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EFFECT OF VERTICAL VIBRATIONS ON THE ONSET OF BINARY CONVECTION

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In the present work the linear stability analysis of double diffusive convection in a binary fluid layer is performed. The major intention of this study is to investigate the influence of time-periodic vertical vibrations on the onset threshold. A regular perturbation method is used to compute the critical Rayleigh number and wave number. A closed form expression for the shift in the critical Rayleigh number is calculated as a function of frequency of modulation, the solute Rayleigh number, Lewis number, and Prandtl number. These parameters are found to have a significant influence on the onset criterion; therefore the effective control of convection is achieved by proper tuning of these parameters. Vertical vibrations are found to enhance the stability of a binary fluid layer heated and salted from below. The results of this study are useful in the areas of crystal growth in micro-gravity conditions and also in material processing industries where vertical vibrations are involved.

Key words: gravity modulation, *g*-jitter, double diffusive convection, perturbation method.

1. Introduction

Double diffusive convection (DDC) occurs when two diffusing components (e.g., temperature and dissolved concentration) contribute to the buoyancy. This is most commonly observed in the layer of a binary fluid mixture heated from below. The problem of DDC in a fluid layer has received significant interest during the past few decades because of its wide spread applications, such as convective heat and mass transfer, solidification of binary mixtures, migration of solutes in water-saturated soils and the migration of moisture through air contained in fibrous insulations and so on. Some of the areas where DDC finds exhaustive applications include oceanography, astrophysics, geophysics, geology, chemistry, and metallurgy. The problem of DDC in a fluid layer has been extensively investigated both theoretically and experimentally (see e.g., Turner, 1973; 1974; 1985; Huppert and Turner 1981; Platten and Legros, 1984; and the references therein).

The study of thermal convection induced by oscillating forces which arise due to either oscillating wall temperatures or complex body forces or a combination of these two has received much attention in the fluid dynamics research community. The complex body forces can arise in a number of different ways, for instance, when a system with a density gradient is subjected to harmonic vertical vibrations, the resulting buoyancy forces will have a complex spatio-temporal structure. The other situations where the gravity fluctuation becomes predominant include buoyancy-driven convection in microgravity conditions occurring in space laboratory experiments, crystal growth, petroleum production, and large-scale atmospheric convection. Owing to several unavoidable sources of residual acceleration experienced by a spacecraft, the gravity field in an orbiting laboratory is not constant in a micro-gravity environment, but is a randomly fluctuating field which is referred to as g-jitter. The vibrations can either substantially enhance or retard heat transfer and thus drastically affect convection (see e.g., Wadih and Roux, 1988). The effect of gravity modulation on a convectively stable configuration can significantly influence the stability of a system by increasing or decreasing its susceptibility to convection. In general, a distribution of a stratifying agency that is convectively stable under constant gravity conditions can be destabilized when a time-dependent component of the gravity field is introduced.

The effect of gravity modulation on the stability of a heated fluid layer was first examined by Gresho and Sani (1970) and Gershuni *et al*. (1970). Murray *et al*. (1991) considered the effect of gravity modulation on the onset of convection for the unidirectional solidification problem. Saunders *et al*. (1992) studied the effect of gravity modulation on the stability of a horizontal double-diffusive layer. Clever *et al*. (1993) studied the problem of two dimensional oscillatory convection in a gravitationally modulated fluid layer. Farooq and Homsy (1996) investigated linear and nonlinear convection in a vertical slot in the presence of gravity modulation. Malashetty and Padmavathi (1997) studied the effect of small amplitude gravity modulation on the onset of convection in the fluid and porous layers. Li (2001) performed a stability analysis of modulated-gravity-induced thermal convection in a heated fluid layer subject to an applied magnetic field. Shu *et al*. (2005) examined the effect of modulation of gravity and thermal gradients on natural convection in a cavity numerically and experimentally. An experimental study on the response of Rayleigh-Benard convection to gravity modulation was carried out by Rogers *et al*. (2005).

Yu *et al*. (2007) made an experimental investigation of a horizontal stably stratified fluid layer, including its subsequent nonlinear evolution under steady and modulated gravity, using two-dimensional numerical simulations. Dyko and Vafai (2007) investigated the effect of gravity modulation on convection in an annulus between two horizontal coaxial cylinders. Zenkovskaya and Rogovenko (1999) investigated filtration convection subject to high frequency oscillations in an arbitrary direction using the averaging method. Malashetty and Swamy (2011) asymptotically analyzed the linear stability of a rotating horizontal fluid and fluid-saturated porous layer heated from below for the case of small-amplitude gravity modulation.

