In this paper, thermal transfer with natural convection in a tilted annular cylinder with a Cu-water nanofluid has been numerically studied. The hot interior and cold exterior elliptical surfaces of the enclosure were maintained at constant temperatures $T_h$ and $T_c$, respectively. The governing equations were solved by the stream function-vorticity approach. The finite volume approach was utilized to discretise the controlling equations. The volume fraction range of the nanoparticles and the Rayleigh number was as follows: $0.005 < \phi < 0.08$ and $10^4 < Ra < 10^6$, respectively. The inclination angles were $\gamma = 30^\circ, 45^\circ$ and $60^\circ$. Results were given as isotherm contours, streamlines, average and local Nusselt numbers. The results indicate that the thermal transfer ratio increases with an increase in the tilt angle, regardless of the nanoparticle size values. and the impact of the inclination angle on the heating transfer rate is more important the higher the Rayleigh number and the more convection there is.

Key words: elliptic cylinder, nanofluid, the Rayleigh number, heat transfer, inclination angle.

1. Introduction

Heat transfer is most commonly used in solar energy collectors, electronic circuit cooling, the cooling of nuclear reactors, and cooling systems for transmission cables. Many studies deal with natural convection heat transfer from nanofluids inside uncomplicated geometries, such as closed enclosures with various shapes (triangular, square, rectangular, etc.) [1-11]. In an inclined annular enclosure containing with Al$_2$O$_3$/water hybrid nanofluid heat transfer through natural convection was digitally studied by Al-Juboori [12]. It was shown that angular distributions for the internal and external cylinders are influenced by the volume fraction of the nanofluid, the Rayleigh number, of the local Nusselt number and tilt angles. Various studies have used magnetohydrodynamic/hydrodynamic and heat transfer methods to study fluid flow in a porous medium with different geometric configurations [13-15]. Laidoudi et al. [16] numerically examined the heat transfer of a
Newtonian laminar fluid by natural convection contained in two concentric surfaces. The findings confirmed that increasing the tilt angle improves the thermal transfer rate of the inner surface for all values of the aspect ratio. The isotherms and streamlines in an annular double-dimensional channel created by two elliptical confocal cylinders that were differentially heated and contained water-based nanofluid and silver nanoparticles were examined numerically by Bouzerzour et al. [17]. The findings indicate the addition of silver nanoparticles significantly improves the overall thermal transfer compared to the basic fluid, with the impact being more sensitive at higher Rayleigh number. The influences of thermal transfer in a cold outer circular cavity, which contains a hot oriented inner elliptical cylindrical cavity, filled with water and copper-based nanoparticles were studied by Sheikholeslami et al. [18]. The results show that as the nanoparticle volume fraction, thermal Rayleigh number and tilt angle rise, the Nusselt number increases. It was also shown that an increase in the Rayleigh number leads to a reduction of the thermal transfer improvement. The lowest thermal transfer enhancement ratio occurs when the Rayleigh number is high. Mejri et al. [19] studied numerically natural convection inside an inclined triangular cavity containing water. In an outer cold circular enclosure containing an inner inclined hot elliptical circular cylinder filled with air, Ghasemi et al. [20] numerically studied the natural convection. The results revealed that the heat transfer ratio increases as the Rayleigh number increases. In addition, it was noted that the tilt angle has a considerable impact on the thermal transfer ratio depending on the thermal Rayleigh number. Park et al. [21] numerically investigated the normal convection between a cold outer inclined square enclosure and a hot inner circular cylinder. The findings show that the Rayleigh numbers, enclosure tilt angle, and inner cylinder size affect the isotherms, streamlines, and number of cells in the cavity, which significantly improve the thermal transfer ratio. There have also been some worthwhile recent studies on the flow of nanofluids [22-23]. Sheremet et al. [24] investigated numerically the influences of the cylinder radius, tilt angle, Rayleigh number on thermal transfer and fluid flow in a container resulting from the temperature change between a hot inner circular cylinder and a cold outer tilted square container where the tilt angle was confined between 0 and 45 degrees. Mahfouz et al. [25] examined the problem of thermal transfer in an enclosed space created between two elliptical confocal surfaces. It was found that a number of factors, the most important of which are the Prandtl number and the Rayleigh number affect the convection inside the enclosure whose inner wall has been heated. Sultan et al. [26] conducted a comparative study on the effect of three different Cu, TiO₂ and Ag nanofluids on heat transfer. Bouras et al. [27-30] studied natural convection by investigating temperature changes and the Nusselt number for different values of the Rayleigh number in different annular spaces (trapezoidal, square, elliptical and semi-elliptical). They used the Businesque approximation based on the finite volume approach. This work is a numerical investigation on natural convection heating transfer in an inclined elliptical annular envelope containing water and copper-based nanoparticles. The external cold elliptical surface was kept at a fixed temperature $T_c$ and the internal hot elliptical surface was kept at a fixed temperature $T_h$. Volume fraction values and Rayleigh numbers were $0 \leq \phi \leq 0.08$ and $10^3 \leq Ra \leq 10^{5}$. The inclination angles were $\gamma = 30^\circ, 45^\circ, \text{and} 60^\circ$. The effect of tilt the angle, volume fraction and Rayleigh number on the thermal transfer ratio is discussed.

