IJME

Int. J. of Applied Mechanics and Engineering, 2023, vol.28, No.1, pp.58-70
DOI: 10.59441/ijame-2023-0006

RAYLEIGH WAVE PROPAGATION IN ISOTROPIC SANDY LAYER SLIDING OVER ISOTROPIC SANDY SEMI-INFINITE MEDIUM WITH SLIDING CONTACT

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The present study aims to investigate Rayleigh wave propagation in an isotropic sandy layer overlying an isotropic sandy semi-infinite medium, with interface considered to be imperfect (slide contact and dislocation like model). Expressions for displacement components are obtained using the variable separation method. The dispersion frequency equation for the Rayleigh wave propagating in sandy media is derived using suitable boundary conditions. Particular cases, such as when the interface is in smooth contact and when sandy media are replaced by elastic media, are also discussed. Using MATLAB software, the effects of the imperfectness parameter (slide contact and dislocation like model) and sandy parameter on the Rayleigh waves' phase velocity are investigated and compared with the already obtained results of the dislocation like model. The present study may find useful applications in geophysics, civil engineering and soil mechanics.

Key words: Rayleigh wave, sandy layer, imperfect interface, dispersion equation, phase velocity.

1. Introduction

Theoretical studies regarding seismic wave propagation play an important role in providing a rich and vast amount of information about the Earth's interior. Studies involving seismic wave propagation are important to seismologists and earthquake engineers since these studies contribute to determining the nature and cause of earthquakes as well as understanding the earth's crust.

Materials of the Earth may not always be isotropic and elastic. The Earth's crust. consisting of sedimentary layers is not perfectly elastic but can be considered as sandy layers. A sandy layer may be defined as the layer consisting of sand particles not retaining moisture or water vapour. Sand boils occurred due to the 7.1 Richter scale Loma Prieta earthquake of 1989, causing the liquefaction of superficial sandy materials. So, the assumption of sandy layer plays an important role in predicting seismic behavior and is very important to seismologists. Various researchers studied seismic or Rayleigh wave propagation in sandy or elastic media. Rayleigh waves propagation considering isotropic an elastic solid and stratified media was investigated by Rayleigh [1] and Bromwich [2]. Weiskopf [3] explored the dynamics for sandy soil introducing the sandiness parameter 'η' and gave the relationship $\frac{E}{\mu}=2\eta(1+\nu)$ where μ, ν and E denote Lame's constant, Poisson's ratio and Young's modulus respectively. Kar *et al.* [4] studied Love wave propagation in a sandy medium, discussing the effects of irregularity. Abd-Alla [5] investigated Rayleigh wave propagation considering an orthotropic elastic half-space. Kuznetsov [6] investigated Love waves propagation in layered monoclinic media. Dispersion relations were derived using the Modified Transfer Matrix method. Effects of gravity field and initial stresses on Rayleigh wave propagation considering magnetoelastic half-space were discussed by Abd-Alla *et al.* [7]. Viswakarma and Gupta [8] investigated Rayleigh wave propagation in the Earth's crustal layer for the sandy and elastic half-space cases, obtaining the effects of inhomogeneity and rigid boundary. Pal *et al.* [9] explored Rayleigh wave propagation considering a sandy half-space and an anisotropic layer and

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derived the Rayleigh wave propagation dispersion frequency equation. Sahu *et al.* [10] examined Rayleigh wave propagation considering an orthotropic half-space with impacts of pre-stresses and self-weight and a liquid layer. Kuznetsov [11] investigated variation of Stoneley waves velocity using generalized Wiechert condition by introducing two dimensionless parameters instead of one.

Rayleigh wave propagation in a geometry containing sandy media surrounded by couple stress media and orthotropic half-space was investigated by Mandi *et al.* [12]. Rayleigh wave propagation considering two cases, a heterogeneous sandy layer overlying an isotropic elastic half-space and isotropic elastic layer overlying on isotropic half-space with perfect contact was investigated by Kuznetsov [13]. Kuznetsov [14] investigated acoustic guided wave propagation in stratified media consisting of a sandy layer bounded by an isotropic layer and isotropic half-space.

