MAGNETOHYDRODYNAMIC FLOW AROUND AN ELONGATED CYLINDER IN NON-DARCIAN POROUS REGIME WITH HIGHER ORDER CHEMICAL REACTION AND SORET/DUFOUR EFFECTS

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Here, we consider magnetohydrodynamic flow of an incompressible, time independent fluid past an elongated cylinder surrounded in a non-Darcian porous regime with magnetic flux supplied at an acute angle. The Soret/Dufour effects and the higher order chemical reactions are also included in the present study. The subsequent governing equations are resolved using the MATLAB-bvp4c method. The flow velocity appears to decrease with the growth of the Reynolds number, inertia parameter, magnetic field and angle of inclination of the magnetic flux, but improves with the Darcy number. The inertia parameter enhances the fluid temperature and skin friction. Further order of chemical reaction, Soret/ Dufour number plays a significant role in the system.

Key words: inertia parameter, magnetohydrodynamic, Soret number, Dufour number, porous medium, elongating cylinder, bvp4c.

1. Introduction


Chemical reactions between foreign bodies and liquids occur in many important technological processes such as fuel combustion, iron production, glass and ceramic production, etc. Many industrial processes involve the transfer of flow and mass through the surface. Dispersed species can be produced or engrossed due to certain chemical species reacting with the surrounding fluid which can significantly affect the flow rate and therefore the characteristics and standard of the ultimate outcome. Cortell [7] examined the influence of chemical reaction on a time independent second-grade fluid flow past a semi-infinite impervious elongated sheet. Hayat et al. [8], Das [9] investigated the significance of chemical reaction for non-Newtonian liquids. M. Ramzan et al. [10] discussed the MHD fluid flow with the inclusion of a melting heat transfer with nonlinear chemical reaction across a slim needle in a porous system. Sarojamma et al. [11], Ly et al. [12], Shafique et al. [13] studied the impact of linear chemical reactions and authors like Sharma and Borgohain [14], Mythili and Sivaraj [15], Hosseinzadeh [16], Muthamilselvan [17], Ramzan et al. [18] considered the impact of nonlinear chemical reaction on different flow problems.

The Soret effect occurs due to mass flux induced by heat differences while the Dufour effect occurs due to concentration gradient. The Soret effects associated with heat transfer and mass transfer with the consideration of radiation and magnetic field of a convective flow passing through a permeable stretched sheet

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were analysed by El-Aziz [19]. Cheng [20] conducted a study on the Soret/Dufour effects in a free convection flow along a porous medium. EL-Kabeir [21] investigated the Soret/Dufour impacts on a stretched sheet using chemically reactive species. A numerical approach to chemically reacting laminar flow was carried out by Makinde et al. [22]. Mathematical investigation of a magnetohydrodynamic flow through upright permeable plates with the Soret and Dufour effect was carried out by Hasan and Hossain [23]. Recently, Das and Dorjee [24], Kumara et al. [25], Das [26], Balla et al. [27], and Kodi and Mopuri [28] contributed to this study.

In this study, our aim is to investigate the influence of the non-Darcian and non-linear chemical reaction on an incompressible magnetohydrodynamic flow over an elongated cylinder surrounded in a porous regime. This paper is an extension of the work of Sharma and Borgohain [14] which incorporates the influences of an inclined magnetic field and non-Darcian porous medium.

2. Mathematical formulation

Consider a magnetohydrodynamic flow of a time independent, incompressible viscous fluid in an elongated cylinder surrounded by a non-Darcian porous regime. The magnetic flux is supplied at an acute angle $\beta$ with a strength $B_0$ and a $n$-th order chemical reaction takes place in the presence of the heat source/sink parameter. The $z^*$-axis is taken along the cylinder axis and the $r^*$-axis is chosen in a radial direction and the cylinder radius is taken as $b$ in the radial direction (Fig.1). Here, Forchheimer’s extension is employed to designate the fluid flow in the permeable medium. The ambient temperature away from the cylinder surface is $T_{\infty}^*$ and the cylinder surface is kept at a constant temperature $T_W^*$ with $T_W^* > T_{\infty}^*$.

