UNSTEADY CONVECTION FLOW OF A MAGNETOMICROPOLAR FLUID PAST A VERTICAL POROUS PLATE

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The paper deals with an unsteady two dimensional laminar slip flow of a viscous incompressible magnetomicropolar fluid past a semi infinite porous plate embedded in a porous medium. The flow is under the influence of a transverse magnetic field and heat source/sink. The free stream velocity follows an exponentially increasing or decreasing small perturbation law. The porous surface absorbs the fluid with time varying suction velocity. Expressions are obtained for velocity and temperature fields, mean angular velocity, skin friction and the Nusselt number.

Key words: heat source/sink, magnetic field, porous medium, slip velocity, unsteady, vertical plate.

1. Introduction

In the literatures, extensive research has been mode on a steady/unsteady free convection heat transfer flow in a porous medium. Details of the flow of Newtonion fluids through a porous medium were given in Kokac *et al*. (1991), Nield and Bejan (1992). Yamamoto and Yoshidha (1976) considered a flow with convection acceleration on a plane porous wall specifically for the flow outside the vortex layer. Chawala and Singh (1979), Raptis *et al*. (1981) studied flows through a porous medium considering generalized Darcy's law.

In all the above-mentioned research papers generalized Darcy's law is derived without taking into account the angular velocity of the fluid particles. Aero *et al.* (1965) derived and solved the flow equations of the fluid in which angular velocity of the fluid particles was considered. These fluids are known as polar fluids in the literature and are more general than ordinary fluids (Lukaszewicz, 1999). Micropolar fluids are fluids with microstructure and belong to a class of fluids with asymmetrical stress tensor. Physically, they

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represent fluids consisting of randomly oriented particles suspended in a viscous medium. Ahmedi (1976) studied the flow of a micropolar fluid. Recently, Kim (2001) studied the basic problem of a viscous incompressible micropolar fluid with fluctuating stream velocity which was also studied by Stokes (1851) for a Newtonion fluid. Because of their practical importance the unsteady flow problems were studied extensively by a number of researchers. Soundelgeker (1973) studied the time dependent flow problems on an infinite vertical plate, when it is cooled or heated by free convection currents. El Arabawy and Hassen (2003) studied the effects of suction/injection on a flow of a micropolar fluid past a continuously moving plate in the presence of radiation. Chu *et al.* (2002) solved the problems of micropolar fluid flows along a vertical wavy surface with a discontinuous temperature profile. Perhaps the most widely known aspects of non equilibrium gas flow in the fact that the velocity of a gas close to a surface describing slip accurately, however is still an active area of research.

  In this paper it is proposed to study an unsteady two dimensional laminar slip flow of a viscous incompressible magnetomicropolar fluid past a semi infinite porous plate embedded in a porous medium. We consider the free stream to consist of a mean velocity and temperature with a superimposed exponentially small variation with time.

2. Formulation of the problem

  Consider an unsteady two dimensional slip flow of a laminar, incompressible, magnetomicropolar fluid past a semi infinite porous plate embedded in a porous medium and subjected to the presence of an applied pressure gradient. The permeability of the porous medium is assumed as constant.
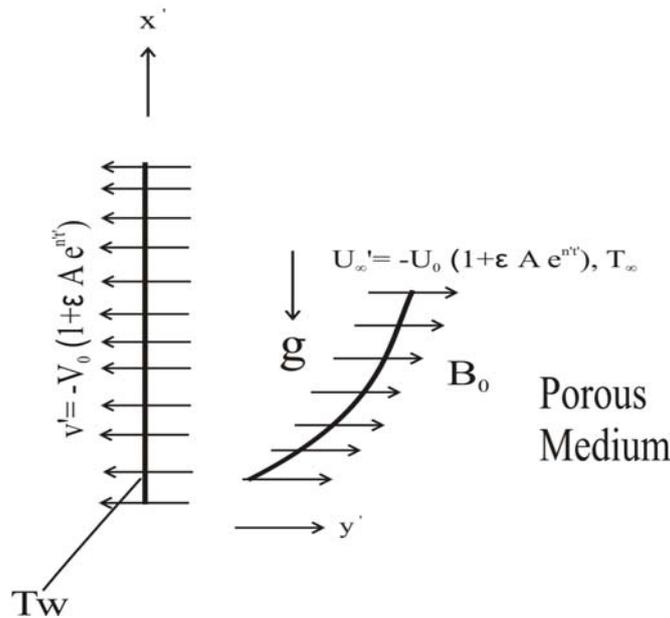
Fig. Physical model and coordinate system of the problem

