]; Chen and Shao, [9]) it is proven that the realization of the conditions for assembly, adjacency, and coaxiality for an $A[A]I$ gear is possible in accordance with the analyzed design requirements and constraints regarding gear ratio, selection of the number of teeth on the gears, and the arrangement of components. In the gear designation with A, external meshing is indicated, with I - internal meshing, with $[A]$ mutual meshing is indicated, and the element in which the closed loop is placed is marked.

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The object of this study is a given gear mechanism with twelve intermeshing satellites (Fig.1). The mechanism consists of one central gear z_1, called the Sun; twelve intermeshing satellites-six internal z_2 and six external z_3, and a ring z_4. The mechanism is implemented according to the method proposed in (Drewniak *et al.* [10]) for the synthesis and calculation of planetary gears with intermeshing satellites. Additionally, an original test stand has been designed and manufactured.

The objective of this work is to synthesize a compound epicyclic mechanism with a structure that integrates the given gear mechanism with twelve intermeshing satellites, achieving high values of output torques. To achieve this objective, it is first necessary to investigate the possible kinematic configurations of the given gear mechanism with intermeshing satellites to compare the magnitudes of the transmitted torques. Then, to propose principal schemes of compound epicyclic gear mechanisms driven by a single electric motor, with additional gear transmissions added to the kinematic chain of the examined configurations. Finally, to perform kinematic synthesis of mechanisms with optimal gear ratios, achieving the highest values of output torques.

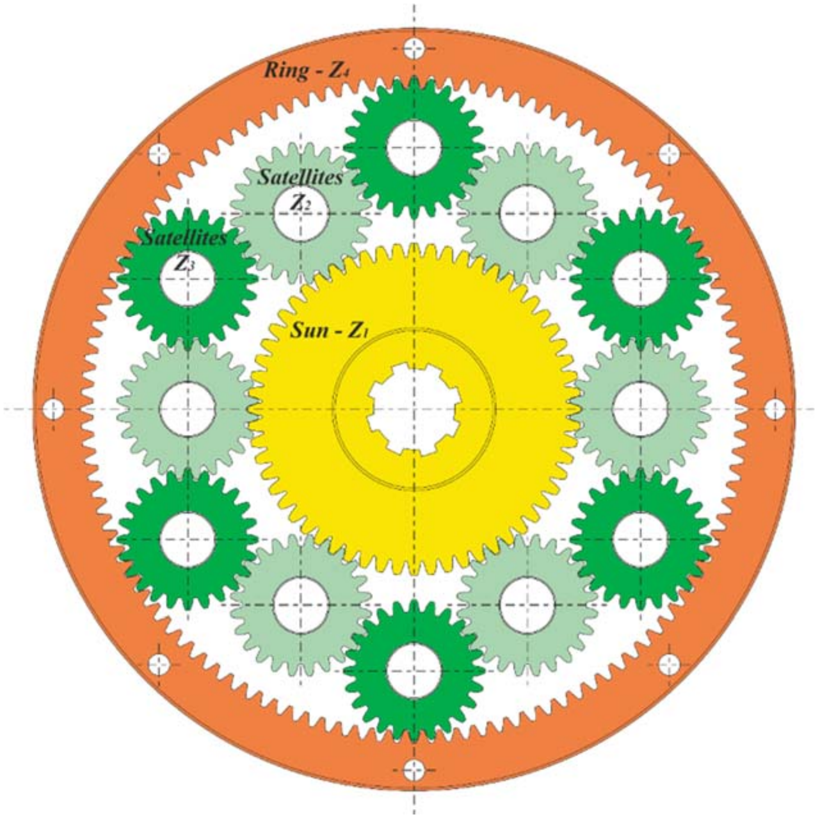


Fig.1. Parametrically modeled gears.

2. Kinematic study of the given gear mechanism with twelve intermeshing satellites

Epicyclic gear mechanisms are divided into two groups according to the number of degrees of freedom: planetary and differential. Planetary gears have one degree of freedom, where the central gear z_1 or the ring gear z_4 are stationary. Differential gear mechanisms have more than one degree of freedom, where all members move. To achieve a specific angular velocity of the output member, for example, the ring gear z_4, the angular velocities of the membes z_1 and H must be specified.

This allows the angular velocity of the output member to change with a constant angular velocity of the drive by altering the angular velocity of member *H*. Similarly, the motion of members z_1 and z_4 can be specified to achieve a certain motion of *H*.

The kinematic study of gear mechanisms involves determining their gear ratio, i.e., the ratio of the angular velocities (revolutions) of the input and output members of the respective mechanism (Shanmukhasundaram *et al.* [11]; An *et al.* [12]; Brumercik *et al.* [13]; Pennock and Alwerdt, [14]; Marciniec *et al.* [15]; Tsai, Huang, *et al.* [16]).