![Fig.1. Physical diagram.](image)

The governing equations, under the above assumptions (following [14]) become:

$$\frac{\partial w^*}{\partial z^*} + u^* \frac{\partial w^*}{\partial r^*} = 0,$$

$$w^* \frac{\partial w^*}{\partial z^*} + u^* \frac{\partial w^*}{\partial r^*} = \frac{\partial^2 w^*}{\partial r^*^2} + \frac{\nu}{\sqrt{k}} \frac{\partial w^*}{\partial r^*} - \frac{C_b}{\rho} w^* \frac{\sigma B_0^2}{\rho} \sin^2 \beta - \frac{\nu}{K} w^*, $$

$$w^* \frac{\partial u^*}{\partial z^*} + u^* \frac{\partial u^*}{\partial r^*} = - \frac{1}{\rho} \frac{\partial p^*}{\partial r^*} + \nu \frac{\partial^2 u^*}{\partial r^*^2} + \frac{\nu}{\sqrt{k}} \frac{\partial u^*}{\partial r^*} - \frac{\nu}{K} u^*, $$
\[ w^* \frac{\partial T^*}{\partial z} + u^* \frac{\partial T^*}{\partial r} = \alpha \left( \frac{\partial^2 T^*}{\partial r^2} + \frac{l}{r} \frac{\partial T^*}{\partial r} \right) + \frac{DK_T}{C_s C_p} \left( \frac{\partial^2 C^*}{\partial r^2} + \frac{l}{r} \frac{\partial C^*}{\partial r} \right) + \frac{Q}{\rho c_p} (T^* - T_{\infty}^*), \quad (2.4) \]

\[ w^* \frac{\partial C^*}{\partial z} + u^* \frac{\partial C^*}{\partial r} = D \frac{\partial^2 C^*}{\partial r^2} + D \frac{l}{r} \frac{\partial C^*}{\partial r} + \frac{DK_T}{T_m} \left( \frac{\partial^2 T^*}{\partial r^2} + \frac{l}{r} \frac{\partial T^*}{\partial r} \right) - k_f \left(C^* - C_{\infty}^*\right)^n. \quad (2.5) \]

Boundary conditions are:

\[ u^* = 0, \quad w^* = W_W^*, \quad T^* = T_W^*, \quad \frac{dT^*}{dr} + \frac{K_T}{T_m} \frac{dT^*}{dr} = 0 \quad \text{at} \quad r^* = b, \]

\[ w^* \to 0, \quad T^* \to T_{\infty}^*, \quad C^* \to C_{\infty}^* \quad \text{as} \quad r^* \to \infty. \quad (2.6) \]

Here, \( W_W^* = 2cz^* \), \( c \) represents a positive constant.

Now, introducing (Sharma and Borgohain [14]) dimensionless variables as:

\[ \eta = \left( \frac{r^*}{b} \right)^2, \quad u^* = -cb \left( \frac{f' (\eta)}{\sqrt{\eta}} \right), \quad w^* = 2cz^* f' (\eta), \quad (2.7) \]

\[ \phi (\eta) = \frac{C^* - C_{\infty}^*}{C_{\infty}^*}, \quad \theta (\eta) = \frac{T^* - T_{\infty}^*}{T_W^* - T_{\infty}^*}, \]

using (2.7), equations (2.2), (2.4), (2.5) reduces to,

\[ \eta f^{''} + f^* + R_e \left[ f f'' - f' \right] - \frac{l}{D_a} \frac{f'}{R_e} F f' \frac{1}{4} H^2 \sin^2 \beta = 0, \quad (2.8) \]

\[ \eta \theta^* + \theta' + D_e \left( \eta \psi^* + \phi' \right) + R_e P_f \left( f \theta' + S \theta \right) = 0, \quad (2.9) \]

\[ \eta \phi^* + \phi' + R_e S \left( \theta' + \eta \theta' \right) - \gamma S_c \phi'' = 0 \quad (2.10) \]

and (2.6) becomes:

\[ f = 0, \quad f' = 1, \quad \theta = 1, \quad \phi' + S \theta \theta' = 0 \quad \text{at} \quad \eta = 1. \]

\[ f' \to 0, \quad \theta \to 0, \quad \phi \to 0 \quad \text{as} \quad \eta \to \infty \quad (2.11) \]