Under the assumptions the flow field is governed by the following equations

$$\frac{\partial v'}{\partial y'} = 0 , \tag{2.1}$$

$$\frac{\partial u'}{\partial t'} + v' \frac{\partial u'}{\partial y'} = -\frac{1}{\rho}\frac{\partial p'}{\partial x'} + g\beta_f\left(T - T_\infty\right) + \left(\nu + \nu_r\right)\frac{\partial^2 u'}{\partial y'^2} - \frac{\nu}{K'}u' + 2\nu_r\frac{\partial \omega'}{\partial y} - \frac{\sigma B_0^2 u'}{\rho}, \tag{2.2}$$

$$\frac{\partial \omega'}{\partial t'} + v' \frac{\partial \omega'}{\partial y'} = \frac{\gamma}{\rho j'}\left(\frac{\partial^2 \omega'}{\partial y'^2}\right),$$

(2.3)

$$\frac{\partial T'}{\partial t'} + v' \frac{\partial T'}{\partial y'} = \alpha \frac{\partial^2 T'}{\partial y'^2} + \frac{S'}{\rho C_\rho}(T - T_\infty),$$

(2.4)

x' is the dimensional distance along the plate and y' is normal to the plate, u' and v' are the dimensional velocity components along x' and y', respectively, ρ is the density, ν is the kinematic viscosity, ν_r is the kinematic rotational viscosity, β_f is the coefficient of volumetric expansion of the working fluid, K' is an empirical constant called permeability of the porous medium, j' is the micro inertia density, ω is the component of angular velocity vector normal to the *xy*-plane, γ is the spin gradient viscosity, S is the heat source parameter, B_0 is the magnetic field, T is the temperature , and α is the fluid thermal diffusivity.
The boundary conditions are as follows

$$y' = 0 \ ; \qquad u' = U_0 + L_1 \frac{\partial u'}{\partial y'}, \qquad T = T_w + \varepsilon(T_w - T_\infty)e^{n't'}, \qquad \omega' = -\frac{1}{2}\frac{\partial u'}{\partial y'},$$

(2.5)

$$y' \to \infty \ ; \qquad u' \to U'_\infty = U_0\left(1 + \varepsilon e^{n't'}\right), \quad T' \to T_\infty, \qquad \omega' \to 0.$$

Equation of continuity Eq.(2.1), shows that the suction velocity normal to the plate is a function of time only and we shall take it in the form

$$v' = -V_0\left(1 + \varepsilon A e^{n't'}\right)$$

(2.6)

where A is any real constant, $A\ \varepsilon << 1$ and there is small variation with time. V_0 is the scale of suction velocity which has a non zero positive constant.
Outside the boundary layer, Eq.(2.2) gives

$$-\frac{1}{\rho}\frac{\partial p'}{\partial x'} = \frac{dU'_\infty}{dt'} + \frac{\nu}{K'}U'_\infty + \frac{\sigma}{\rho}B_0^2 U'_\infty.$$

Now, we introduce the dimensionless quantities as

$$y = \frac{y'V_0}{\nu}, \qquad t = \frac{V_0^2 t'}{\nu}, \qquad M = \frac{\sigma B_0^2 \nu}{\rho V_0^2}, \qquad u = \frac{u'}{U_0}, \qquad v = \frac{v'}{V_0}, \qquad K = \frac{K'V_0^2}{\upsilon^2},$$

$$\theta = \frac{T - T_\infty}{T_w - T_\infty}, \qquad U_\infty = \frac{U'_\infty}{U_0}, \qquad \omega = \frac{\nu}{U_0 V_0}\omega', \qquad \beta = \frac{\nu_r}{\nu}, \qquad \text{Pr} = \frac{\upsilon \rho C_p}{\kappa} = \frac{\upsilon}{\alpha},$$