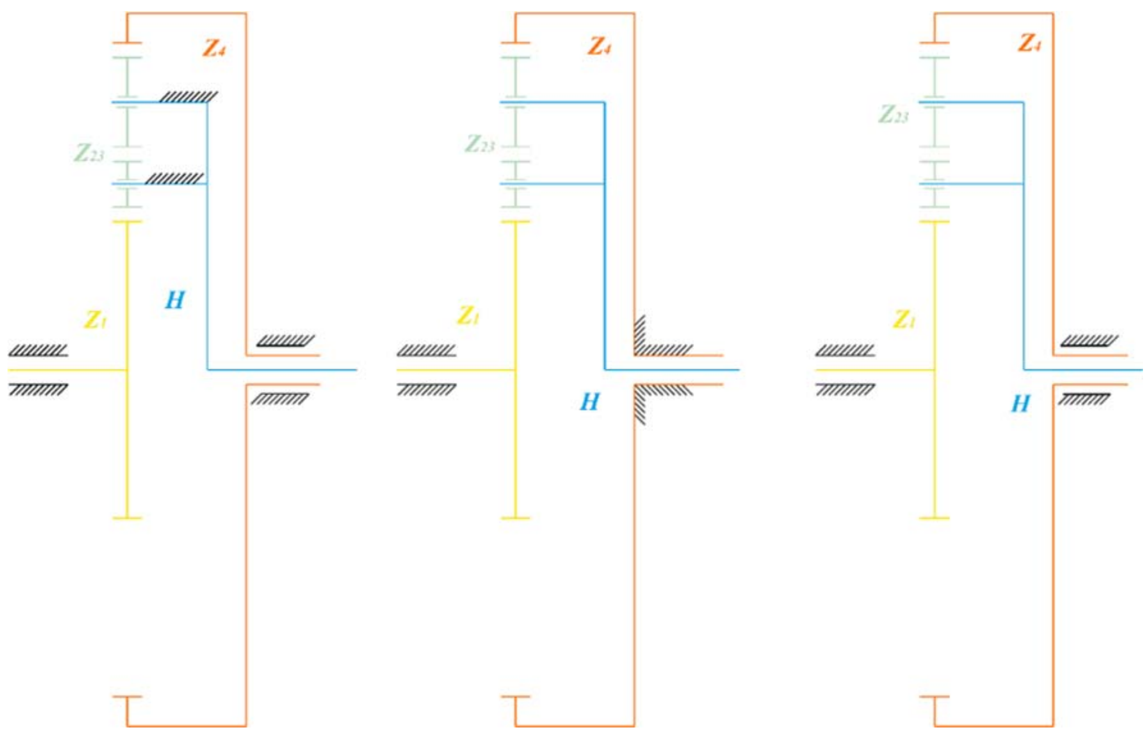
The analytical determination of the gear ratios in an epicyclic mechanism requires its kinematic conversion into a simple mechanism. This necessitates stopping the carrier. This method is known as Willis's method (Xue [17]; Morlin *et al.* [18]; Liu *et al.* [19]; Talpasanu *et al.* [20]). For this purpose, all members of the mechanism are given an angular velocity equal in magnitude and opposite in direction to the angular velocity of the carrier, i.e., an inversion with respect to the carrier is performed.

The structure of the examined gear mechanism with intermeshing satellites $A[A]I$ allows the following kinematic configurations with input gear *1*:

- Simple gear mechanism (SGM) – the carrier is stationary. In this configuration, gears 2 and 3 perform simple motion - rotation around their own axis (Fig.2a). The mechanism has one degree of freedom. The output member is gear 4;

- Planetary gear mechanism (PGM) – here, gear *4* is stationary, and the carrier *H* is movable. In this configuration, gears *2* and *3* perform complex motions - rotation around their own axis and rotation around the axis of the carrier (Fig.2b). The mechanism has one degree of freedom. The output member is the carrier *H*;

- Differential gear mechanism (DGM) – here, all members are movable. In this configuration, gears *2* and *3* perform complex motions - rotation around their own axis and rotation around the axis of the carrier (Fig.2c). The mechanism has two degrees of freedom. The output member can be the carrier *H* or gear *4*.



a) SGM b) PGM c) DGM

Fig.2. Kinematic schemes of configurations of a gear mechanism with intermeshing satellites.

2.1. Kinematic study of the given gear mechanism with twelve intermeshing satellites

2.1.1. Gear ratio of the simple gear mechanism (SGM)

The gear ratio of the simple gear mechanism (SGM) is determined by the following relationship:

$$i_{14} = \frac{\omega_1}{\omega_4} = -(1)^b \frac{z_2}{z_1} \frac{z_3}{z_2} \frac{z_4}{z_3} = \frac{z_4}{z_1} = i_{ob}, \tag{2.1}$$

where b is the number of gears with external meshing - $b = 2$.
For the angular velocity of the output of the examined kinematic configuration, the following is obtained:

$$\omega_{out} = \omega_4 = \frac{\omega_1}{i_{ob}}. \tag{2.2}$$

2.1.2. Gear ratio of the planetary gear mechanism (PGM)

The gear ratio of the planetary gear mechanism (PGM) is determined by Willis's relationship:

$$i_{14}^{(H)} = \frac{\omega_1 - \omega_H}{-\omega_H} = \omega_{ob},$$

$$1 - i_{1H} = i_{ob}, \tag{2.3}$$

$$i_{14}^{(H)} = \frac{\omega_1}{\omega_H}.$$

For ω_{out} the following is obtained:

$$\omega_{out}^{H} = \omega_H = \frac{\omega_1}{1 - i_{ob}}. \tag{2.4}$$

2.1.3. Gear ratio of the differential gear mechanism (DGM)

The gear ratio of the differential gear mechanism (DGM) is determined by Willis's relationship:

$$i_{14}^{(H)} = \frac{\omega_1 - \omega_H}{\omega_4 - \omega_H} = i_{ob}. \tag{2.5}$$

For output member Eq.(2.4) from Eq.(2.5), the following is obtained:

$$\omega_{out} = \omega_4 = \frac{\omega_1 - \omega_H}{i_{ob}} + \omega_H. \tag{2.6}$$

And for output member *H*:

$$\omega_{out} = \omega_H = \frac{i_{ob}\omega_4 - \omega_1}{(i_{ob} - 1)}. \tag{2.7}$$

To determine the transmitted torque in the examined kinematic configurations of the gear mechanism with intermeshing satellites $A[A]I$, the following relationship is used:

$$M_{out} = \frac{P_{dv}}{\omega_{out}} \eta_{gm} , \tag{2.8}$$

where P_{dv} is the power of the electric motor; η_{gm} - coefficient of efficiency of the gear mechanism.

2.2. Numerical solutions

The transmitted torque is determined for the three configurations of the gear mechanism with $z_1 = 60; z_2 = 24; z_3 = 24; z_4 = 120$; $\eta_{gm} = 0.985$. For each configuration, it is assumed that gear *1* is driven by a motor with:

$$P_{dv} = P_H = 2.2kW, \quad n_{dv} = 1450 \min^{-1},$$
$$\omega_{dv} = \frac{\pi \, n_{dv}}{30} = 151.76 \approx 152 \left[s^{-1} \right]. \tag{2.9}$$

Table 1 shows the obtained results for the simple gear mechanism and Tab.2 for the planetary gear mechanism.