The main objective of this article is to analyze the effect of small amplitude gravity modulation on the onset of a binary fluid layer for a wide range of values of frequency of the modulation, solute Rayleigh number, Lewis number, and Prandtl number. It is believed that the results of this study are useful in the areas of crystal growth in micro-gravity conditions and in material processing industries where vertical vibrations are involved.

2. Formulation of the problem

An initially quiescent infinite horizontal binary fluid layer of height d in the presence of harmonic vertical vibrations is considered, so that the gravity varies periodically with time. Thus,

$$\boldsymbol{g} \equiv \left(0, 0, -g(t)\right), \qquad \text{where} \qquad g(t) = g_0\left(1 + \varepsilon\cos\bar{\omega}t\right),$$

with g_0 the constant gravity in an otherwise unmodulated system. A Cartesian reference frame is chosen with the origin in the lower boundary and the z-axis vertically upwards. The temperatures T_l and T_u with $T_l > T_u$ and solute concentrations S_l and S_u with $S_l > S_u$ are maintained respectively on lower and upper impermeable, stress-free, isothermal and isohaline boundaries held at $z = 0$ and $z = d$. Under Boussinesq assumption, the basic governing equations are

$$\nabla \cdot \boldsymbol{q} = 0, \tag{2.1}$$

$$\rho_0\left(\frac{\partial \boldsymbol{q}}{\partial t} + \left(\boldsymbol{q}\cdot\nabla\right)\boldsymbol{q}\right) + \nabla p - \rho\boldsymbol{g} = \mu\nabla^2\boldsymbol{q}, \tag{2.2}$$

$$\frac{\partial T}{\partial t} + \left(\boldsymbol{q}\cdot\nabla\right)T = \kappa_T\nabla^2 T, \tag{2.3}$$

$$\frac{\partial S}{\partial t} + \left(\boldsymbol{q}\cdot\nabla\right)S = \kappa_S\nabla^2 S, \tag{2.4}$$

$$\rho = \rho_0 \left[1 - \beta_T (T - T_0) + \beta_S (S - S_0) \right]. \tag{2.5}$$

The basic state is assumed to be quiescent so that

$$\boldsymbol{q}_b \equiv (0,0,0), \qquad p = p_b(z,t), \qquad \rho = \rho_b(z), \qquad T = T_b(z), \qquad S = S_b(z)$$

where
$$p_b(z,t) = \rho_0 g(t) \left[(\beta_T T_l - \beta_S S_l) z - \frac{1}{2d} \left(\beta_T (T_l - T_u) - \beta_S (S_l - S_u) \right) z^2 \right], \tag{2.6}$$

$$T_b(z) = T_l - \frac{1}{d}(T_l - T_u) z, \tag{2.7}$$

$$S_b(z) = S_l - \frac{1}{d}(S_l - S_u) z, \tag{2.8}$$

$$\rho_b = \rho_0 \left[1 - \beta_T (T_b - T_0) + \beta_S (S_b - S_0) \right]. \tag{2.9}$$

To study the stability, one can superpose infinitesimal perturbations on the basic state. Then

$$\boldsymbol{q} = \boldsymbol{q}_b + \boldsymbol{q}', \quad p = p_b(z) + p', \quad \rho = \rho_b(z) + \rho', \quad T = T_b(z) + T', \quad S = S_b(z) + S'. \tag{2.10}$$

The basic governing equations are then simplified and the product terms in perturbation variables are neglected to obtain

$$\nabla \cdot \boldsymbol{q}' = 0, \tag{2.11}$$

$$\frac{\partial \boldsymbol{q}'}{\partial t} + \frac{1}{\rho_0} \nabla p' - (\beta_T T' + \beta_S S') g_0 (1 + \varepsilon \cos \bar{\omega} t) \boldsymbol{k} - \frac{\mu}{\rho_0} \nabla^2 \boldsymbol{q}' = 0, \tag{2.12}$$

$$\frac{\partial T'}{\partial t} - w'(T_l - T_u)/d = \kappa_T \nabla^2 T', \tag{2.13}$$

$$\frac{\partial S'}{\partial t} - w'(S_l - S_u)/d = \kappa_S \nabla^2 S'. \tag{2.14}$$

Here $\boldsymbol{k}$ denotes the unit vector in the z-direction. The boundaries are assumed to be stress-free, isothermal and isohaline so that

$$w' = \partial^2 w'/\partial z^2 = T' = S' = 0 \qquad \text{at} \qquad z = 0 \qquad \text{and} \qquad d. \tag{2.15}$$