(2.7)

$$n = \frac{n'\nu}{V_0^2}, \qquad G = \frac{\upsilon g \beta_f (T_w - T_\infty)}{U_0 V_0^2}, \qquad j = \frac{V_0^2}{\upsilon^2}j', \qquad h = \frac{L_1 V_0}{\upsilon}, \qquad S = \frac{S'\upsilon}{\rho C_p V_0^2},$$

and the spin gradient viscosity γ is defined as

$$\gamma \ = \left(\mu + \frac{\wedge}{2}\right) j' = \ \mu j' \left(1 + \frac{1}{2}\beta'\right)$$

where $\beta' = \dfrac{\wedge}{\mu}$ is the dimensionless viscosity ratio and $\wedge$ is the coefficient of gyro viscosity (or vortex viscosity) management.

In view of Eqs (2.6) and (2.7), the governing Eq.(2.2) to Eq.(2.4) reduce to the following non-dimensional form

$$\frac{\partial u}{\partial t} - \left(1 + A\,\varepsilon\,e^{nt}\right)\frac{\partial u}{\partial y} = \frac{dU_\infty}{dt} + \frac{1}{K}\left(U_\infty - u\right) + \left(1 + \beta\right)\frac{\partial^2 u}{\partial y^2} + G\theta + 2\beta\frac{\partial \omega}{\partial y} - M\left(u - U_\infty\right), \qquad (2.8)$$

$$\frac{\partial \omega}{\partial t} - \left(1 + A\,\varepsilon\,e^{nt}\right)\frac{\partial \omega}{\partial y} = \frac{1}{\eta}\frac{\partial^2 \omega}{\partial y^2}, \qquad (2.9)$$

$$\frac{\partial \theta}{\partial t} - \left(1 + A\,\varepsilon\,e^{nt}\right)\frac{\partial \theta}{\partial y} = \frac{1}{\Pr}\frac{\partial^2 \theta}{\partial y^2} + S\theta, \qquad (2.10)$$

with the corresponding boundary conditions

$$y = 0; \qquad u = 1 + h\frac{\partial u}{\partial y}, \qquad \theta = 1 + \varepsilon e^{nt}, \qquad \omega = -\frac{1}{2}\frac{\partial u}{\partial y},$$

$$(2.11)$$

$$y \to \infty; \qquad u \to U_\infty = 1 + \varepsilon e^{nt}, \qquad \omega \to 0, \qquad \theta \to 0$$

where

$$\eta = \frac{\mu j'}{\gamma} = \left(\frac{2}{2 + \beta'}\right)$$

3.   Solution of the problem

To solve the partial differential Eqs (2.9) to (2.11), we assume for small ε

$$f(y,t) = f_0(y) + \varepsilon e^{nt} f_1(y) + ... \qquad (3.1)$$

where f stands for u, ω and θ.

Substituting expressions (3.1) in Eqs (2.8) to (2.10) and equating the harmonic and non harmonic terms and neglecting the coefficient of $o\left(\varepsilon^2\right)$, we get the following sets of ordinary differential equations.

$$(1+\beta)u_0'' + u_0' - \left(\frac{1}{K} + M\right)u_0 = -G\theta_0 - \left(\frac{1}{K} + M\right) + 2\beta\omega_0' , \tag{3.2}$$

$$(1+\beta)u_1'' + u_1' - \left(\frac{1}{K} + M + n\right)u_1 = -G\theta_1 - \left(\frac{1}{K} + M\right) + 2\beta\omega_1' - Au_0' , \tag{3.3}$$