As the Earth is a layered medium, various interfacial conditions such as irregularities or an imperfect interface do have a significant effect on seismic wave propagation and such studies provide a rich amount of information regarding the seismic behavior of the Earth. This imperfectly bonded surface is the actual contact between the layer and half-space as bonding between the interfaces are often affected by various environmental factors and thermal/mechanical loadings. Various researchers studied effects of these interfacial conditions. Hua *et al.* [15] studied effects of the imperfectness parameter on propagation of Love wave considering a geometry consisting of layered graded composite structures. Rayleigh wave propagation in an orthotropic elastic layer overlying an orthotropic elastic space with the interface assumed to be in spring contact finite sliding contact was investigated by Vinh and Anh [16] while Kaur [17] considered same geometry but with sliding contact. Vishwakarma and Xu [18] investigated dispersion of Rayleigh wave considering a sandy layer overlying an orthotropic mantle. Effects of irregular boundaries on upper plane were discussed observing the initial stress and sandiness parameter effect. Dispersion equation for SH wave propagation in a layer of viscoelastic overlying a couple stressed substrate with the interface assumed to be imperfect was derived by Sharma and Kumar [19]. They described the effects of imperfectness, heterogeneity, friction and imperfectness parameter. Kumar *et al.* [20] examined shear wave propagation considering a micropolar elastic half-space and a piezoelectric layer under the effects of initial stresses with an imperfect interface. Kumar and Madan [21] discussed Love wave propagation considering a layer consisting of sand particles overlying an orthotropic semi-infinite media and discussed the effects of imperfectness and sandy parameter. Madan *et al.* [22] investigated propagation of Rayleigh wave an considering orthotropic elastic medium under effects of pre-stresses. An explicit secular equation for perfect and sliding contact was derived.

Effects of imperfect interfacial conditions (arising due to thermal/mechanical loadings or environmental factors) for Rayleigh wave propagation in sandy media remains yet unexplored. So, an effort has been made to study propagation of Rayleigh wave in a geometry comprised of an isotropic sandy layer and isotropic sandy semi-infinite medium. The interface is assumed to be in sliding contact as various interfacial conditions have a significant effect on seismic waves propagation. A parameter G (sliding parameter) with $0 \leq G \leq 1$ is introduced whose extreme values correspond to smooth and perfect contact, respectively. Dispersion equation has been derived for Rayleigh wave propagation using appropriate boundary conditions. Particular cases for smooth contact and for an isotropic layer with sliding contact have also been discussed. MATLAB software has been used for plotting phase velocity against wave number to demonstrate graphically the significant effects of various parameters (sliding and sandiness) involved in the dispersion equation for the sliding contact and dislocation like model.

2. Geometry of the problem

The model comprised of an isotropic sandy layer with thickness 'h', overlying an isotropic sandy semi-infinite medium is considered. The Cartesian co-ordinate system is used for the study. The Rayleigh wave is assumed to be propagating along the x-direction, the z-axis is taken in the increasing depth direction with origin O located at the layer and semi-infinite medium interface. The layer and semi-infinite medium are assumed to be in sliding contact. This contact is shown by using a sliding parameter denoted by $G\left(0 \leq G \leq 1\right)$ and $G = 0$

corresponds to perfect contact and $(0<G<1)$ corresponds to finite sliding contact The geometry for the considered problem is shown in Fig.1.

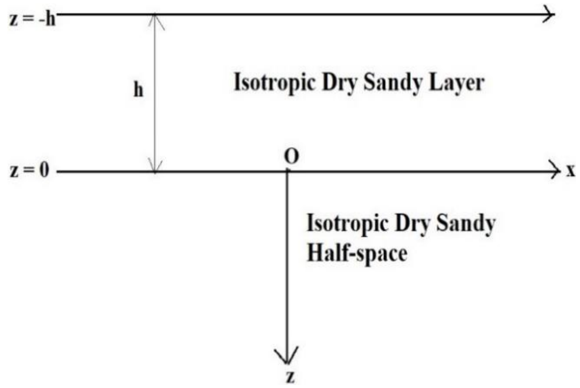

Fig.1. Geometry of the problem.

For a 2-dimensional problem (-*xz* plane), displacements components are assumed independent of *y*, i.e. $\frac{\partial}{\partial y}\approx 0$ and are zero in -*y*-direction.

3. Dynamics of sandy layer and semi-infinite medium

For Rayleigh wave propagation, the dynamical equation of motion without external forces for displacement components u_1 and w_1 along the *x* and *z* direction for a sandy layer is given as (Biot [23]):

$$\frac{\partial\tau^1_{xx}}{\partial x}+\frac{\partial\tau^1_{xz}}{\partial z}=\rho_1\frac{\partial^2 u_1}{\partial t^2}\,, \tag{3.1}$$

$$\frac{\partial\tau^1_{xx}}{\partial x}+\frac{\partial\tau^1_{xz}}{\partial z}=\rho_1\frac{\partial^2 w_1}{\partial t^2} \tag{3.2}$$

where $\tau^1_{xx},\tau^1_{xz}\ ,\tau^1_{zz}$ denotes stress components and ρ_1 denotes the density of material in the sandy layer.
We use stress-displacement relations for the sandy layer (Weiskopf [3]):