Table 1. Kinematic and Torque Results for the Simple Gear Mechanism (SGM)

	input	output	$\omega_{input}[s^{-1}]$	$\omega_{out}[s^{-1}]$	$M_{out}[N.m]$
OGM	gear *1*	gear *4*	*152*	*75.88*	*28.56*

Table 2. Kinematic and Torque Results for the Planetary Gear Mechanism (PGM)

	input	output	$\omega_{input}[s^{-1}]$	$\omega_{out}[s^{-1}]$	$M_{out}[N.m]$
PGM	gear *1*	gear *h*	*152*	*-152*	*-14.3*

Table 3. Kinematic and Torque Results for the Differential Gear Mechanism (DGM1) with Gear 4 as Output

	input	output	$\omega_{input} = \omega_1[s^{-1}]$	$\omega_{input} = \omega_H[s^{-1}]$	$\omega_{out} = \omega_4[s^{-1}]$	$M_{out}[N.m]$
DGM*1*	gear *1*	gear *4*	*152*	*350*	*251*	*8.63*
			152	*300*	*226*	*9.59*
			152	*250*	*201*	*10.78*
			152	*200*	*176*	*12.31*
			152	*152*	*152*	*14.25*
			152	*150*	*151*	*14.35*
			152	*100*	*126*	*17.20*
	carrier *h*		*152*	*50*	*101*	*21.46*
			152	*-50*	*51*	*42.49*

Table 3 cont. Kinematic and Torque Results for the Differential Gear Mechanism (DGM1) with Carrier h as Output

	input	output	$\omega_{input} = \omega_1 [s^{-1}]$	$\omega_{input} = \omega_H [s^{-1}]$	$\omega_{out} = \omega_4 [s^{-1}]$	$M_{out} [N.m]$
DGM*1*	carrier *h*	gear *4*	*152*	*-100*	*26*	*83.35*
			152	*-150*	*1*	*2167*
			152	*-200*	*-24*	*-90.3*
			152	*-250*	*-49*	*-44.22*
			152	*-300*	*-74*	*-29.28*
			152	*-350*	*-99*	*-21.89*

Table 4. Kinematic and Torque Results for the Differential Gear Mechanism (DGM2)

	input	output	$\omega_{input} = \omega_1 [s^{-1}]$	$\omega_{input} = \omega_4 [s^{-1}]$	$\omega_{out} = \omega_H [s^{-1}]$	$M_{out} [N.m]$
DGM*2*	gear *1*	carrier *h*	*152*	*350*	*548*	*3.95*
			152	*300*	*448*	*4.84*
			152	*250*	*348*	*6.23*
			152	*200*	*248*	*8.74*
			152	*152*	*152*	*14.3*
			152	*150*	*148*	*14.64*
			152	*100*	*126*	*45.14*
	gear *4*		*152*	*50*	*-52*	*-41.67*
			152	*-50*	*-252*	*-8.6*
			152	*-100*	*-352*	*-6.16*
			152	*-150*	*-452*	*-4.79*
			152	*-200*	*-552*	*-3.93*
			152	*-250*	*-652*	*-3.32*
			152	*-300*	*-752*	*-2.88*
			152	*-350*	*-852*	*-2.54*

In Figs 3 and 4 graphically show the obtained results for ω_{out} and M_{out} when varying the angular velocity of the carrier from $\omega_H = -350 s^{-1}$ to $\omega_H = 350 s^{-1}$. It can be seen that the transmitted torque increases in the range from $\omega_H = -100 s^{-1}$ to $\omega_H = -200 s^{-1}$.

In Figs 5 and 6 graphically show the obtained results for when varying the angular velocity of the carrier from ω_{out} and M_{out} when varying the angular velocity of the carrier from $\omega_4 = -350 s^{-1}$ to $\omega_4 = 350 s^{-1}$. It can be seen that the transmitted torque increases in the range from $\omega_4 = 50 s^{-1}$ to $\omega_4 = 100 s^{-1}$.

The analysis of the obtained results shows that:

- in the SGM, the transmitted torque is approximately twice the torque of the motor;
- in the PGM, the transmitted torque is approximately equal to the torque of the motor;
- in DGM *1*, a higher torque can be achieved at negative values of $\omega_{input} = \omega_H$;
- in DGM *2*, a higher torque can be achieved when $\omega_{input} = \omega_4 \leq \omega_1$.

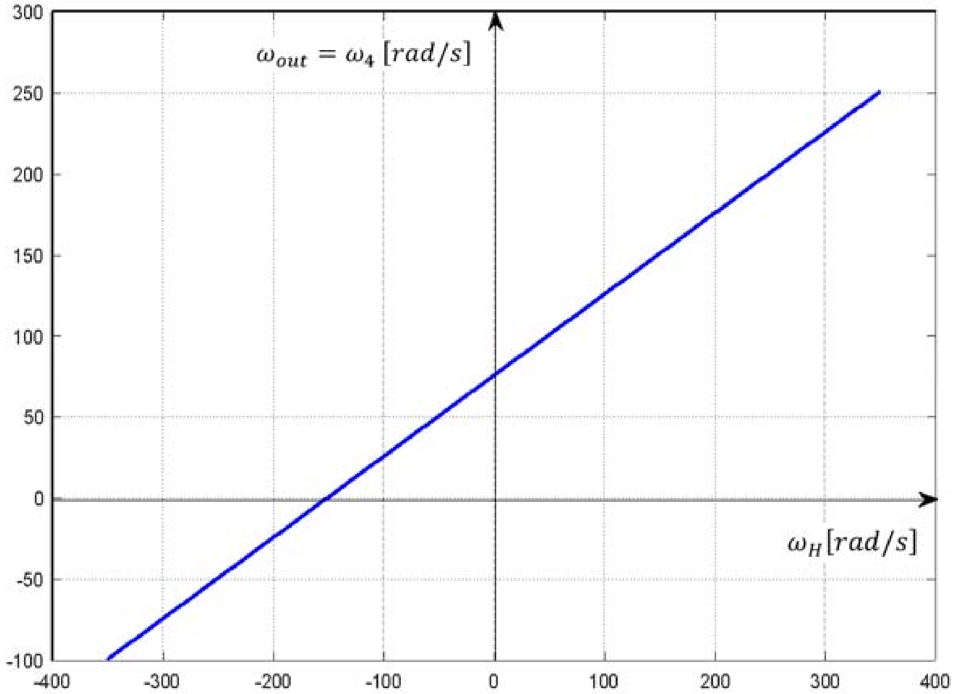


Fig.3. Graphical dependence $\omega_{out}(\omega_H)$; $\omega_{input} = \omega_1 = 152s^{-1}$, $\omega_{input} = \omega_H = -350 \div 350s^{-1}$.