$$\omega_0'' + \eta\omega_0' = 0 , \tag{3.4}$$

$$\omega_1'' + \eta\omega_1' - n\eta\omega_1 = -A\eta\omega_0' , \tag{3.5}$$

$$\theta_0'' + \Pr\theta_0 + S\Pr\theta_0 = 0 , \tag{3.6}$$

$$\theta_1'' + \Pr\theta_1 - (n+S)\Pr\theta_1 = -A\Pr\theta_0' . \tag{3.7}$$

The relevant boundary conditions are given as

$$u_0 = 1 + hu_0', \quad u_1 = hu_1', \quad \theta_0 = 1, \quad \theta_1 = 1, \quad \omega_0 = -\frac{1}{2}u_0', \quad \omega_1 = -\frac{1}{2}u_1' \quad \text{at} \quad y = 0, \tag{3.8}$$

$$u_0 \to 1, \quad u_1 \to 1, \quad \theta_0 \to 0, \quad \theta_1 \to 0, \quad \omega_0 \to 0, \quad \omega_1 \to 0 \quad \text{as} \quad y \to \infty.$$

The prime denotes differentiation with respect to y.

On solving Eqs (3.2) to (3.7) with the help of boundary conditions (3.8), we get the following results

$$u_0 = C_5 e^{-m_7 y} - D_3 e^{-m_3 y} - D_4 C_1 e^{-\eta y} + D_5 , \tag{3.9}$$

$$u_1 = C_6 e^{-m_9 y} + D_6 e^{-m_7 y} + D_7 e^{-m_3 y} + D_8 e^{-\eta y} - D_9 e^{-m_5 y} + D_{10}C_2 e^{-m_1 y} + D_{11} , \tag{3.10}$$

$$\theta_0 = C_3 e^{-m_3 y} , \tag{3.11}$$

$$\theta_1 = C_4 e^{-m_5 y} - D_2 e^{-m_3 y} , \tag{3.12}$$

$$\omega_0 = C_1 e^{-\eta y} , \tag{3.13}$$

$$\omega_1 = C_2 e^{-m_1 y} - D_1 e^{-\eta y} \tag{3.14}$$

where

$$m_1 = \frac{\eta + \sqrt{\eta^2 + 4n\,\eta}}{2}, \qquad m_2 = \frac{-\eta + \sqrt{\eta^2 + 4n\,\eta}}{2} ,$$

$$m_3 = \frac{\Pr + \sqrt{\Pr^2 - 4\Pr S}}{2} , \qquad m_4 = \frac{-\Pr + \sqrt{\Pr^2 - 4\Pr S}}{2},$$

$$m_5 = \frac{\Pr + \sqrt{\Pr^2 + 4\Pr(n+S)}}{2} , \qquad m_6 = \frac{-\Pr + \sqrt{\Pr^2 + 4\Pr(n+S)}}{2},$$

$$m_7 = \frac{1 + \sqrt{1 + 4(1+\beta)\left(\frac{1}{K} + M\right)}}{2} , \qquad m_8 = \frac{-1 + \sqrt{1 + 4(1+\beta)\left(\frac{1}{K} + M\right)}}{2},$$

$$D_1 = \frac{A\eta C_1}{n}, \qquad D_2 = \frac{A\Pr m_3}{m_3{}^2 - m_3\Pr - \Pr(n+S)}, \qquad D_3 = \frac{\mathrm{Gr}}{(1+\beta)m_3{}^2 - m_3 - \left(\frac{1}{K} + M\right)},$$

$$D_4 = \frac{2\eta\beta}{\left\{(1+\beta)\eta^2 - \eta - \left(\frac{1}{K} + M\right)\right\}} , \qquad D_5 = 1,$$

$$D_6 = \frac{AC_5 m_7}{(1+\beta)m_7^2 - m_7 - \left(\frac{1}{K} + M + n\right)}, \qquad D_7 = \frac{(\mathrm{Gr}D_2 - AD_3 m_3)}{(1+\beta)m_3^2 - m_3 - \left(\frac{1}{K} + M + n\right)},$$