$$\tau^1_{xx}=\eta_1\left\{(\lambda_1+2\mu_1)\frac{\partial u_1}{\partial x}+\lambda_1\frac{\partial w_1}{\partial z}\right\}, \tag{3.3}$$

$$\tau^1_{zx}=\eta_1\mu_1\left(\frac{\partial u_1}{\partial z}+\frac{\partial w_1}{\partial x}\right)\left\{(\lambda_1+2\mu_1)\frac{\partial u_1}{\partial x}+\lambda_1\frac{\partial w_1}{\partial z}\right\}, \tag{3.4}$$

$$\tau^1_{zz}=\eta_1\left\{\lambda_1\frac{\partial w_1}{\partial z}+(\lambda_1+2\mu_1)\frac{\partial u_1}{\partial x}\right\} \tag{3.5}$$

where λ_1 and μ_1 denote Lame’s constant, η_1 denotes sandiness parameter.
Using Eqs (3.3), (3.4) and (3.5) in (3.1) and (3.2), we have

$$\eta_1(\lambda_1+2\mu_1)\frac{\partial^2 u_1}{\partial x^2}+\eta_1\mu_1\frac{\partial^2 u_1}{\partial z^2}+\eta_1(\lambda_1+\mu_1)\frac{\partial^2 w_1}{\partial x\partial z}=\rho_1\frac{\partial^2 u_1}{\partial t^2}\,, \tag{3.6}$$

$$\eta_1\frac{\partial^2 w_1}{\partial x^2}+\eta_1(\lambda_1+2\mu_1)\frac{\partial^2 w_1}{\partial z^2}+\eta_1(\lambda_1+\mu_1)\frac{\partial^2 u_1}{\partial x\partial z}=\rho_1\frac{\partial^2 w_1}{\partial t^2}\,. \tag{3.7}$$

Now, assume solution of Eqs (3.6) and (3.7) to be:

$$u_1=\left(Ae^{-kpz}+Be^{kpz}\right)e^{ik(x-ct)}\,, \tag{3.8}$$

$$w_1=\left(Ce^{-kpz}+De^{kpz}\right)e^{ik(x-ct)}\,. \tag{3.9}$$

Using values of u_1 and w_1 in Eqs (3.6) and (3.7) and separating coefficients of e^{-kpz} and e^{kpz}, we have:

$$\left[\rho_1c^2-\eta_1(\lambda_1+2\mu_1)+\eta_1\mu_1p^2\right]A_1-i\eta_1(\lambda_1+\mu_1)pC_1=0\,, \tag{3.10}$$

$$\left[\rho_1c^2-\eta_1(\lambda_1+2\mu_1)+\eta_1\mu_1p^2\right]B_1+i\eta_1(\lambda_1+\mu_1)pD_1=0\,, \tag{3.11}$$

$$\left[\rho_1c^2-\eta_1\mu_1+(\lambda_1+2\mu_1)\eta_1p^2\right]C_1-i\eta_1(\lambda_1+\mu_1)pA_1=0\,, \tag{3.12}$$

$$\left[\rho_1c^2-\eta_1\mu_1+(\lambda_1+2\mu_1)\eta_1p^2\right]D_1+i\eta_1(\lambda_1+\mu_1)pB_1=0\,. \tag{3.13}$$

Writing Eqs (3.10)-(3.13) in a determinant form in order to eliminate A_1, B_1, C_1 and D_1, we must have:

$$\begin{vmatrix} \rho_1c^2-\eta_1(\lambda_1+2\mu_1)+\eta_1\mu_1p^2 & 0 & -i\eta_1(\lambda_1+\mu_1)p & 0 \\ 0 & \rho_1c^2-\eta_1(\lambda_1+2\mu_1)+\eta_1\mu_1p^2 & 0 & -i\eta_1(\lambda_1+\mu_1)p \\ -i\eta_1(\lambda_1+\mu_1)p & 0 & \rho_1c^2-\eta_1(\lambda_1+2\mu_1)+\eta_1\mu_1p^2 & 0 \\ 0 & -i\eta_1(\lambda_1+\mu_1)p & 0 & \rho_1c^2-\eta_1(\lambda_1+2\mu_1)+\eta_1\mu_1p^2 \end{vmatrix}=0$$

On expanding the determinant, we obtain a biquadratic equation in p, given as

$$\begin{aligned}&\eta_1^2\frac{\beta_1^2}{\alpha_1^2}p^4+\left\{\eta_1^2\left(1-\frac{\beta_1^2}{\alpha_1^2}\right)^2+\eta_1\left(\frac{c^2}{\alpha_1^2}-\eta_1\right)+\eta_1\frac{\beta_1^4}{\alpha_1^4}\left(\frac{c^2}{\beta_1^2}-\eta_1\right)\right\}p^2+\\&+\frac{\beta_1^2}{\alpha_1^2}\left(\frac{c^2}{\alpha_1^2}-\eta_1\right)\left(\frac{c^2}{\beta_1^2}-\eta_1\right)=0\end{aligned} \tag{3.14}$$

where

$$\alpha_1^2=\frac{(\lambda_1+2\mu_1)}{\rho_1} \quad \text{and} \quad \beta_1^2=\frac{\mu_1}{\rho_1}\,.$$