2. Fundamental equations

The physical form of the studied problem is shown in Fig.1, which is an inclined elliptical enclosure containing a water-copper nanofluid. The outer elliptical cylinder is kept at an invariant temperature $T_c$ and the inner elliptical boundary wall is kept at a constant high temperature $T_h$. The flow is two-dimensional laminar.

2.1. Mathematical model

The following are the dimensionless formulations of the equations governing heat transfer in annular space by natural convection:
\[
\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = \theta, \quad (2.1)
\]

a) Cross section with boundary conditions. 

b) The mesh diagram.

\[U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = \frac{\mu_{nf}}{\rho_{nf} \alpha_f} \left( \frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2} \right) - \frac{\partial P}{\partial X}, \quad (2.2)\]

\[U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} = \frac{\mu_{nf}}{\rho_{nf} \alpha_f} \left( \frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2} \right) + \frac{(\rho \beta_{nf})}{\rho_{nf} \beta_f} Ra_f Pr \Theta - \frac{\partial P}{\partial Y}, \quad (2.3)\]

\[U \frac{\partial \Theta}{\partial X} + V \frac{\partial \Theta}{\partial Y} = \frac{\alpha_{nf}}{\alpha_f} \left( \frac{\partial^2 \Theta}{\partial X^2} + \frac{\partial^2 \Theta}{\partial Y^2} \right). \quad (2.4)\]

From their dimensional quantities, the dimensional variables are written in the following form:

\[X = \frac{x}{D_h}, \quad Y = \frac{y}{D_h}, \quad U = \frac{uD_h}{\alpha_f}, \quad V = \frac{vD_h}{\alpha_f}, \quad P = \frac{pD_h^2}{\rho \alpha_f^2}, \]

\[\Theta = \frac{T - T_c}{T_h - T_c}, \quad Pr = \frac{v_f}{\alpha_f}, \quad Ra_t = \frac{g \beta_f (T_h - T_c) D_h^3}{\alpha_f v_f}. \]

2.2. Nusselt number calculation

In the case of the external and internal cylinders, the average and the local Nusselt numbers are determined as follows:

\[Nu_h = \frac{k_{nf}}{k_f} \left( \frac{\partial T}{\partial Y} \right)_{Y=0}, \quad (2.5)\]
\[ N_{u_{c}} = -\frac{k_{nф}}{k_{ф}} \left( \frac{\partial T}{\partial Y} \right)_{Y=1}, \]  
\[ \bar{N}_{u_{avg}} = \frac{1}{D_{h}} \int_{0}^{D_{h}} N_{u_{x}} \, dx. \]  

2.3. Nanofluid properties

Except for the variation of the density, which is determined by employing the Boussinesq equations, Tab.1 shows the thermo-physical properties of the nanofluids, whose values are constant.

Table 1. Thermo-physical characteristics of copper nano-particles and water.