$$D_8 = \frac{(AD_4 C_1\eta - 2\beta D_1\eta)}{(1+\beta)\eta^2 - \eta - \left(\frac{1}{K} + M^2 + n\right)}, \qquad D_9 = \frac{\mathrm{Gr}C_4}{(1+\beta)m_5^2 - m_5 - P\left(\frac{1}{K} + M + n\right)},$$

$$D_{10} = \frac{2\beta m_1}{(1+\beta)m_1^2 - m_1 - \left(\frac{1}{K} + M + n\right)}, \qquad D_{11} = \frac{\left(\frac{1}{K} + n\right)}{\left(\frac{1}{K} + M + n\right)},$$

$$K_1 = D_4 + h\eta D_4, \qquad K_2 = 1 + h m_7, \qquad K_3 = 1 + hD_3 m_3 + D_3 - 1,$$

$$K_4 = 1 + \frac{\eta D_4}{2}, \qquad K_5 = \frac{m_7}{2}, \qquad K_6 = \frac{-D_3 m_3}{2}, \qquad K_7 = 1 - \frac{D_{10} m_1}{2}$$

$$K_8 = \frac{m_9}{2}, \qquad K_9 = D_1 + \frac{1}{2}\left[D_6 m_7 + D_7 m_3 + D_8\eta - D_9 m_5\right],$$

$$K_{10} = D_{10} + h m_1 D_{10}, \qquad K_{11} = 1 + h m_9$$

$$C_1 = \frac{K_3K_5 + K_2K_6}{K_1K_5 + K_2K_4}, \qquad C_2 = \frac{K_{12}K_8 + K_9K_{11}}{K_{10}K_8 + K_7K_{11}},$$

$$C_3 = 1, \qquad C_4 = 1 + D_2C_2 = \frac{K_{12}K_8 + K_9K_{11}}{K_{10}K_8 + K_7K_{11}}.$$

Using Eq.(3.1), we obtain the main, angular velocity and temperature as

$$u(y,t) = C_5 e^{-m_7 y} - D_3 e^{-m_3 y} - D_4 C_1 e^{-\eta y} + 1 +$$

$$+ \varepsilon e^{nt} \left(C_6 e^{-m_9 y} + D_6 e^{-m_7 y} + D_7 e^{-m_3 y} + D_8 e^{-\eta y} - D_9 e^{-m_5 y} + D_{10} C_2 e^{-m_1 y} + D_{11} \right), \qquad (3.15)$$

$$\omega(y,t) = \left(C_1 e^{-\eta y} \right) + \varepsilon e^{nt} \left(C_2 e^{-m_1 y} - D_1 e^{-\eta y} \right), \qquad (3.16)$$

$$\theta(y,t) = \left(C_3 e^{-m_3 y} \right) + \varepsilon e^{nt} \left(C_4 e^{-m_5 y} - D_2 e^{-m_3 y} \right). \qquad (3.17)$$

4. Skin friction and Nusselt number

After obtaining velocity distribution, the important parameter of skin friction at the plate can now be obtained as

$$\tau_w = \frac{\tau^*_w}{\rho V_0 U_0} = \left[\frac{\partial u}{\partial y} \right]_{y=0} = \{ -C_5 m_7 + D_3 m_3 - D_4 C_1 \eta \} +$$

$$+ \varepsilon e^{nt} \{ -C_6 m_9 - D_6 m_7 - D_7 m_3 - D_8 \eta + D_9 m_5 - D_{10} C_2 m_1 \}. \qquad (4.1)$$

From Eq.(3.17), we calculate the rate of heat transfer in terms of the Nusselt number as

$$\text{Nu} = x \frac{\left(\partial T / \partial y^* \right)_w}{T_w - T_\infty} = \left[\frac{\partial \theta}{\partial y} \right]_{y=0} = \{ -C_3 m_3 \} + \varepsilon e^{nt} \{ m_5 C_4 + m_3 D_2 \}. \qquad (4.30)$$

5. Discussions and conclusion

The effects of suction velocity varying exponentially in time about a non zero constant mean value on the flow and heat transfer of an incompressible magnetomicropolar fluid along a vertical porous plate were determined for different values of β (rotational parameter), K (permeability parameter), S (heat source parameter), h (slip parameter) and Gr (Grashof number) fixing $\varepsilon = 0.1$, $\eta = 0.1$, $A = 0.5$, $n = 0.1$, $t = 1$.