Let us assume $\pm p_1$ and $\pm p_2$ to be solution of Eq.(3.14), then displacement expression in Eqs (3.8) and (3.9) can be written as:

$$u_1 = \left(A_1 e^{-kp_1 z} + A_2 e^{-kp_2 z} + B_1 e^{kp_1 z} + B_2 e^{kp_2 z} \right) e^{ik(x-ct)}, \tag{3.15}$$

$$w_1 = \left(n_1 A_1 e^{-kp_1 z} + n_2 A_2 e^{-kp_2 z} - n_1 B_1 e^{kp_1 z} - n_2 B_2 e^{kp_2 z} \right) e^{ik(x-ct)} \tag{3.16}$$

where

$$C_j = n_j A_j, \qquad D_j = -n_j B_j$$

and

$$n_j = \frac{\dfrac{c^2}{\alpha_1^2} - \eta_1 + \eta_1 \dfrac{\beta_1^2}{\alpha_1^2} p_j^2}{i\eta_1 \left(1 - \dfrac{\beta_1^2}{\alpha_1^2} \right) p_j} \quad (j = 1,2),$$

with

$$p_1 = \sqrt{1 - \frac{c^2}{\beta_1^2}} \quad \text{and} \quad p_2 = \sqrt{1 - \frac{c^2}{\alpha_1^2}}. \tag{3.17}$$

In a similar manner the displacement expression for the sandy semi-infinite medium can be written as:

$$u_2 = \left(E_1 e^{-kq_1 z} + E_2 e^{-kq_2 z} + F_1 e^{kq_1 z} + F_2 e^{kq_2 z} \right) e^{ik(x-ct)}, \tag{3.18}$$

$$w_2 = \left(n_1' E_1 e^{-kq_1 z} + n_1' E_2 e^{-kq_2 z} - n_1' F_1 e^{kq_1 z} - n_2' F_2 e^{kq_2 z} \right) e^{ik(x-ct)} \tag{3.19}$$

where

$$n_j' = \frac{\dfrac{c^2}{\alpha_2^2} - \eta_2 + \eta_2 \dfrac{\beta_2^2}{\alpha_2^2} q_j^2}{i\eta_2 \left(1 - \dfrac{\beta_2^2}{\alpha_2^2} \right) q_j} \quad (j = 1,2),$$

with

$$q_1 = \sqrt{1 - \frac{c^2}{\beta_2^2}}, \qquad q_2 = \sqrt{1 - \frac{c^2}{\alpha_2^2}}, \qquad \alpha_2^2 = \frac{(\lambda_2 + 2\mu_2)}{\rho_2}$$

and

$$\beta_2^2 = \frac{\mu_2}{\rho_2}. \tag{3.20}$$

Also, we have displacement vanishing as the depth increases, i.e. u_2, $w_2 \to 0$ as $z \to \infty$. So, F_1 and F_2 must be zero. Then from Eqs (3.18) and (3.19):

$$u_2 = \left(E_1 e^{-kq_1 z} + E_2 e^{-kq_2 z}\right) e^{ik(x-ct)}, \tag{3.21}$$

$$w_2 = \left(n_1' E_1 e^{-kq_1 z} + n_2' E_2 e^{-kq_2 z}\right) e^{ik(x-ct)}. \tag{3.22}$$

Equations (3.21) and (3.22) represent expressions for displacement components for the semi-infinite medium.

4. Boundary conditions and dispersion equation

To examine Rayleigh wave propagation in an isotropic sandy layer overlying an isotropic sandy semi-infinite medium with sliding contact at the interface, a parameter, $G(0 \le G \le 1)$ termed as the sliding parameter, is introduced. The boundary conditions involving stresses, displacements and the sliding parameter may be presented as:

i. Displacement vanishes at the rigid upper boundary plane. i.e. $u_1 = 0$ and $w_1 = 0$ at $z = -h$.

ii. For the slide contact at the interface $z = 0$, appropriate conditions are

$$(1-G)\tau_{zx}^2 + FkGu_2 = FkGu_1; \qquad w_1 = w_2,$$

$$\tau_{zx}^1 = G\tau_{zx}^2; \qquad \tau_{zz}^1 = \tau_{zz}^2 \qquad \text{at} \qquad z = 0.$$

where F is a constant quantity having dimension force per unit area.