By operating double curl on Eq.(2.12) we eliminate p' and then make all equations non-dimensional by scaling length, time, velocity, pressure, temperature and solute concentration, respectively, with d, d^2/κ_T, $\varphi\kappa_T/d$, $\mu\varphi\kappa_T/d^2$, ΔT and ΔS, to obtain

$$\left(\mathrm{Pr}^{-1}\,\partial/\partial t-\nabla^{2}\right)\nabla^{2}w-\left(1+\varepsilon\cos\omega t\right)\nabla_{1}^{2}\left(\mathrm{Ra}_{T}T-\mathrm{Ra}_{S}S\right)=0, \tag{2.16}$$

$$\left(\partial/\partial t-\nabla^{2}\right)T-w=0, \tag{2.17}$$

$$\left(\partial/\partial t-\mathrm{Le}^{-1}\nabla^{2}\right)S-w=0. \tag{2.18}$$

Further, the boundary conditions (2.15) become

$$w=\partial^{2}w/\partial z^{2}=T=S=0 \qquad \text{at} \qquad z=0,\ 1. \tag{2.19}$$

Coupling between Eqs (2.16)-(2.18) is resolved to obtain

$$\begin{aligned}&\Big[\left(\mathrm{Pr}^{-1}\,\partial/\partial t-\nabla^{2}\right)\left(\partial/\partial t-\nabla^{2}\right)\left(\partial/\partial t-\mathrm{Le}^{-1}\nabla^{2}\right)\nabla^{2}+\\&-\mathrm{Ra}_{T}\left(\partial/\partial t-\mathrm{Le}^{-1}\nabla^{2}\right)\left(1+\varepsilon\cos\omega t\right)\nabla_{1}^{2}+\mathrm{Ra}_{S}\left(\partial/\partial t-\nabla^{2}\right)\left(1+\varepsilon\cos\omega t\right)\nabla_{1}^{2}\Big]w=0.\end{aligned} \tag{2.20}$$

Then the boundary conditions (2.19) become

$$w=\partial^{2}w/\partial z^{2}=\partial^{4}w/\partial z^{4}=\partial^{6}w/\partial z^{6}=0, \qquad \text{at} \qquad z=0,\ 1. \tag{2.21}$$

According to the normal mode technique disturbances can be expressed as

$$w=W(z,t)e^{i(lx+my)+\sigma t}. \tag{2.22}$$

Here $W(z,t)$ is a periodic function of time with the same period as the gravity modulation $\sigma=\sigma_{r}+i\sigma_{i}$ is the growth rate of the disturbances. Here we consider only the synchronous mode $(\text{i.e.,}\sigma=0)$ Then Eqs (2.20)-(2.21) yield

$$\begin{aligned}&\Big[\left(\mathrm{Pr}^{-1}\,\partial/\partial t-\left(D^{2}-a^{2}\right)\right)\left(\partial/\partial t-\left(D^{2}-a^{2}\right)\right)\left(\partial/\partial t-\mathrm{Le}^{-1}\left(D^{2}-a^{2}\right)\right)\left(D^{2}-a^{2}\right)+\\&+\left\{\mathrm{Ra}_{T}\left(\partial/\partial t-\mathrm{Le}^{-1}\left(D^{2}-a^{2}\right)\right)-\mathrm{Ra}_{S}\left(\partial/\partial t-\left(D^{2}-a^{2}\right)\right)\right\}\left(1+\varepsilon\cos\omega t\right)a^{2}\Big]W=0,\end{aligned} \tag{2.23}$$

and

$$W=D^{2}W=D^{4}W=D^{6}W=D^{8}W=0 \qquad \text{at} \qquad z=0,\ 1, \tag{2.24}$$

with $D\equiv\partial/\partial z$. This is an eigenvalue problem with eigenvalue Ra_{T} and eigenfunctions W.

3. Perturbation method

The gravity field under the influence of vibrations deviates from the constant gravity field by a small quantity ε. Since the above eigenvalue problem contains this small parameter ε by the theory of small perturbations one can expand the eigenfunctions W and eigenvalue Ra_{T} in the form of a perturbation series

$$\left(W,\,\mathrm{Ra}_T\right)=\left(W_0,\,R_0\right)+\varepsilon\left(W_1,\,R_1\right)+\varepsilon^2\left(W_2,\,R_2\right)+\cdots. \tag{3.1}$$

Here W_0 and R_0 are respectively the eigenfunctions and eigenvalues of the unmodulated system and W_i and R_i, $(i\geq 1)$ are the corrections to W_0 and R_0 in the presence of gravity modulation.