where: \( P_r \approx \frac{v}{\alpha} \), the Prandtl number; \( S_r = \frac{DK_T \left( T_W^* - T_{\infty}^* \right)}{\nu T_W^* T_{\infty}^*} \), the Soret number; \( D_f = \frac{DK_f}{C_s C_p \left( T_W^* - T_{\infty}^* \right)} \).
the Dufour number; \( H = \frac{B_0 b}{\sqrt{\frac{\sigma}{\mu}}} \), the magnetic parameter; \( S = \frac{Q b^2}{4\sqrt{\nu C_p}} \), the heat source parameter; \( S_e = \frac{\nu}{D} \), the Schmidt number; \( \gamma = \frac{k_b \frac{e^*}{\sigma}}{4\nu} \), the chemical reaction parameter; \( D_a = \frac{4K}{b^2} \), the Darcy parameter; \( F = \frac{c_b e^*}{\sqrt{k}} \), the inertia parameter, \( R_e = \frac{c b^2}{2\nu} \), the Reynolds number.

3. Results and discussion

Equations (2.8)-(2.10) with conditions (2.11) are solved by using MATLAB-bvp4c method. The following parameter values are used for the calculation:

\[
R_e = 1, \quad D_f = 0.15, \quad S_e = 0.4, \quad \gamma = 0.02, \quad P_r = 7, \quad n = 2, \quad F = 0.2.
\]

\[
H = 2, \quad \eta = 2, \quad \beta = \frac{\pi}{3}, \quad S = 0.4, \quad D_a = 1, \quad S_e = 0.6.
\]

Here it should be mentioned that the present problem reduces to the work of Sharma and Borgohain [14] when \( F = 0, \quad H = 0 \) and \( S = 0 \). The results compared for \(-\phi'(1)\) are presented in Tab.1 and are in good agreement.
Figures 2-6 are plotted for fluid velocity against \( \eta \) to exhibit the influence of inclination of the magnetic flux \( (\beta) \), the Hartmann number \( (H) \), inertia parameter \( (F) \), Reynolds number \( (R_e) \) Darcy number \( (D_a) \), respectively. The figures show that the velocity declines exponentially from its highest on the cylindrical surface to its smallest estimate at the boundary layer edge. Figure 2 depicts that the velocity declines for the growing estimates of \( \phi \) from \( \beta = 0 \) to \( \beta = \frac{\pi}{2} \) through \( \beta = \frac{\pi}{4} \) and thus \( f'(\eta) \) decays. The sketch of the \( H \) effect in Fig.3 shows that \( H \) slows the velocity to an appreciable level, because of the resistive magnetic attraction exerted by the Lorentz force. The inertia parameter \( F \) which is resistive in nature reduces the flow velocity as shown in Fig.4. Figure 5 depicts that the augmentation in \( R_e \) reduces the flow velocity. This may be due to the increase in the Reynolds number; the viscous forces become less important and hence the flow velocity will get reduced.

Figure 6 shows that the higher the Darcy number \( D_a \), the higher the flow velocity. As we increase the Darcy number, the capacity of the porous space increases, which subsequently increases the velocity profile.

Figures 7-13 show the variation of temperature for the inclination of the magnetic flux \( (\beta) \), inertia parameter \( (F) \), Darcy number \( (D_a) \), heat source parameter \( (S) \), Schmidt number \( (S_c) \), Dufour number \( (D_F) \) and Prandtl number \( (P_r) \) respectively. The Figures show that the temperature drops exponentially from its peak on the surface of the cylinder to its lowest value at the boundary layer edge.
Figure 7 shows that an increasing value of $\beta$ enhances the temperature inside the boundary layer. Figure 8 explains that the temperature decays with growing $D_a$ values, although a different trend is prominent for increasing $F$ values as illustrated in Fig.9.

Figure 10 shows that the temperature increases as the heat source ($S$) improves. Physically, the external heat source improves thermal conductivity, which leads to an amplification in temperature. Figure 11 shows that an increase in $S_c$ from $S_c = 0.75$ to $S_c = 1.78$ through $S_c = 1.16$ increases the temperature profile, while an increase in $D_F$ (Fig.12) makes the temperature drop.

Figure 13 shows that the thickness of the boundary layer reduces for the rising values of $P_r$ from $P_r = 4$ to $P_r = 13.4$ through $P_r = 7$. This reduction in temperature is due to the dominance of kinematic viscosity over thermal diffusivity.