Applying the boundary conditions and using expressions for displacements and stresses for the sandy layer and semi-infinite medium, we obtain the following set of six homogeneous equations in terms of A_1, A_2, B_1, B_2, E_1 and E_2:

$$A_1 e^{kp_1 h} + A_2 e^{kp_2 h} + B_1 e^{-kp_1 h} + B_2 e^{kp_2 h} = 0, \tag{4.1}$$

$$A_1 n_1 e^{kp_1 h} + A_2 n_2 e^{kp_2 h} - B_1 n_1 e^{-kp_1 h} - B_2 n_2 e^{kp_2 h} = 0, \tag{4.2}$$

$$\begin{aligned} &A_1 FG + A_2 FG + B_1 FG + B_2 FG + E_1 \left\{(1-G)\left(q_1 - in_1'\right)\eta_2\mu_2 - FG\right\} + \\ &+ E_2 \left\{(1-G)\left(q_2 - in_2'\right)\eta_2\mu_2 - FG\right\} = 0 \end{aligned} \tag{4.3}$$

$$A_1 n_1 + A_2 n_2 - B_1 n_1 - B_2 n_2 - n_1' E_1 - n_2' E_2 = 0, \tag{4.4}$$

$$\begin{aligned} &A_1 \eta_1\mu_1 (-p_1 + in_1) + A_2 \eta_1\mu_1 (-p_2 + in_2) + B_1 \eta_1\mu_1 (p_1 - in_1) + \\ &+ B_2 \eta_1\mu_1 (p_2 - in_2) + E_1 G\eta_2\mu_2 \left(q_1 - in_1'\right) + E_2 \eta_2\mu_2 G\left(q_2 - in_2'\right) = 0, \end{aligned} \tag{4.5}$$

$$\begin{aligned}&A_1\eta_1\left\{\lambda_1 i-(\lambda_1+2\mu_1)n_1p_1\right\}+A_2\eta_1\left\{\lambda_1 i-(\lambda_1+2\mu_1)n_2p_2\right\}+\\&+B_1\eta_1\left\{\lambda_1 i-(\lambda_1+2\mu_1)n_1p_1\right\}+B_2\eta_1\left\{\lambda_1 i-(\lambda_1+2\mu_1)n_2p_2\right\}-\\&+E_1\eta_2\left\{\lambda_2 i-(\lambda_2+2\mu_2)n_1'q_1\right\}-E_2\eta_2\left\{\lambda_2 i-(\lambda_2+2\mu_2)n_2'q_2\right\}=0.\end{aligned}\tag{4.6}$$

To get the non-trivial solution of this homogeneous system from (4.1)-(4.6), we must have,

$$\left|a_{ij}\right|=0,\ i,\ j=1 \text{ to } 6 \tag{4.7}$$

where a_{ij} are coefficients of A_1, A_2, B_1, B_2, E_1 and E_2 in the system of six equations represented by Eqs (4.1)-(4.6). The real part of Eq.(4.7) represents the dispersion frequency equation for the Rayleigh wave propagating in an isotropic sandy layer overlying an isotropic sandy semi-infinite medium with sliding contact.

5. Special Cases

Case I. When $G=0$, then Eq.(4.7) represents the frequency equation for Rayleigh wave propagation in a sandy layer overlaying a sandy semi-infinite medium with smooth contact.

Case II. When $\eta_1=\eta_2=1$, then Eq.(4.7) becomes the frequency equation for propagation of Rayleigh wave in an isotropic elastic layer overlaying an isotropic elastic semi-infinite medium with a sliding contact interface.

Case III. When $\eta_1=\eta_2=G=1$, then Eq.(4.7) becomes the frequency equation for Rayleigh wave propagation in an isotropic elastic layer overlying an isotropic elastic semi-infinite medium with a perfect interface.

6. Numerical computations and discussion

Propagation of the Rayleigh wave considering a geometry comprised of an isotropic sandy layer and isotropic sandy semi-infinite medium has been examined, with the interface assumed to be in sliding contact. A parameter *G* (sliding parameter) with $0 \le G \le 1$ is introduced such that $G=0$ corresponds to smooth contact of the layer and half-space, $G=0$ corresponds to perfect contact and $0<G<1$ corresponds to finite sliding contact. The frequency equation, involving various parameters such as the sliding parameter, sandiness, has been derived for the considered model. In addition, the previously obtained frequency equation for Rayleigh waves in the dislocation type model is used to compare the sliding contact, smooth contact and dislocation type imperfection model (η_T, η_N denoting the imperfectness parameter in tangential and normal direction). To examine the effects of various parameters, the following data have been considered (Gubbins [24], at the depth of *10km* and *500 km* approximately):
For the sandy layer:

$$\rho_1=2802\,kg/m^3,\quad \mu_1=32.3\,GPa,\quad \lambda_1=42.9\,GPa.$$

For the sandy half-space:

$$\rho_2=3865\,kg/m^3,\quad \mu_2=104.6\,GPa,\quad \lambda_2=154.5\,GPa.$$

Graphs are plotted to show the effects of the sliding parameter, sandiness parameter for the layer and half-space on phase velocity variation against the wave number using the real part of Eq.(4.7) with help of

MATLAB software. Fundamental modes are plotted to observe the effects. Graphs are also compared with the dislocation like imperfection model.