In Fig.1, the velocity distribution is plotted against y taking air (Pr = 0.71) as a fluid. It is observed that the velocity increases with the increase of h, S, M, and K, but when the rotational parameter is increased the velocity has decreases.

Moreover, the velocity increases for an increase in Gr but a fall in velocity is observed for negative values of Gr. We may deduce that these results are consistent with the physical observations for the case of cooling or heating of the surface by natural convection.

The variation of temperature against y is illustrated in Fig.2. Here, it is interesting to note that for the case of air an increase in the source parameter increases the temperature but for water the temperature decreases with an increase of the source parameter.

Pr = 0.71, n = 0.1, t = 1.0, A = 0.5, η = 0.1, ∈ = 0.1,

S.No.	β	M	K	S	h	Gr
1	0.2	0.0	1.0	0.0	0.0	2.0
2	0.2	3.0	1.0	0.1	0.2	2.0
3	0.2	3.0	5.0	0.1	0.2	2.0
4	0.2	3.0	1.0	0.05	0.2	2.0
5	0.2	3.0	1.0	0.1	0.1	2.0
6	0.2	3.0	1.0	0.1	0.2	5.0
7	0.2	4.0	1.0	0.1	0.2	2.0
8	0.2	3.0	1.0	0.1	0.2	–2.0
9	0.2	3.0	∞	0.1	0.2	2.0
10	0.5	3.0	1.0	0.1	0.2	2.0
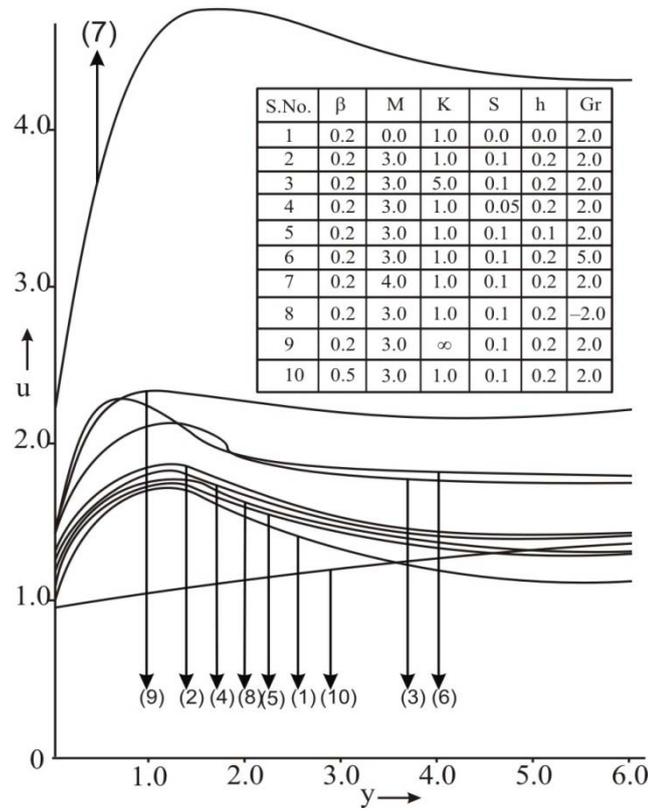
Fig.1.Velocity distribution plotted against *y* for different values of β, *M*, *K*, *S*, *h* and Gr.