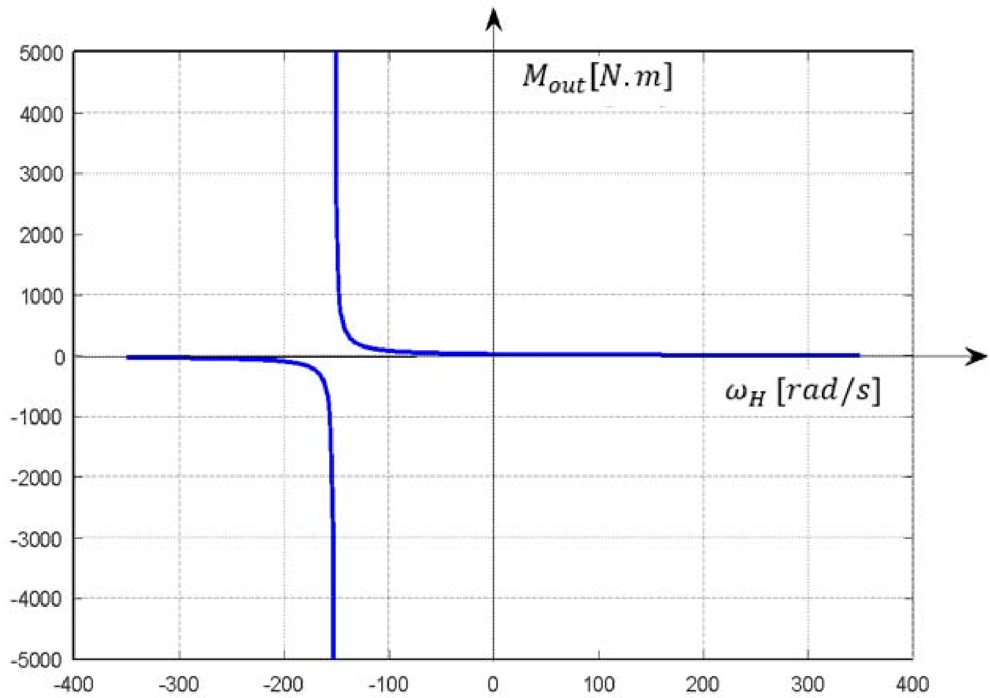


Fig.4. Graphical dependence $M_{out}(\omega_4)$.

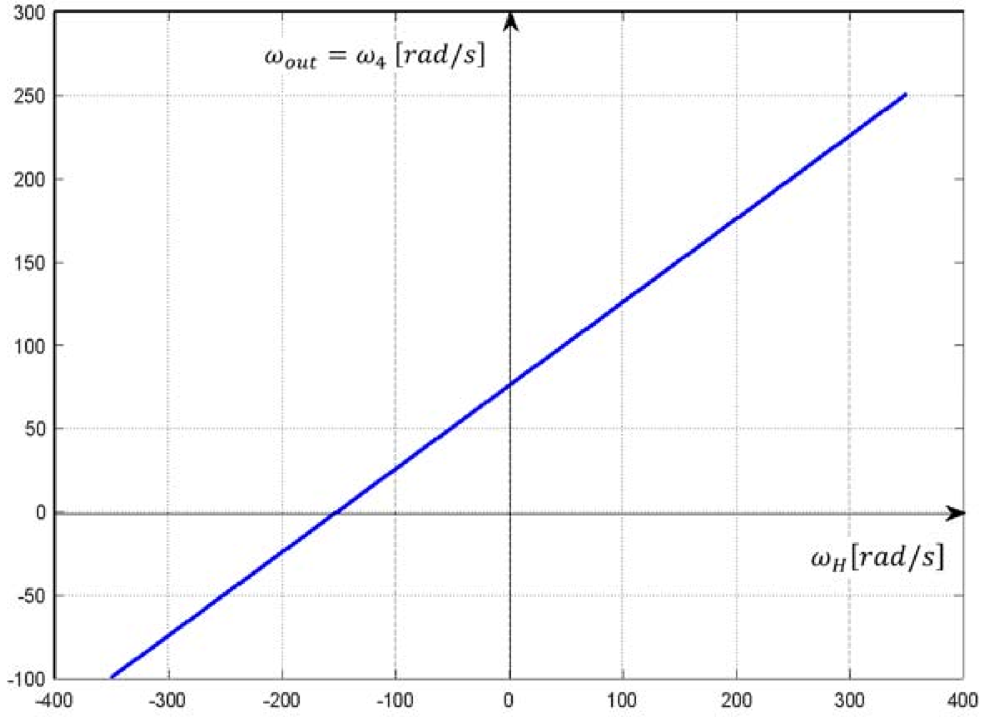


Fig.5. Graphical dependence $\omega_{out}(\omega_4)$; $\omega_{input} = \omega_1 = 152s^{-1}$, $\omega_{input} = \omega_4 = -350 \div 350s^{-1}$.