A substitution of Eq.(3.1) into Eq.(2.23) and comparison of corresponding terms, results in

$$LW_0=0, \tag{3.2}$$

$$LW_1=-a^2R_0L_3\cos\omega tW_0-a^2R_1L_3\sin\pi z+a^2\mathrm{Ra}_SL_2\cos\omega tW_0, \tag{3.3}$$

$$\begin{gathered}LW_2=-a^2R_1L_3\cos\omega tW_0-a^2R_2L_3W_0-a^2R_0L_3\cos\omega tW_1-a^2R_1L_3W_1+\\+a^2\mathrm{Ra}_SL_2\cos\omega tW_1\end{gathered} \tag{3.4}$$

where $\quad L\equiv L_1L_2L_3\left(D^2-a^2\right)+a^2R_0L_3-a^2\mathrm{Ra}_SL_2,$

$$L_1\equiv\mathrm{Pr}^{-1}\left(\partial/\partial t\right)-\left(D^2-a^2\right),\qquad L_2\equiv\partial/\partial t-\left(D^2-a^2\right),\qquad L_3\equiv\partial/\partial t-Le^{-1}\left(D^2-a^2\right).$$

Each of W_i is required to satisfy the boundary conditions (2.24). Equation (3.2) obtained at zeroth order is the one used in the study of DDC in a fluid layer with a constant gravity field. A marginally stable solution of that problem is $W_0=\sin\pi z$ and the corresponding eigenvalue is

$$R_0=\mathrm{Ra}_S\mathrm{Le}+\left(\pi^2+a^2\right)^3\Big/a^2, \tag{3.5}$$

which assumes the minimum value R_{0c} for $a=a_0$, obtained by solving

$$3a_0^2\left(\pi^2+a_0^2\right)^2-\left(\pi^2+a_0^2\right)^3=0. \tag{3.6}$$

These are the classical results obtained for DDC in a binary fluid layer (Turner, 1973). Further, these equations yield the values $R_{0c}=27\pi^4/4=657.5$ and $a_0=\pi/\sqrt{2}$, for $\mathrm{Ra}_S=0$ which are associated with the classical Rayleigh-Benard problem (Chandrasekhar, 1981).

On substituting the zeroth order solution into Eq.(3.3) one can get,

$$LW_1=-a^2\left[\left(R_0\tilde{L}_3-\mathrm{Ra}_S\tilde{L}_2\right)\mathrm{Re}\left\{e^{-i\omega t}\right\}+R_1\delta^2\mathrm{Le}^{-1}\right]\sin\pi z \tag{3.7}$$

where $\tilde{L}_2=\delta^2-i\omega$, $\tilde{L}_3=\delta^2\mathrm{Le}^{-1}-i\omega$, $\delta^2=\pi^2+a^2$. The above equation is inhomogeneous and its solution poses a problem, because of the presence of resonance term. The mathematical properties and solvability conditions of the differential equations with time periodic coefficients have been extensively discussed by Yakubovich and Starzhinskii (1975). If this equation is to have a solution, the right-hand side must be orthogonal to the null space of the operator L. This requires that the steady part of the right-hand side must be orthogonal to its steady state solution W_0, i.e., $\sin\pi z$. Since $\cos\omega t$ varies sinusoidally with time, the only steady term is $-a^2 R_1 \delta^2 \mathrm{Le}^{-1} \sin\pi z$, so that

$$\int_0^1 \left(-a^2 R_1 \delta^2 \mathrm{Le}^{-1} \sin\pi z\right)\sin\pi z\, dz = 0\,,$$

which gives $-(0.5)a^2 R_1 \delta^2 \mathrm{Le}^{-1} = 0$, indeed $R_1 = 0$. This in turn implies that all the odd coefficients, *viz.*, R_1, R_3, ... in Eq.(3.1) are zero because a change of the sign of ε shifts the time origin by half period but does not change the physical problem. Now Eq.(3.12) is solved by inverting the operator L term-by-term. Consequently,

$$W_1 = -a^2\left(R_0\tilde{L}_3 - \mathrm{Ra}_S\tilde{L}_2\right)\mathrm{Re}\left\{\sum_{n=1}^{\infty} e^{-i\omega t}\sin n\pi z \big/ L(\omega,n)\right\} \tag{3.8}$$

where $L(\omega,n) = B_1 + i\,B_2$,

with $B_1 = \delta_n^2\,\mathrm{Pr}^{-1}\left[a^2\,\mathrm{Pr}\left(\mathrm{Le}^{-1}R_0 - Ra_S\right) - \mathrm{Le}^{-1}\,\mathrm{Pr}\,\delta_n^6 + \delta_n^2\omega^2\left(1+\mathrm{Pr}+\mathrm{Le}^{-1}\right)\right]$,