Figure 2 shows the effects of the sliding contact (*G*) and imperfectness parameter (η_T, η_N). Variation of phase velocity against the wave number to observe the effects of the sliding and dislocation parameter is shown for the three cases, i) for sliding contact ii) for dislocation-type model iii) for sliding contact but when the layer and half-space becomes isotropic elastic, i.e. $\eta_1 = \eta_2 = 1$. Three different values of the sliding parameter and dislocation-type imperfectness parameter are used for plotting with values of η_1 and η_2 taken as *1.5* for Fig. 2(a). It can be seen that phase velocity decreased with increase in sliding and dislocation parameter.

Figure 3 shows the effects of the sandiness factor (η_1) for the layer. Variation is shown for the sliding contact, smooth contact and dislocation-type model, respectively. Phase velocity is plotted against the wave number using three values of η_1 with value of $G = 0.5$ and *0* and $\eta_T = \eta_N = 0.4$ for Fig. 3(a),(b) and (c). The sandiness parameter for the layer first increased but then decreased the phase velocity of the Rayleigh wave for the sliding and smooth contact model but it increased for the dislocation-type model.

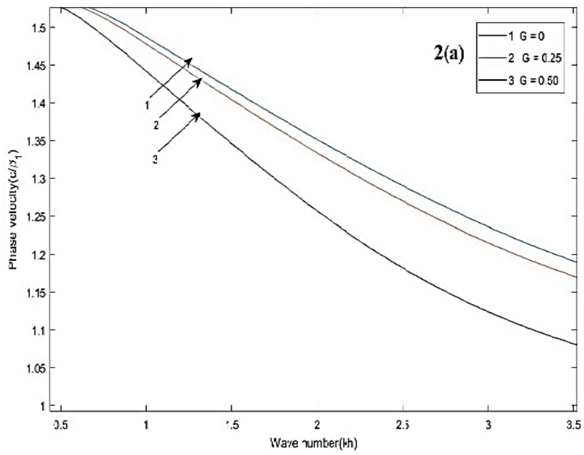

Fig.2a. Imperfectness parameter effect, using variations of phase velocity (c/β_1) against wave number (kh) for sandy media case with sliding contact

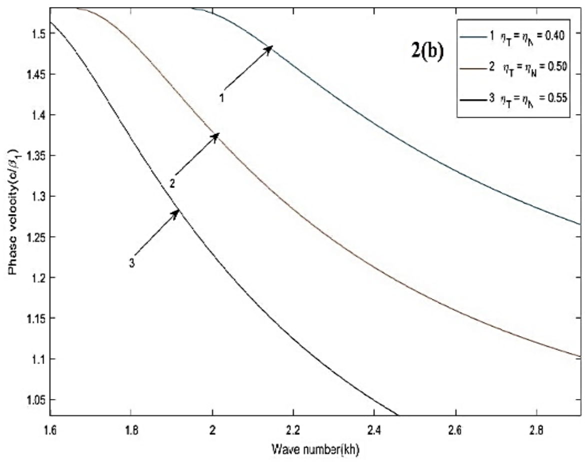

Fig.2b. Imperfectness parameter effect, using variations of phase velocity (c/β_1) against wave number (kh) for sandy media with dislocation-type model, with sliding contact.

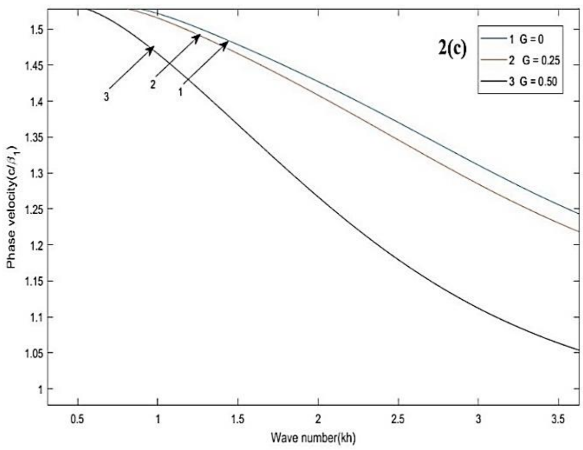

Fig.2c. Imperfectness parameter effect, using variations of phase velocity (c/β_1) against wave number (kh) for for isotropic elastic case $(\eta_1 = \eta_2 = 1)$, with sliding contact.