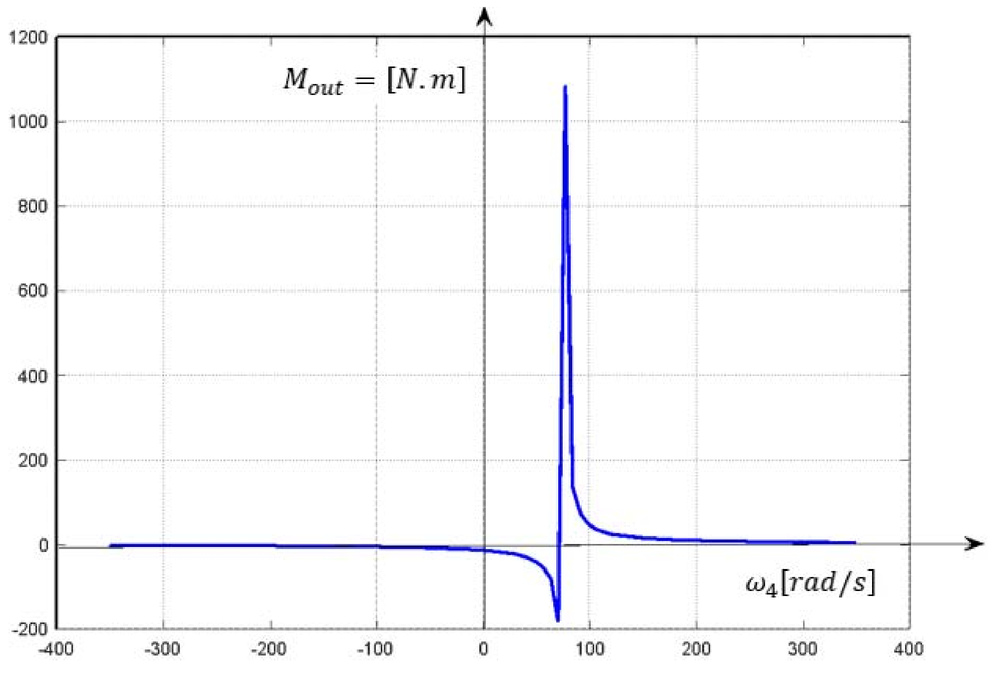


Fig.6. Graphical dependence $M_{out}(\omega_4)$.

3. Synthesis of a compound epicyclic gear mechanism

3.1. Principal schemes

The additional gear transmissions must have coaxial input and output members. They can form a simple or planetary gear mechanism (SGM or PGM). Figure 7 shows the principal schemes of the synthesized epicyclic mechanisms.

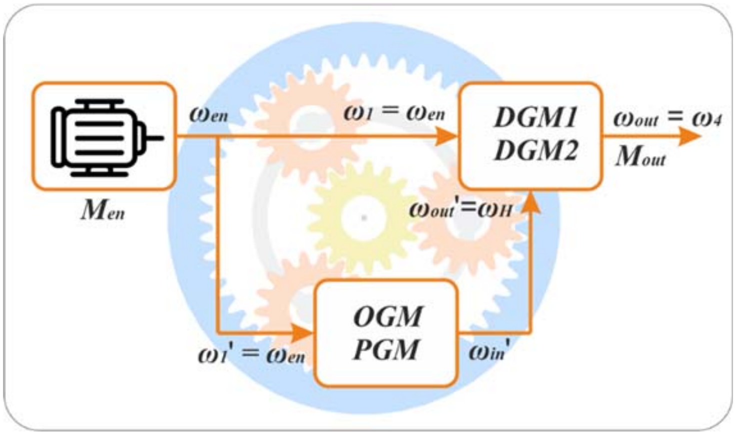


Fig.7. Principal schemes of compound epicyclic gear mechanisms (EGM).

3.2. Kinematic synthesis of the epicyclic gear mechanism (EGM*1*) based on the kinematic configuration of DGM*1*

The analysis of the results shows that only gear transmissions forming a planetary gear mechanism can be connected to the main kinematic chain of DGM*1*, as the synthesis of an SGM with coaxial input and output members and a negative gear ratio within the required range is impossible.

The synthesis consists of selecting the kinematic scheme of the mechanism and determining the number of teeth and the number of satellite gears that provide the prescribed gear ratio and satisfy the conditions for coaxiality, adjacency, assembly, and proper meshing.

The task of the synthesis is to compose the initial equations reflecting the specified conditions for the synthesis of the chosen kinematic scheme and to solve them jointly.

$$-1.1 \le i_{o'} \le -1,$$
$$-1 \le i_{o'} \le -0.9. \qquad (2.10)$$

Condition for the gear ratio.

The first step in meeting this condition is the correct selection of the kinematic scheme.

Figure 8 proposes a kinematic scheme of a planetary gear mechanism (PGM) that can achieve a gear ratio $i_{o'}$ within the range (2). The mechanism consists of:

- a central movable gear $z_{1'}$, mounted on the central shaft, driven by the same electric motor;
- a central stationary gear $z_{3'}$;
- a satellite gear block - $z_{2'} - z_{2''}$;
- a carrier, H', which will be the input to DGM1.

The gear ratio of the mechanism is determined by the following relationship:

$$i_{1'3'}^{(H')} = (-1)^b \frac{z_{2'}}{z_1} . \frac{z_{3'}}{z_{2''}} = i_{o'} . \tag{2.11}$$

Condition for coaxiality.
The condition for coaxiality for the mechanism in Fig.8, with identical modules for both meshing gears, is as follows:

$$z_{1'} + z_{2'} = z_{2''} + z_{3'} . \tag{2.12}$$

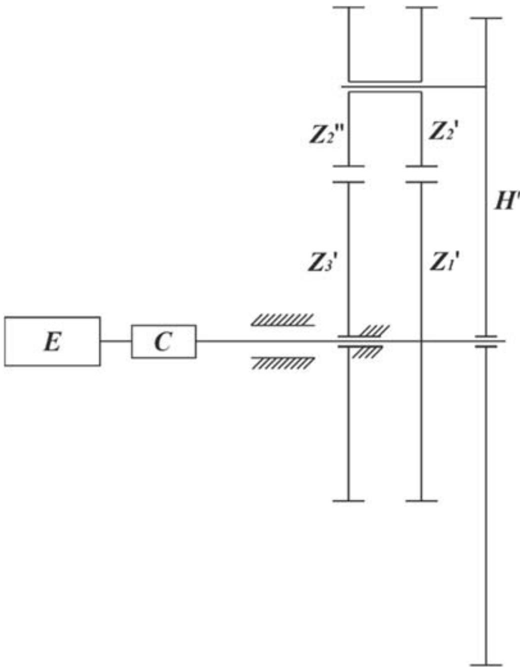


Fig.8. Kinematic Scheme of the PGM.

Condition for adjacency.
This condition excludes the possibility of overlap with the addendum circles of two adjacent satellite gears. For the selected kinematic scheme, it has the following form:

$$\sin\frac{180^o}{s'} \geq \frac{z_{C'} + 2}{\sum z} , \tag{2.13}$$

where: s' is the number of satellite blocks $z_{2'} - z_{2''}$; $z_{C'}$ - is the larger of $z_{2'}$ and $z_{2''}$; $\sum z = z_{1'} + z_{2'}$ and $z_{2'} \geq z_{2''}$; $\sum z = z_{3'} + z_{2''}$ and $z_{2''} \geq z_{2'}$.