Figures 14-17 illustrate the variation in concentration profiles against $\eta$ under the influence of $\gamma$, $S_c$, $S_r$ and $D_F$ respectively. The concentration values are lower near the surface of the cylinder than those of the boundary layer edge, but increase sharply and reach a maximum around $\eta = 2$. Then they slowly decrease, reaching a constant value away from the surface. Figure 14 shows that the chemical reaction parameter ($\gamma$) reduces the concentration profile. An enhancing effect of $S_c$ on the concentration profile is seen in Fig.15. Physically, growth of $S_c$ causes a decrease in molecular diffusivity which leads to an augmentation in concentration. The impact of the Soret number (Fig.16) and Dufour number (Fig. 17) causes a growth in the concentration profile.
The skin friction coefficient is proportional to $f''(1)$. Figures 18-19 represent the graph of $f''(1)$ against $H$ for $R_e$ and $F$. It is observed that the skin friction decreases significantly with the amplification of the magnetic parameter and $R_e$ (Fig. 18) and $F$ (Fig. 19).

The Nusselt number is proportional to $-\theta'(1)$. Figures 20 and 21 represent the graph of $-\theta'(1)$ against $S$ for $R_e$ and $P_r$ respectively. We observe that the Nusselt number decreases significantly with increasing $S$ but an opposite impact is observed for $R_e$ and $P_r$. 
The Sherwood number is proportional to $-\phi'(l)$. Figure 22 represents the graph of $-\phi'(l)$ against $n$ for different values of the Soret numbers. We see that the Sherwood number decreases for $0 \leq n < 1.2$ after which it remains almost stagnant. Also, the Sherwood number decreases due to the growth in the Soret number.

**Table 1. Comparison table for \(-\phi'(l)\).**

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<th>$P_r$</th>
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**Fig. 20. Local Nusselt number vs. $S$.**

**Fig. 21. Local Nusselt number vs. $S$.**
4. Conclusion

Main observations are:
1. The flow velocity declines with the growth of inertia, the magnetic field, angle of inclination of the magnetic flux, Reynolds number but rises with the Darcy number.
2. The temperature profile can be improved by increasing the inclination of the magnetic flux, inertia parameter, Schmidt number, and heat source, while the Dufour, Darcy and Prandtl numbers show the opposite trend.
3. The concentration profile can be amplified by escalating the Schmidt number, Soret number, Dufour number, whereas the chemical reaction parameters show the opposite trend.
4. A considerable decline in the skin friction is observed with the augmentation of the magnetic parameters and $Re$.
5. The Nusselt number shows a significant decrease as $S$ increases, but an opposite impact is observed for $Re$ and $Pr$.
6. The Sherwood number has a significant impact on the chemical reaction’s order and on the Soret number.

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Nomenclature

$u$, $w$ – fluid velocity along the $z$- and $r$-directions
$K$ – permeability parameter
$K_T$ – thermal diffusion ratio $(m^2 / s)$
$D$ – mass diffusivity
$T^*$ – temperature $(K)$
$T_\infty^*$ – ambient temperature $(K)$
$T_w^*$ – constant temperature
$T_m$ – mean fluid temperature
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\[ C^* - \text{concentration} \left( \text{mol} / \text{m}^3 \right) \]
\[ C_{\infty}^* - \text{ambient concentration} \left( \text{mol} / \text{m}^3 \right) \]
\[ C_p - \text{specific heat capacity} \left( \text{J} \cdot \text{kg}^{-1} \cdot \text{K}^{-1} \right) \]
\[ C_s - \text{concentration susceptibility} \]
\[ K_f - \text{chemical reaction parameter} \]
\[ n - \text{chemical reaction order} \]
\[ p^* - \text{pressure} \left( \text{Pa} \right) \]
\[ R_e - \text{Reynolds number} \]
\[ P_r - \text{Prandtl number} \]
\[ S_c - \text{Schmidt number} \]
\[ S_r - \text{Soret number} \]
\[ D_d - \text{Darcy number} \]
\[ D_F - \text{Dufour number} \]
\[ H - \text{Hartmann’s number} \]
\[ S - \text{heat source parameter} \]
\[ \nu - \text{kinematic viscosity} \left( \text{m}^2 / \text{s} \right) \]
\[ \gamma - \text{non-dimensional chemical reaction parameter} \]
\[ \alpha - \text{thermal diffusivity} \left( \text{m}^2 / \text{s} \right) \]
\[ \rho - \text{density} \left( \text{kg} / \text{m}^3 \right) \]

References


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