n = 1.0, t = 1.0, ∈ = 0.1, A = 0.5

S.No.	S	Pr
1	0.0	0.71
2	0.0	1.0
3	0.05	0.71
4	0.0	7.0
5	0.05	7.0
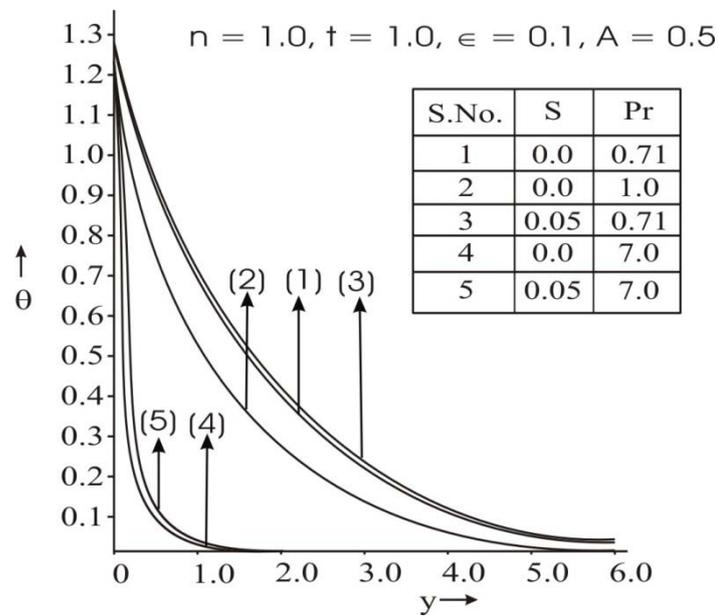
Fig.2. Temperature distribution plotted against *y* for different values of *S* and Pr.

Moreover, an increase in Pr decreases the temperature of the fluid. The reason is that smaller values of the Prandtl number are equivalent to increasing thermal conductivity and therefore heat is able to diffuse away from the heated surface more rapidly.

In Fig.3, the non dimensional angular velocity vector is plotted for different values of S and Gr.

S.No.	S	Gr
1	0.0	2.0
2	0.0	0.0
3	0.0	−2.0
4	0.1	2.0
5	0.1	0.0
6	0.1	−2.0
7	0.05	2.0
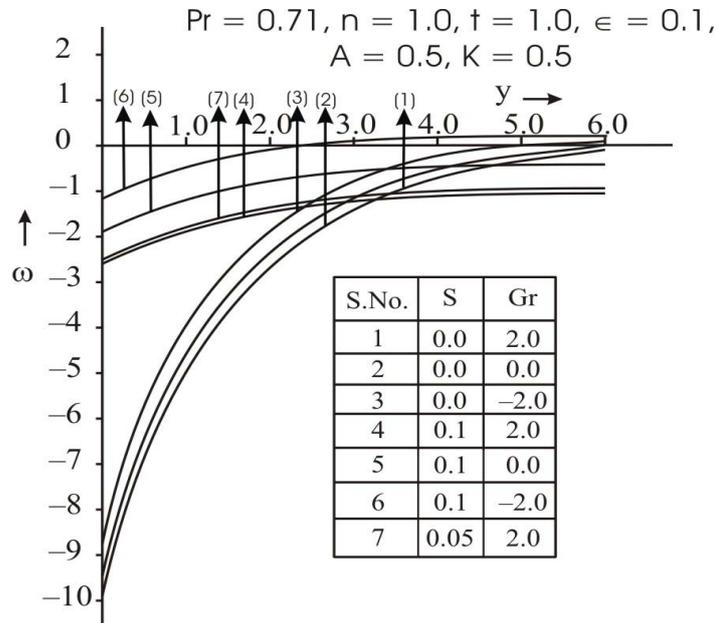
Fig.3. Angular velocity ω plotted against y for different values of S and Gr.

It is noted that angular velocity decreases with the increase of S and Gr.
The skin friction τ is plotted against β at the cooling plate in Fig.4.