<table>
<thead>
<tr>
<th></th>
<th>[ K \left( \text{W.m}^{-1}\text{K}^{-1} \right) ]</th>
<th>[ C_{p} \left( \text{J.Kg}^{-1}\text{K}^{-1} \right) ]</th>
<th>[ \rho \left( \text{Kg.m}^{-3} \right) ]</th>
<th>[ \mu \left( \text{Kg.m}^{-1}\text{s}^{-1} \right) ]</th>
<th>[ \beta \left( \text{K}^{-1} \right) ]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pure water</td>
<td>0.613</td>
<td>4179</td>
<td>997.1</td>
<td>0.001002</td>
<td>21×10^{-5}</td>
</tr>
<tr>
<td>Copper</td>
<td>401</td>
<td>385</td>
<td>8933</td>
<td>/</td>
<td>1.67×10^{-5}</td>
</tr>
</tbody>
</table>

The thermal expansion coefficient, heat capacity, effective density, thermal diffusivity, efficient heat conductivity and the dynamic viscosity of the nano-fluid are solved as follows:

\[ (\beta \rho)_{nф} = \phi (\beta \rho)_{ф} - (1 + \phi)(\beta \rho)_{ф}, \]  
\[ (\rho C_{p})_{nф} = - (\phi - 1)(\rho C_{p})_{ф} + \phi (\rho C_{p})_{ф}, \]  
\[ (\rho)_{nф} = \phi \rho_{ф} + (1 - \phi) \rho_{ф}, \]  
\[ (\alpha)_{nф} = \frac{K_{nф}}{(\rho C_{p})_{nф}}, \]  
\[ (\mu)_{nф} = \frac{\mu_{ф}}{(1 - \phi)^{2.5}}, \]  
\[ K_{nф} = K_{ф} \frac{\left( K_{ф} + 2K_{ф} \right) - 2\phi \left( K_{ф} - K_{ф} \right)}{\left( K_{ф} + 2K_{ф} \right) + \phi \left( K_{ф} - K_{ф} \right)}. \]

2.4. Boundary condition

Boundary conditions of the present study can be written as follows:

- Outer wall
  \[ V=U= 0, \]
\[ \Theta(X, Y) = 0 . \]  \hspace{1cm} (2.15)

- Inner wall

\[ V = U = 0 , \]  \hspace{1cm} (2.16)

\[ \Theta(X, Y) = 1 . \]  \hspace{1cm} (2.17)

### 2.5. Validation of results

An extensive mesh testing procedure was carried out to look for a mesh-independent solution. There is an increase in the mesh size from $100 \times 100$ to $225 \times 225$. Figure 2 shows the variations in the mean Nusselt numbers of the inner surface with the grid number at $Ra=10^4$, the inclination angle at 30 deg. Consequently, it is decided to select $200 \times 200$.

![Fig.2. Convergence with mesh refinement at $Ra=10^4$ along the thermal internal cylinder for the average Nusselt number.](image)

The numerical results are compared with the case of heat transfer with pure natural convection within two concentric elliptical cylinders [31] and [32] for the validation of the mathematical and numerical model. The mean Nusselt number $Nu_{avg}$ is presented in Tab.2. In both cases, it can be seen that these values are in excellent agreement.

**Table 2. Comparison of our results for the outer and inner average Nusselt numbers with those obtained by other researchers.**

<table>
<thead>
<tr>
<th>$e_1$</th>
<th>$\theta$</th>
<th>$Ra_t$</th>
<th>$Nu_o$</th>
<th>$Nu_o$</th>
<th>$Nu_{avg}$</th>
<th>$Nu_i$</th>
<th>$Nu_{avg}$</th>
<th>$Nu_{avg}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Our results)</td>
<td>(Our results)</td>
<td>[31]</td>
<td>[32]</td>
<td>[31]</td>
<td>[32]</td>
<td>[31]</td>
<td>[32]</td>
<td>[31]</td>
</tr>
<tr>
<td>0.90</td>
<td>0</td>
<td>$10^4$</td>
<td>1.17</td>
<td>1.19</td>
<td>1.15</td>
<td>3.46</td>
<td>3.53</td>
<td>3.54</td>
</tr>
<tr>
<td>0.86</td>
<td>90</td>
<td>$10^4$</td>
<td>1.37</td>
<td>1.35</td>
<td>1.39</td>
<td>3.73</td>
<td>3.68</td>
<td>3.70</td>
</tr>
<tr>
<td>0.86</td>
<td>90</td>
<td>$4 \times 10^4$</td>
<td>1.90</td>
<td>1.93</td>
<td>1.87</td>
<td>5.20</td>
<td>5.34</td>
<td>5.27</td>
</tr>
</tbody>
</table>
3. Results and discussion

In this investigation, we studied the natural convection thermal transfer of a copper-water nanofluid in an inclined-elliptical annular enclosure. The nanoparticle volume fraction range and the Rayleigh number are defined as $0 \leq \phi \leq 0.08$ and $10^3 \leq Ra \leq 10^5$. The angles of inclination are equal to $\gamma = 30^\circ, 45^\circ$ and $60^\circ$. The findings are shown through local Nusselt numbers, current lines and isotherms.