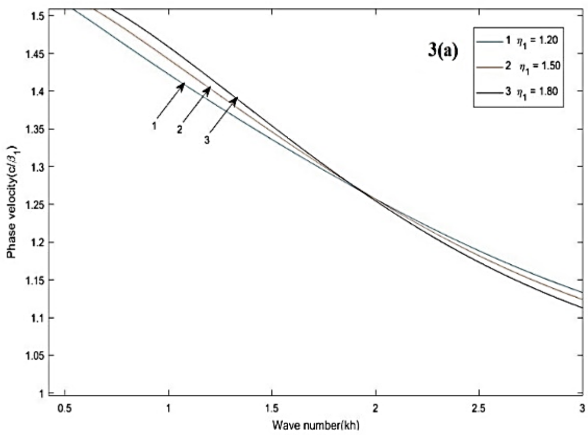

Fig.3a. Sandy parameter (η_1) effect, using phase velocity (c/β_1) variations against wave number (kh) for sliding contact.

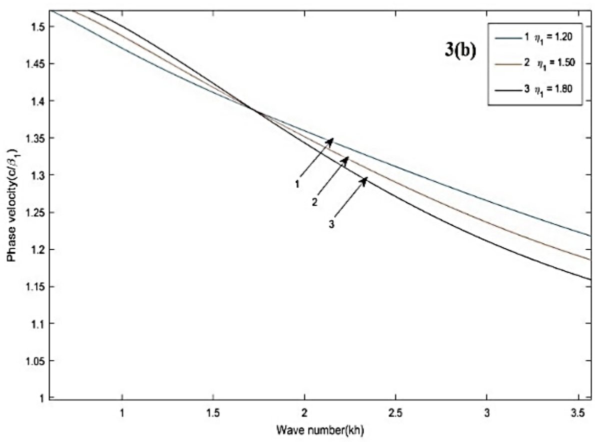

Fig.3b. Sandy parameter (η_1) effect, using phase velocity (c/β_1) variations against wave number (kh) for smooth contact.

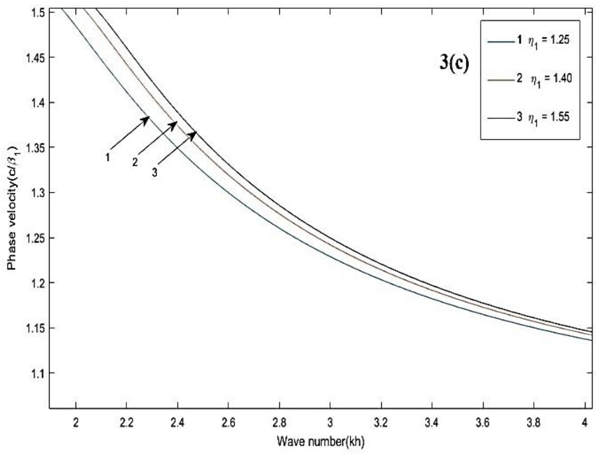

Fig.3c. Sandy parameter (η_1) effect, using phase velocity (c/β_1) variations against wave number (kh) for dislocation like model.

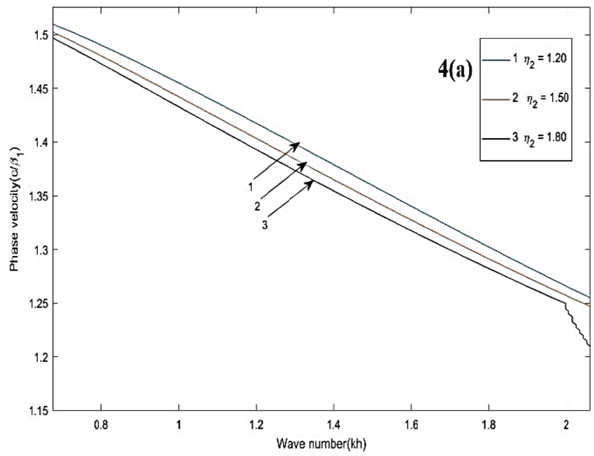

Fig.4a. Effects of sandy parameter (η_2) using phase velocity (c/β_1) variations against wave number (kh) for sliding contact.