S.No.	S	M	Gr
1	0.1	3.0	2.0
2	0.0	3.0	2.0
3	0.05	3.0	2.0
4	0.1	5.0	2.0
5	0.1	3.0	3.0
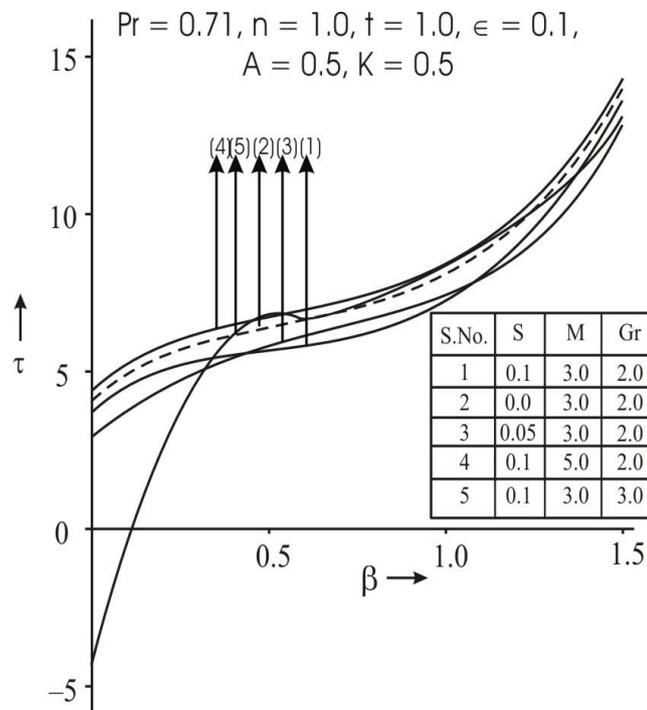
Fig.4. Skin friction (τ) plotted against β for different values of S, M and Gr.

It is observed that an increase in *M* and Gr increases τ, but an increase in *S* decreases the skin friction. For the Nusselt number (Nu) at the plate *y = 0*, we observed from Fig.5, that it increases with the increase of *S*, and decreases with the increase of Pr.
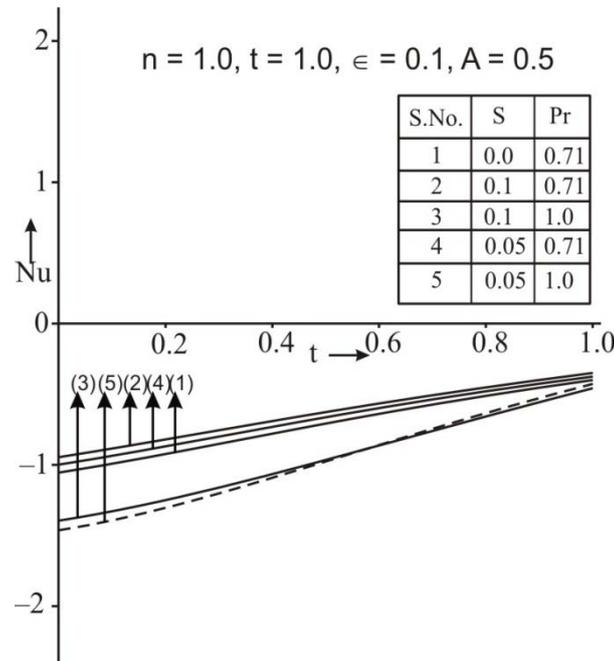
Fig.5. Nusselt Number Nu plotted against *t* for different values of *S* and Pr.

Acknowledgements

Nomenclature

A – suction velocity amplitude
B_0 – magnetic field
C_p – specific heat at constant pressure
E – scalar constant $(<< 1)$
Gr – Grashof number
g – acceleration due to gravity
K – permeability of the porous medium
Nu – Nusselt number
n – dimensionless exponential index
Pr – Prandtl number
S – heat source parameter
T – temperature
t – dimensionless time
U_0 – scale of free stream velocity
u, v – component of velocity along and perpendicular to the plate, respectively
V_0 – scale of suction velocity

x, y – distances along and perpendicular to the plate, respectively
α – fluid thermal diffusivity
β' – dimensionless viscosity ratio
β_f – coefficient of volumetric expansion of the working fluid

γ – spin gradient viscosity
θ – dimensionless temperature
κ – thermal conductivity
μ – fluid dynamic viscosity
ν – fluid kinematic viscosity
ν_r – fluid kinematic rotational viscosity
ρ – fluid density
ω – angular velocity vector
$\wedge$ – coefficient of gyro viscosity

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