$B_2 = \omega\,\mathrm{Pr}^{-1}\left[\delta_n^2\left(\delta_n^4\mathrm{Le}^{-1} + \delta_n^4\,\mathrm{Pr}\left(1+\mathrm{Le}^{-1}\right) - \omega^2\right) + a^2\,\mathrm{Pr}\left(\mathrm{Ra}_S - R_0\right)\right]$ and $\delta_n^2 = n^2\pi^2 + a^2$.

With the zeroth and first order solutions, Eq.(3.4) now looks like

$$LW_2 = -a^2\delta^2\mathrm{Le}^{-1}R_2\sin\pi z - a^2\left(R_0\tilde{\tilde{L}}_3 - \mathrm{Ra}_S\tilde{\tilde{L}}_2\right)\mathrm{Re}\left\{e^{-i\omega t}\right\}W_1 \tag{3.9}$$

where $\tilde{\tilde{L}}_2 = \delta_n^2 - 2i\omega$, $\tilde{\tilde{L}}_3 = \delta_n^2 - 2i\omega$. The aim of Eq.(3.9) is merely to find R_2, the first non-zero correction to Ra_T which characterizes the influence of g-jitter. The solvability condition of Eq.(3.9), i.e., the time-independent part of right-hand side must be an orthogonal steady state solution, gives,

$$R_2 = -2\,\mathrm{Le}\,\delta^{-2}\left(R_0\tilde{\tilde{L}}_3 - \mathrm{Ra}_S\tilde{\tilde{L}}_2\right)\mathrm{Re}\left\{\int_0^1 \overline{e^{-i\omega t}W_1}\sin\pi z\, dz\right\} \tag{3.10}$$

where the over bar indicates the time average. Now, from Eq.(3.8), one can obtain

$$\mathrm{Re}\left\{\overline{e^{-i\omega t}W_1}\sin\pi z\right\} = -\left[a^2\left(R_0 L_3 - \mathrm{Ra}_S L_2\right)\right]^{-1}\overline{W_1 L W_1}\,. \tag{3.11}$$

The time-average is computed by using Eqs (3.8) and (3.7) and then the resultant of Eq.(3.11) is substituted into Eq.(3.10) to obtain

$$R_2 = \frac{a^2\mathrm{Le}}{2\delta^2}\sum_{n=1}^{\infty}\frac{B_1\left(B_3^2 - \omega^2 B_4^2\right) + 2\omega B_2 B_3 B_4}{B_1^2 + B_2^2}\,, \tag{3.12}$$

with $B_3 = \delta^2 \left(\mathrm{Le}^{-1} R_0 - \mathrm{Ra}_S \right)$, $B_4 = \mathrm{Ra}_S - R_0$. If desired Eq.(3.9) could now be solved for W_2, and the procedure may be continued to obtain further corrections to W and Ra_T. However since we required to estimate only the first non-trivial correction to Ra_T, we shall stop at this step. The value of Ra_T obtained by this procedure is the eigenvalue corresponding to the eigenfunctions W, which, though oscillating, remains bounded in time. Since Ra_T is a function of the wave number a and the amplitude of modulation ε, accordingly we expand

$$\mathrm{Ra}_T(a, \varepsilon) = R_0(a) + \varepsilon^2 R_2(a) + \varepsilon^4 R_4(a) + \cdots, \tag{3.13}$$

$$a = a_0 + \varepsilon^2 a_2 + \cdots \tag{3.14}$$

where R_0 and a_0 are respectively the Rayleigh number and wave number for the unmodulated system. Ra_T as a function of the wave number a has a least value Ra_{Tc} which occurs at $a = a_c$ and the critical wave number occurs when $\partial \mathrm{Ra}_T / \partial a = 0$, that implies

$$\frac{\partial R_0}{\partial a_0} + \varepsilon \left(\frac{\partial^2 R_0}{\partial a_0^2} \right) a_1 + \varepsilon^2 \left[\frac{1}{2} \left(\frac{\partial^3 R_0}{\partial a_0^3} \right) a_1^2 + \left(\frac{\partial^2 R_0}{\partial a_0^2} \right) a_2 + \frac{\partial R_2}{\partial a_0} \right] + \cdots = 0 . \tag{3.15}$$