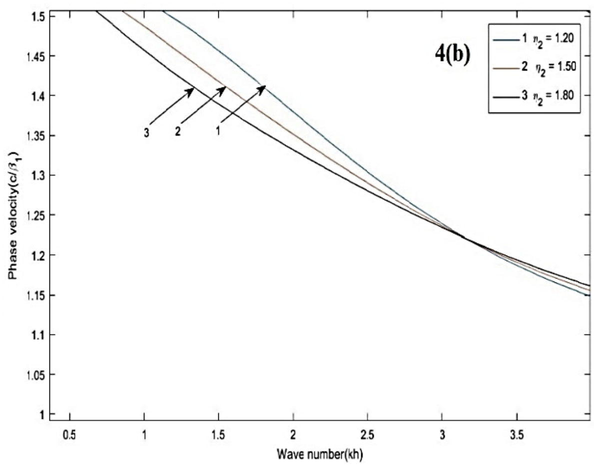

Fig.4b. Effects of sandy parameter (η_2) using phase velocity (c/β_1) variations against wave number (kh) for smooth contact.

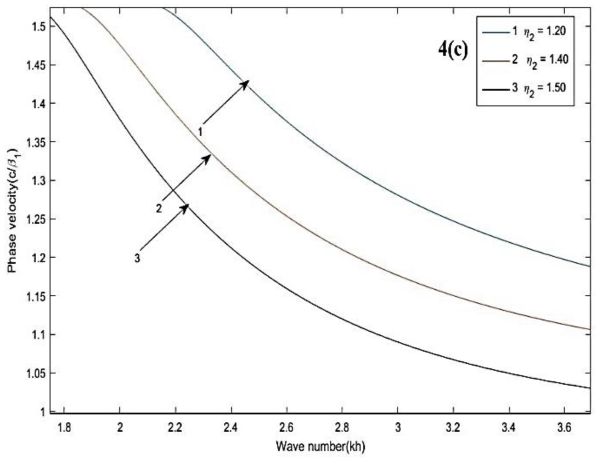

Fig.4c. Effects of sandy parameter (η_2) using phase velocity (c/β_1) variations against wave number (kh) for dislocation like model.

Figure 4 shows the effects of the sandiness parameter (η_2) for the semi-infinite medium. Variation is shown for the contact, smooth contact and dislocation like model respectively using three different values of $\eta_2 = 1$ with value of $G = 0.5$ and 0 and η_T and $\eta_N = 0.5$ for Fig. 4(a), (b) and (c). It has been found that the sandiness parameter acts against the Rayleigh wave phase velocity for the sliding and dislocation-type model but shows a different behaviour for smooth contact.

7. Conclusion

A mathematical analysis of Rayleigh wave propagation in a sandy layer overlying a sandy half-space has been studied with the interface assumed to be imperfect dislocation like model. The dispersion frequency equation has been obtained by applying appropriate boundary conditions for the geometry. Earlier obtained results for Rayleigh wave propagation in sandy media for dislocation type model are compared with the results of sliding contact. Graphs have been plotted to show the impacts of imperfectness (both sliding contact and dislocation type model) and the sandiness factor (η_1 and η_2) on Rayleigh waves phase velocity. The conclusions for can be summarized as:

1) The imperfectness parameter acts against the phase velocity of the Rayleigh wave for the isotropic sandy layer and isotropic elastic layer case.
2) The sandiness parameter for the layer has significantly affected the phase velocity. As the sandiness parameter increases, the phase velocity increases for some wave number value but then decreases both for sliding and smooth contact and phase velocity increases for dislocation-type model.
3) The sandiness parameter for the semi-infinite medium also significantly affected the phase velocity of Rayleigh waves for sliding contact, smooth contact and dislocation-type model.

Theoretical studies regarding seismic wave propagation considering layered media have various applications in geophysics, civil engineering and in understanding the effects and causes of earthquakes. The Rayleigh wave causes more damage during earthquakes in comparison to other surface waves due to its ground roll motion. The present results could be used in field applications.

Acknowledgement

The author(s) would like to thank the reviewer for kind and valuable suggestions to improve the quality of the paper. One of the authors, Naveen Kumar, is thankful to the University Grants Commission (UGC), New Delhi, for providing fellowship in the form of JRF.

Nomenclature

u_i, w_i – displacement components for the layer and half-spaces for $i = 1,2$ respectively

η_1 , η_2 – sandiness parameter for the layer and half-space

ρ_1, ρ_2 – density of material in the layer and half-space

λ_i, μ_i – Lame's constants for the layer and half-space for $i = 1,2$ respectively

$\tau_{xx}^{(i)}, \tau_{xx}^{(i)}$, $\tau_{zz}^{(i)}$ – stress components for the layer and half-spaces for $i = 1,2$ respectively

G – sliding contact parameter

η_T , η_N – imperfectness parameter along tangential and normal direction

kh – wave number

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Received: September 9, 2022
Revised: January 4, 2023