Condition for assembly.
The condition for assembly arises from the sequential assembly of s' number of satellite gears, which must fit into the spaces between the teeth of the central gears. For the selected kinematic scheme, it has the following form:

$$\frac{z_{1'}.z_{2''} - z_{3'}.z_{2'}}{s'.D} = C\,, \tag{2.14}$$

where: D is the greatest common divisor of $z_{2'}$ and $z_{2''}$; C is an integer.

Equations (2.11), (2.12), (2.13) and (2.14) are the initial equations reflecting the specified conditions for the synthesis of the chosen kinematic scheme. By solving them simultaneously, multiple variants have been obtained.

Kinematic scheme of epicyclic gear mechanism 1

The epicyclic gear mechanism EGM1 is obtained by connecting the kinematic chain G*1*-G*2*-G*3*-G*4*-*H* of the synthesized planetary mechanism to the main kinematic chain G*1′*-G*2′*-G*2″*-G*4′*-*H* of the existing test stand for a gear mechanism with intermeshing satellites. Figure 9 shows the kinematic scheme of the mechanism.

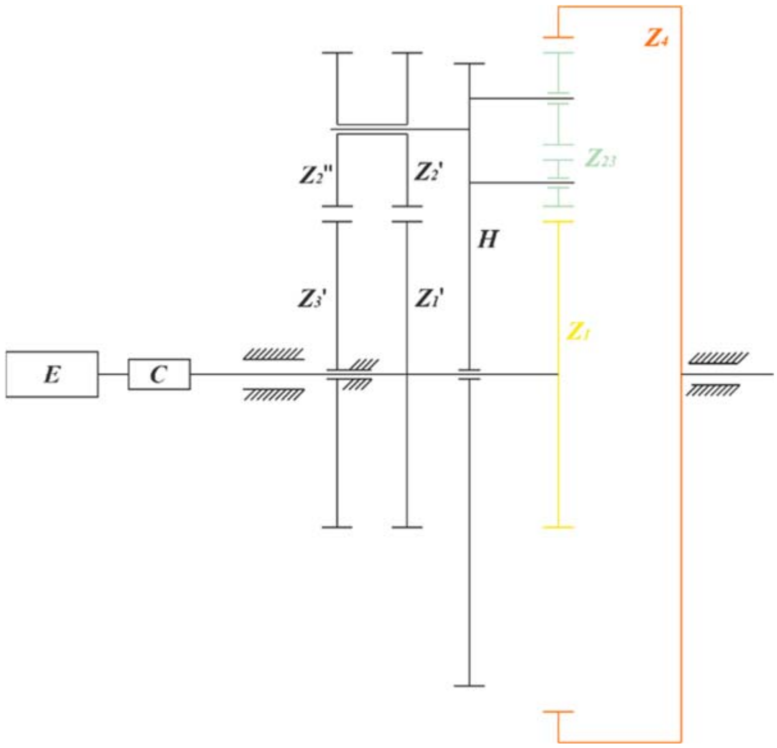


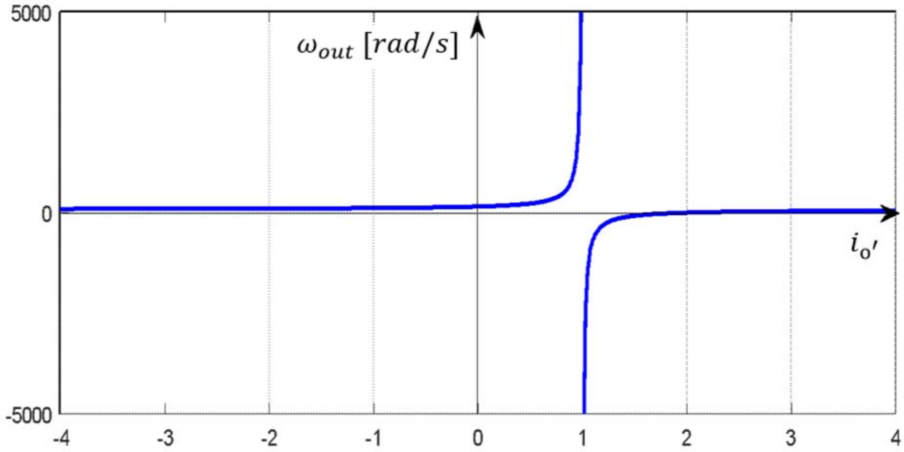
Fig.9. Kinematic Scheme of EGM1.

Figures 10a and 10b show the obtained graphical dependencies $\omega_{out}(i_{o'})$ and $M_{out}(i_{o'})$ respectively, when varying $i_{o'} = -4 \div 4$.

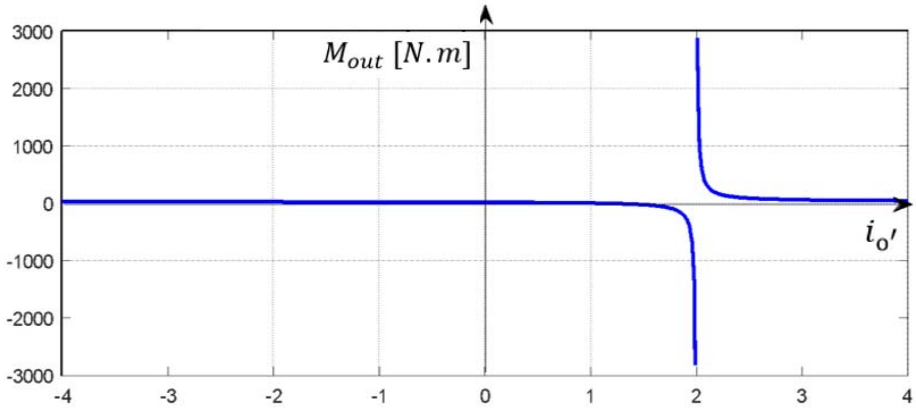
Table 5 presents the geometric and force characteristics of the synthesized mechanism.