The critical thermal Rayleigh number is then given by

$$\begin{aligned} \mathrm{Ra}_{Tc}(a, \varepsilon) &= R_{0c} + \varepsilon^2 R_{2c} + \varepsilon^4 R_{4c} + \cdots \\ &= R_0(a_0) + \varepsilon \left(\partial R_0 / \partial a_0 \right) a_1 + \varepsilon^2 \left[\frac{1}{2} \left(\partial^2 R_0 / \partial a_0^2 \right) a_1^2 + \left(\partial R_0 / \partial a_0 \right) a_2 + R_2(a_0) \right] + \cdots . \end{aligned} \tag{3.16}$$

However, on equating the coefficients of like powers of ε on both sides of Eq.(3.16) we get

$$\partial R_0 / \partial a_0 = 0 , \qquad a_1 = 0 , \qquad a_2 = -\left(\partial R_2 / \partial a_0 \right) \Big/ \left(\partial^2 R_0 / \partial a_0^2 \right), \tag{3.17}$$

so that

$$\mathrm{Ra}_{Tc}(a, \varepsilon) = R_0(a_0) + \varepsilon^2 R_2(a_0) + \cdots . \tag{3.18}$$

It is only when one wishes to evaluate R_4, a_2 must be taken into account (Venezian, 1969). If $R_{2c} = \left(R_2 \right)_{a=a_0}$ is positive, the g-jitter stabilizes the system, while if R_{2c} is negative, the effect of vibration is destabilizing. The influence of R_{2c} relative to R_{0c} is revealed in terms of R_{2c} / R_{0c}.

4. Results and discussion

The onset of DDC in a fluid layer subject to a time-periodically varying gravitational force, is investigated analytically using the linear stability theory. Due to g-jitter there is a shift in the onset criteria.

The critical Rayleigh number and the wave number are computed using the regular perturbation technique based on the assumption that the amplitude of imposed modulation is very small. Because of this the present analysis is restricted only to the first order correction to the critical Rayleigh number, *viz*., R_{2c}. The influence of various governing parameters on the onset of DDC is revealed through Figs 1-3.

When ω is very small, the period of modulation becomes sufficiently large and the disturbances grow to a large extent and therefore, the entire system under consideration becomes unstable. This is justified by the magnitude of R_{2c}, which is found to be sufficiently small or even negative in some cases. On the otherhand, when ω is very large the effect of gravity modulation is confined only to a narrow boundary layer near the boundary. This is due to the fact that the high frequencies correspond to renormalization of the static gravity field. Thus, outside this thickness the buoyancy force takes a mean value tending towards the equilibrium sate value of the unmodulated case. Therefore, the value of R_{2c} approaches asymptotically to zero. Hence the effect of gravity modulation is significant only for the moderate values of ω.

The variation of R_{2c} with ω is revealed through Figs 1-3. It is observed that R_{2c} is negative for small ω while for moderate values of ω, there is a considerable increase in the value of R_{2c}. Thus, the low frequency gravity modulation destabilizes the system whereas enhances the stability when ω is quite large. The system becomes most stable when R_{2c} attains a maximum value corresponding to a specific frequency $\omega = \omega^*$. If ω is increased beyond ω^*, we notice that R_{2c} goes on decreasing and becomes independent of ω when the frequency is sufficiently large. Thus, the critical Rayleigh number tends to its equilibrium value of unmodulated state since R_{2c} tends asymptotically to zero.

The influence of Ra_S on the gravity modulated binary fluid layer is displayed in Fig.1. When $\mathrm{Ra}_S = 0$, the variation of R_{2c} is found to be similar to that of a single component case. In this case R_{2c} is positive over the entire range of values of ω. This indicates the stabilizing effect of gravity modulation on the onset of thermal convection in a viscous fluid layer. However, when $\mathrm{Ra}_S \neq 0$, R_{2c} is negative for small values of ω. Thus the presence of a second diffusing agent, namely, the solute concentration leads the gravity modulation to advance the convection as compared to the unmodulated case. For a moderate frequency the stabilizing effect is noticed and at $\omega = \omega^*$, the system becomes most stable due to both gravity modulation and the solute gradient. Further it is found that ω^* increases with Ra_S.