3.1. Isotherms and streamlines

Figures 3, 4 and 5 illustrate the streamlines and isotherms for various values of volume fraction and different inclined enclosure equal to $\gamma = 30^\circ, 45^\circ$ and $60^\circ$, respectively, and for a Rayleigh number equal to $Ra = 10^4$.

\[ \phi = 0\% \]

\[ \phi = 4\% \]

\[ \phi = 8\% \]

Fig. 3. Streamlines and isotherms for different values of the volume fraction at $Ra = 10^4$ and $\gamma = 30^\circ$. 
It has been observed that for all values of the tilt angle and volume fraction of nanoparticles, the isotherms have the shape of a mushroom. The temperature is transmitted to the external elliptical wall from the internal elliptical surface. In the case of an inclined enclosure $\gamma = 30^\circ$ when $\phi = 0$, the flow structure is formed by two cells rotating slowly on opposing directions, and the values of the flow function are very small with values equal to $\psi_{\text{max}} = 0.0028$. For $\phi = 0.04$ and $\phi = 0.08$, the flow structure is formed by two cells that rotate quickly in opposite directions, and the values of the stream function increase significantly to be equal $\psi_{\text{max}} = 0.0036$ and $\psi_{\text{max}} = 0.0045$, respectively. Indeed, it has been noted that when the nanoparticles volume fraction increases the values of the streamlines also rise. In the case of an inclined enclosure $\gamma = 45^\circ$, it was observed that for all values of the volume fraction of nanoparticles $\phi = (0, 0.04 \text{ and } 0.08)$, the flow structure is formed by two cells that rotate rapidly in opposite directions. Under these conditions, the values of the flow
function are very high with values equal to $\psi_{\text{max}} = 0.003$, $\psi_{\text{max}} = 0.004$, and $\psi_{\text{max}} = 0.005$, respectively. In the case, of $\gamma = 60^\circ$ with $\phi = (0, 0.04 \text{ and } 0.08)$, the values of the flux function are $\psi_{\text{max}} = (0.0032, 0.004, \text{ and } 0.005)$, respectively. Thus, it was found that for all tilt angles, if the tilt angle rises, the values of the flux distribution increase. The effect of volume fractions of copper nanoparticles and tilt angle on the flux functions is shown in Figs 3-5. From these results, it can be concluded that as the size of the nanoparticles and the tilt angle increase, the values of streamlines also increase, resulting in an increase in the rate of thermal transfer.

Fig.5. Stream-function and isotherms for various values of the volume fraction at $Ra = 10^4$ and $\gamma = 60^\circ$.

3.2. Heat transfer rate

Figure 6 illustrates the effect of the Rayleigh number on the average Nusselt number at several values of the tilt angle when $\phi = (0, 0.04 \text{ and } 0.08)$. From these results, it was observed that whatever the value of
the inclination angle, the mean Nusselt number increases exponentially with the increase of the Rayleigh number. Indeed, it has been observed that with a small Rayleigh number the thermal transfer is done through conduction at the heated wall.

We can also see that the effect of the inclination angle is most pronounced for larger Rayleigh numbers where the dominant mode of thermal transfer is convection. We can also see that an increase in nanoparticle size enhances the mean Nusselt number.

Figure 6. Influence of the Rayleigh number over the mean Nusselt number at several angles where \( \phi = (0, 0.04 \text{ and } 0.08) \).

Figure 7 presents the mean Nusselt number versus the Rayleigh number variation for different nanoparticle volume fractions and different inclined enclosures \( \gamma = 30^\circ, 45^\circ, \) and \( 60^\circ \). From these results, it has been observed that for each volume fraction value, the mean Nusselt number increases with the rise of the Rayleigh number. It is also noted that the average Nusselt number increases as the volume fraction of nanoparticles increases. Thus, as the volume fraction and the tilt angle increase, the heat transfer rate also increases.