Table 5. Geometric and Force Characteristics of the Synthesized Epicyclic Gear Mechanism EGM1

z_1	z_2	z_3	z_4	$z_{1'}$	$z_{2'}$	$z_{2''}$	$z_{3'}$	$P_{dv}[kW]$	$M_{dv}[N.m]$	$M_{out}[N.m]$
60	24	24	120	48	72	50	70	2.2	14.3	313.64



a) Graphical dependence $\omega_{out}(i_{o'})$.



b) Graphical dependence $M_{out}(i_{o'})$.

Fig.10. Numerical solutions of the kinematic dependencies for EGM1 under $i_{o'} = -4 \div 4$.

3.3. Kinematic synthesis of the epicyclic gear mechanism (EGM2) based on the kinematic configuration of DGM2

In this kinematic configuration, the output link is the carrier *H*. The mechanism will achieve a higher torque when the gear z_1 and the gear z_4 rotate in the same direction. Therefore, additional gear trains must be connected to the primary kinematic chain to drive the gear z_4 in the same direction as the gear z_1. The principal scheme of the synthesized mechanism is similar to that shown in Figure 7.
Similarly, the ranges of the transmission ratio $i_{o'}$ of the additional gear mechanisms, in which high torque values are achieved, are determined. For the simple gear mechanism, it is obtained as follows:

$$1 \le i_{o'} \le 2, \quad 2 \le i_{o'} \le 3 \tag{2.15}$$

and for the planetary gear mechanism (PGM):

$$-1.5 \le i_{o'} \le -1,$$
$$-1 \le i_{o'} \le -0.5. \qquad (2.16)$$

The analysis of the results shows that only gear trains forming a simple gear mechanism (SGM) can be connected to the primary kinematic chain of DGM2.

- Condition for transmission ratio.

In Fig.11, a kinematic scheme of a simple gear mechanism (SGM) is proposed, which can realize a transmission ratio $i_{o'}$ within the range of Eq.(2.15). The mechanism consists of:

- a central movable gear $z_{1'}$, mounted on the central shaft, driven by the same motor;
- two satellite gears $z_{2'}$ and $z_{3'}$;
- an outer ring $z_{4'}$ which will be the input to DGM2.

The transmission ratio of the mechanism is determined by the following relationship:

$$i_{ob} = i_{1'4'} = (-1)^b \frac{z_{2'}}{z_{1'}} . \frac{z_{3'}}{z_{2'}} . \frac{z_{4'}}{z_{3'}} = \frac{z_{4'}}{z_{1'}} = i_{o'} , \qquad (2.17)$$

where the number of gears with external meshing is $b = 2$.

- Coaxial condition.

The coaxial condition for the mechanism in Fig.11, with identical modules for both meshing, is as follows:

$$z_{2'} + z_{3'} = 0.5(z_{4'} - z_{1'}) . \qquad (2.18)$$

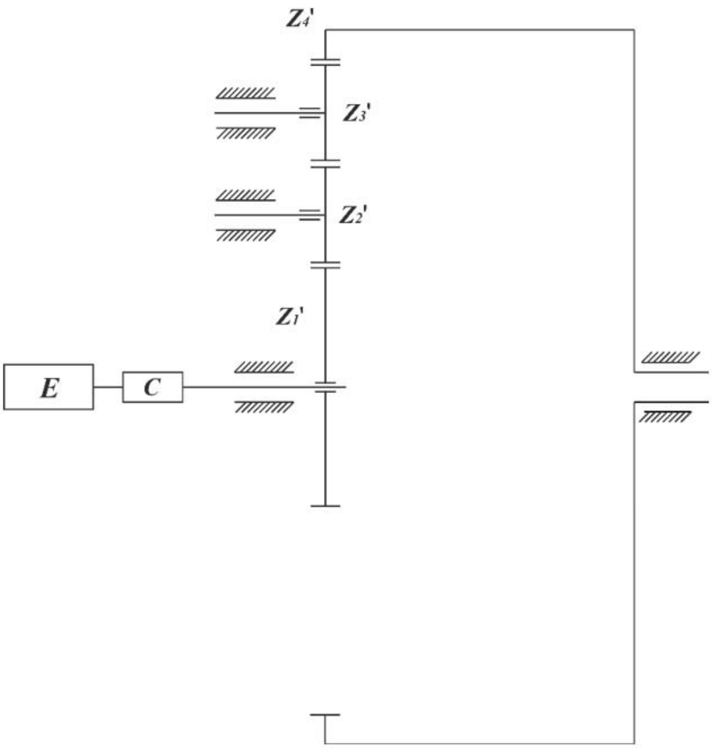


Fig.11. Kinematic scheme of the simple gear mechanism (SGM).

- Condition for adjacency.

For the selected kinematic scheme, this condition is as follows:

$$\sin\frac{180^o}{s'} \geq \frac{z_{2'}+2}{z_{1'}+z_{2'}} . \tag{2.19}$$

- The assembly condition.

The assembly condition is as follows:

$$\frac{z_{4'}-z_{1'}}{s'} = C . \tag{2.20}$$

In Eqs (2.17), (2.18), (2.19), and (2.20) are the fundamental equations reflecting the conditions for the synthesis of the chosen kinematic scheme. The determination of the number of teeth is carried out by simultaneously solving these equations.

Kinematic scheme of epicyclic gear mechanism 2

The epicyclic gear mechanism EGM2 is obtained by connecting the kinematic chain G*1*-G*2*-G*3*-G*4*-*H* (Fig.1b) of the existing test stand for a gear mechanism with intermeshing satellites to the kinematic chain G*1*'-G*2*'-G*3*' of the synthesized planetary mechanism (Fig.11). Figure 12 shows the kinematic scheme of the mechanism.