In Fig.2 the effect of Le on the stability of the binary fluid layer is exhibited. In the DDC experiments the most common solutes used with water are sugar and salt. The value of Le is *83* and *243* respectively for water-salt and water-sugar mixture (Bejan, 1993). Therefore, we consider a range *0-300* for the values of Le to encompass a variety of binary mixtures. The role of Le is to stabilize the system. The frequency ω^* at which the system is most stable is independent of Le. When Le = *0*, we observed that R_{2c} is negatively very small over the entire domain of ω. Thus in this case the gravity modulation shows a very weak destabilizing effect. Figure 3 depicts the variation of R_{2c} with ω for different values of Pr. It is reported that the influence Pr is to enhance the stabilizing effect of Ra_S and Le. This figure also indicates that ω^* increases with Pr.

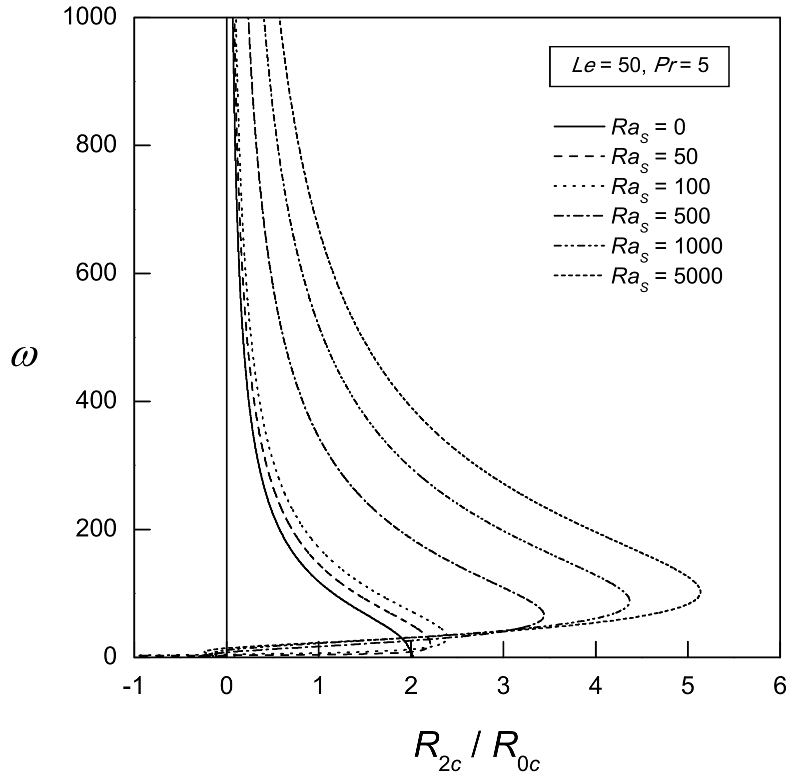

Fig.1. Variation of R_{2c} with ω for different values of Ra_s.

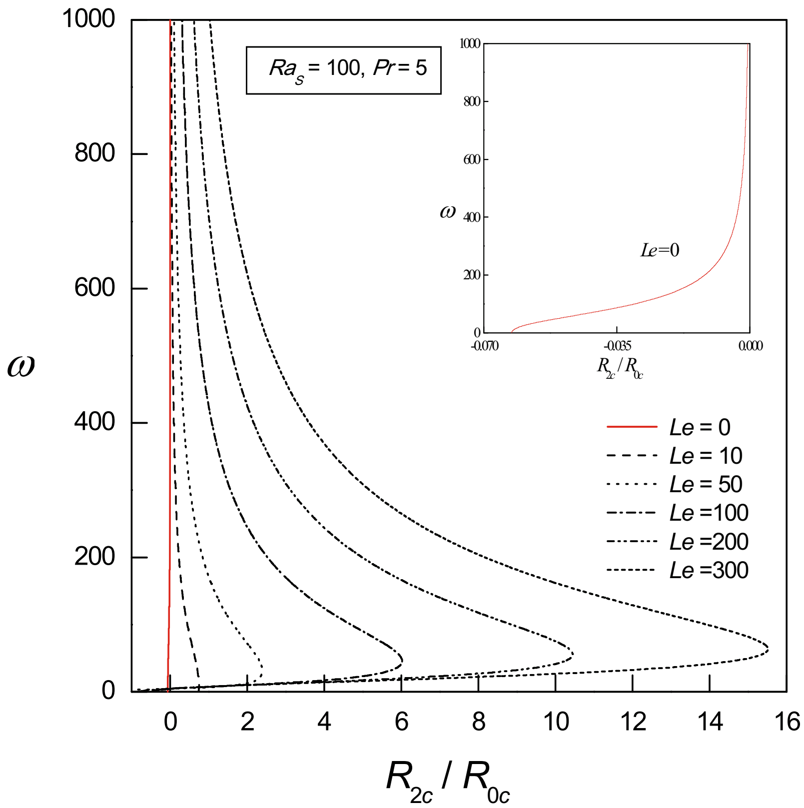

Fig.2. Variation of R_{2c} with ω for different values of Le.