Figures 8 and 9 present the distributions of the local Nusselt number on the outer and inner surfaces of the ring for different tilt angles when \( \phi = 0, \phi = 0.04 \) and \( \phi = 0.08 \) at \( Ra=10^4 \). It has been noticed that the local Nusselt number concerning the outside surface of the annulus is very low when \( 180^\circ < \theta \leq 225^\circ \), while it is very high when \( 0^\circ \leq \theta < 90^\circ \). The local Nusselt number on the inner surface of the annulus is very low when \( 0^\circ < \theta \leq 90^\circ \) while it is very high when \( 180^\circ \leq \theta < 225^\circ \). It is also noted that as the inclination angle increases the local Nusselt number increases. Indeed, it has been noticed that the exterior and interior Nusselt numbers show an inverse distribution.
Fig. 7. Evolution of the average Nusselt number as a function of the Rayleigh number for different volume fractions of nanoparticles when $\gamma = 30^\circ$, $45^\circ$ and $60^\circ$.

Fig. 8. Variation of the local Nusselt number for different tilt angles when $\phi = 0$, $0.04$ and $0.08$ at $Ra=10^4$. 
Fig. 9. Variation of local the Nusselt number for different tilt angles when $\theta = 0$, 0.04 and 0.08 at $Ra=10^4$.

4. Conclusion

In this study, natural convection heat transfer was investigated in an inclined elliptical annular shell filled with a Cu-water nanofluid. The effects of the inclination angle of the elliptical annulus, Rayleigh number and particle size of nanoparticles on the flow and heat transfer properties were investigated.

The obtained results indicate the following:
- For all tilt angles, the streamlines increase with increasing nanoparticle volume fraction.
- Addition of nanoparticles significantly improved the heat transfer.
- The isotherms and the stream functions are not symmetrically distributed for all tilt angles.
- The impact of the inclination angle on the heating transfer rate is more important the higher the Rayleigh number is and convection is the dominant way of heat transfer.
- The mean Nusselt number increases as the Rayleigh number rises for the different inclination angles.
- The thermal transfer ratio rises as the tilt angle increases regardless of the nanoparticle volume fraction values.

These results will be used in industrial applications involving thermal transfer through natural convection in forms similar to the one studied. These effects can also be used to create cockpit heaters and heat exchangers. In the future, it is proposed to study heating and mass transfer between elliptical cylinders, as well as use other nanofluids to find the best ones.

Acknowledgements

The authors thank the management of Department of Physics, Faculty of Sciences, University Mohamed Boudiaf of M'sila for their continuous support to carry out this research work.
Nomenclature

\(A\) – major axe of the elliptic cylinder, \([m]\)
\(a\) – thermal diffusivity, \([m^2/s]\)
\(B\) – minor axis of the elliptic cylinder, \([m]\)
\(C_p\) – specific heat at constant pressure, \([J/kg K]\)
\(e\) – eccentricities of ellipse
\(g\) – gravitational acceleration, \([m/s^2]\)
\(k\) – thermal conductivity, \([W/m K]\)
\(Nu\) – local Nusselt number, \([-]\)
\(Nu_{avg}\) – average Nusselt number, \([-]\)
\(P\) – length or width of the square enclosure, \([m]\)
\(p\) – pressure, \([N/m^2]\)
\(Pr\) – Prandtl number, \([-]\)
\(\rho\) – local fluid density, \([kg/m^3]\)
\(Ra\) – Rayleigh number, \([-]\)
\(T\) – local temperature, \([K]\)
\(T_0\) – reference temperature, \([K]\)
\(T_c,T_h\) – cold and hot wall temperature, \([K]\)
\(\Delta T\) – temperature difference, \([K]\)
\(U\) – velocity component in the \(x\)-direction, \([m/s]\)
\(V\) – velocity component in the \(y\)-direction, \([m/s]\)
\(v\) – Cartesian coordinate in the vertical direction, \([m]\)
\(x\) – Cartesian coordinate in the horizontal direction, \([m]\)
\(\beta\) – volumetric coefficient of thermal expansion, \([K^{-1}]\)
\(\rho_0\) – characteristic density at reference temperature, \([kg/m^3]\)
\(\Omega\) – nanoparticle volume fraction

Subscripts

\(c\) – cold
\(f\) – fluid
\(h\) – hot
\(nf\) – nanofluid
\(p\) – solid particles

References


Received: December 26, 2022
Revised: July 28, 2023