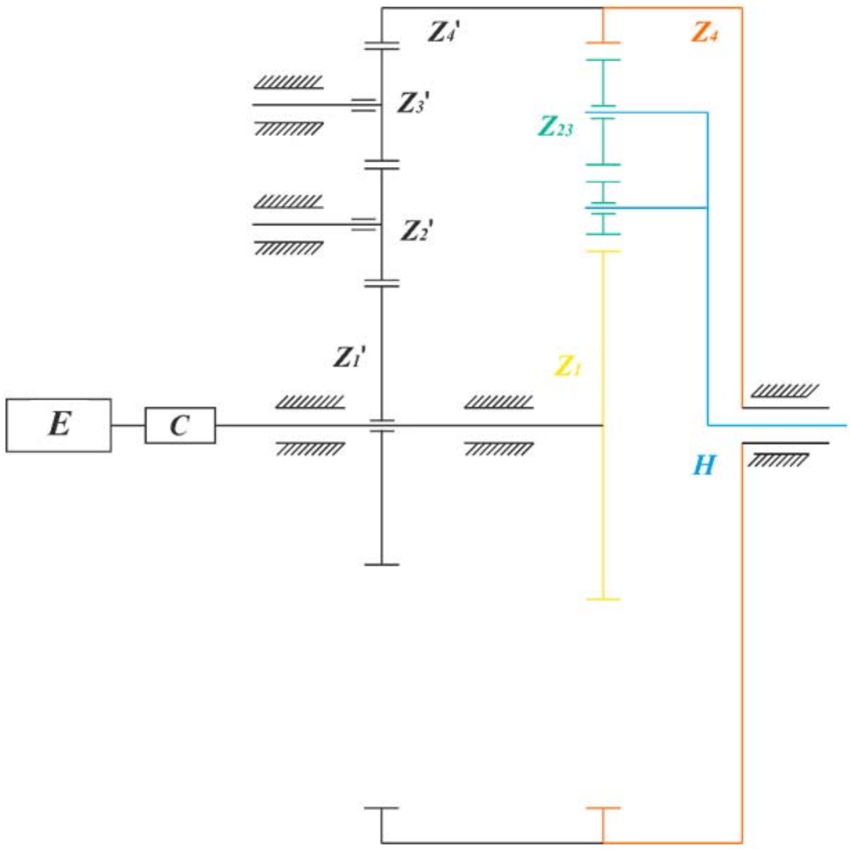
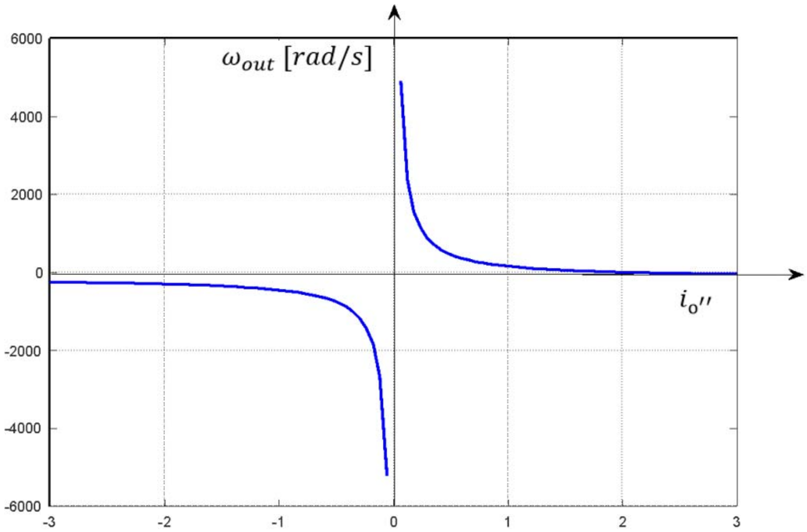


Fig.12. Kinematic scheme of EGM2.

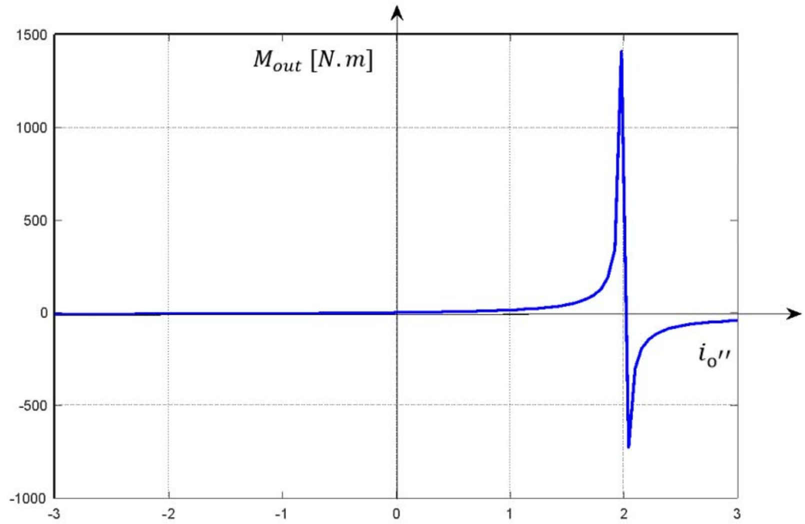
Table 6 presents the geometric and force characteristics of the synthesized mechanism.

Table 6. Geometric and Force Characteristics of the Synthesized Epicyclic Gear Mechanism EGM2

z_1	z_2	z_3	z_4	$z_{1'}$	$z_{2'}$	$z_{3'}$	$z_{4'}$	$P_{dv}[kW]$	$M_{dv}[N.m]$	$M_{out}[N.m]$
60	*24*	*24*	*120*	*67*	*17*	*17*	*135*	*2.2*	*14.3*	*1924.64*



a) Graphical dependency $\omega_{out}(i_{o''})$.



b) Graphical dependency $M_{out}(i_{o''})$.

Fig.13. Numerical solutions of the kinematic dependencies for EGM2 under $i_{o''} = -3 \div 3$.

4. Conclusion

The study addresses the problem of optimizing gear mechanisms for high torque transmission, with a particular focus on compound epicyclic mechanisms with intermeshing satellites. The goal is to achieve maximum output torque while maintaining compact dimensions and high efficiency. The main findings of the research are:

The highest values of transmitted torques can be achieved by adding additional gear trains that form a simple gear mechanism to the kinematic chain of DGM2.

- The synthesized epicyclic mechanism can realize an output torque that is a hundred or more times greater than the input torque.

- A parametric geometric model will be created in SolidWorks to construct the epicyclic mechanism (EGM2) and simulate its operation and loading.

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Nomenclature

OGM – ordinary gear mechanism

PGM – planetary gear mechanism

DGM*1* and DGM*2* – differential gear mechanisms

EGM*1* and EGM*2* – epicyclic gear mechanism

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