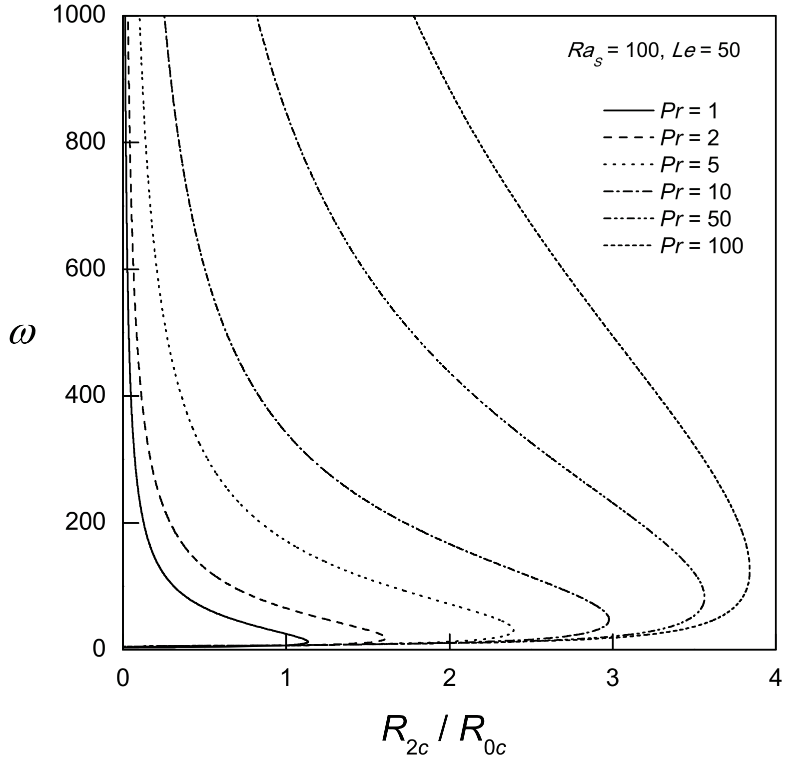

Fig.3. Variation of R_{2c} with ω for different values of Pr.

5. Conclusions

The onset of gravity modulated DDC in a binary fluid layer is investigated analytically. The variation of correction of the Rayleigh number with frequency of *g*-jitter is shown graphically. The effect of *g*-jitter is significant only for small and moderate values of ω, while $R_{2c} \to 0$ as $\omega \to \infty$. In general, R_{2c} is positive over the entire realm of ω, indicating the inhibition of DDC as compared to the unmodulated system. However, for very small frequency R_{2c} is found to be negative. Le and Pr reinforce the stabilizing effect of *g*-jitter and Ra_S. When Le = *0*, *g*-jitter shows a very weak destabilizing effect. The system becomes most stable at a specific frequency $\omega = \omega^*$ and ω^* increases with Ra_S and Pr and it is independent of Le.

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Nomenclature

a – overall horizontal wave number, $\sqrt{l^2 + m^2}$
Le – Lewis number, κ_T / κ_S
l, m – wave numbers in x- and y-direction
Pr – Prandtl number, ν / κ
p – pressure

$\boldsymbol{q}$ – velocity vector, (u,v,w)

Ra_S – solute Rayleigh number, $\beta_S g(S_l - S_u)d^3/\nu\kappa_T$

Ra_T – thermal Rayleigh number, $\beta_T g(T_l - T_u)d^3/\nu\kappa_T$

S – solute concentration

T – temperature

t – time

x, y, z – space coordinates

β – expansion coefficient

ε – amplitude of modulation

κ – diffusivity

μ – dynamic viscosity

ν – kinematic viscosity

ρ – density

ω – dimensionless frequency of modulation, $\bar{\omega}d^2/\kappa_T$

$\bar{\omega}$ – frequency of modulation,

∇_1^2 – $\partial^2/\partial x^2 + \partial^2/\partial y^2$

∇^2 – $\nabla_1^2 + \partial^2/\partial z^2$

Subscripts/Superscripts

$'$ – perturbed quantity

b – basic state

c – critical

0 – reference value

S – solute

T – thermal

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