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EFFECTS OF THERMAL DIFFUSION AND VISCOUS DISSIPATION ON UNSTEADY MHD FREE CONVECTION FLOW PAST A VERTICAL POROUS PLATE UNDER OSCILLATORY SUCTION VELOCITY WITH HEAT SINK

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The thermal diffusion and viscous dissipation effects on an unsteady MHD free convection heat and mass transfer flow of an incompressible, electrically conducting, fluid past an infinite vertical porous plate embedded in a porous medium of time dependent permeability under oscillatory suction velocity in the presence of a heat absorbing sink have been studied. It is considered that the influence of a uniform magnetic field acts normal to the flow and the permeability of the porous medium fluctuates with time. The dimensionless governing equations for this investigation have been solved numerically by using the efficient Galerkin finite element method. The numerical results obtained have been presented through graphs and tables for the thermal Grashof number $(G_r > 0)$ corresponding to the cooling of the porous plate and $(G_r < 0)$ corresponding to heating of the porous plate to observe the effects of various material parameters encountered in the problem under investigation. Numerical data for skin-friction, Nusselt and Sherwood numbers are tabulated and then discussed.

Key words*:* MHD, vertical plate, oscillatory suction velocity, heat sink, Galerkin FEM.

1. Introduction

The study of natural convection induced by the simultaneous action of buoyancy forces from thermal and mass diffusion is of considerable interest in many industrial applications such as geophysics, oceanography, drying processes and solidification of binary alloys. The effect of a magnetic field on free convection flows is important in liquid metals, electrolytes and ionized gasses. The thermal physics of MHD problems with mass transfer is of interest in power engineering and metallurgy. When free convection flows occur at high temperature, radiation effects on the flow become significant.

Sreekanth *et al.* (1996) investigated the effect of permeability variation on a free convective flow past a vertical porous wall in a porous medium when the permeability varies in time. Singh *et al.* (1999) studied a hydro-magnetic free convective and mass transfer flow of a viscous stratified fluid considering variation in permeability with direction. Acharya *et al*. (2000) presented magnetic field effects on the free convection and mass transfer flow through a porous medium with constant suction and constant heat flux. Sahoo *et al*. (2003) analyzed on MHD unsteady free convective flow past an infinite vertical plate with constant suction and heat sink. Singh *et al*. (2003) presented the effects of oscillatory suction velocity on free convection and mass transfer flow of a viscous fluid past an infinite vertical porous plate. Seddek and Salama (2007) presented the effects of temperature dependent viscosity and thermal conductivity on an unsteady MHD convective heat transfer past an infinite porous moving plate with variable suction. Hossain and Mandal (2007) presented effects of mass transfer and free convection on an unsteady MHD flow past a vertical porous plate with constant suction. Raji Reddy and Sri Hari (2009) studied a numerical solution of an unsteady flow of a radiating and chemically reacting fluid with time dependent suction. Soundalgekar (1972) analyzed the viscous dissipation heat on the two-dimensional unsteady free convective flow past an infinite vertical porous plate when the temperature oscillates in time and there is constant suction at the plate. Cookey *et al*. (2003) studied the influence of viscous dissipation and radiation on an unsteady MHD free convection flow past an infinite heated vertical plate in a porous medium with time dependent suction. Ramachandra Prasad and Bhaskar Reddy (2008) presented radiation and mass transfer effects on an unsteady MHD convection flow past a semi-infinite vertical permeable moving plate embedded in a porous medium with viscous dissipation. Ramana Reddy *et al.* (2011) studied on MHD free convection heat and mass transfer flow of a viscous fluid past a semi-infinite vertical moving porous plate embedded in a porous medium in the presence of heat generation taking thermal diffusion into account. Prabhakar Reddy and Anand Rao (2011) studied the numerical solution of thermal diffusion effect on an unsteady MHD free convective mass transfer flow past a vertical porous plate with Ohmic dissipation. Prabhakar Reddy and Anand Rao (2011) studied the radiation and thermal diffusion effects on an unsteady MHD free convection mass transfer flow past an infinite vertical porous plate with the Hall current and a heat source.

The aim of this study is to investigate the effects of thermal diffusion and viscous dissipation on an unsteady MHD free convection involving heat and mass transfer fluid flow past an infinite vertical porous plate with permeability variation under oscillatory suction velocity in the presence of a heat absorbing sink. The problem is governed by the system of coupled non-linear partial differential equations whose exact solutions are difficult to obtain. So, the Galerkin finite element method has been adopted for its solution, which is more economical from computational point of view.

2. Mathematical analysis

A two-dimensional unsteady free convection flow of a laminar, incompressible, electrically conducting, viscous dissipative fluid flow past an infinite vertical porous plate embedded in a porous medium of time dependent permeability and oscillatory suction velocity in the presence of a heat absorbing sink has been considered. The $x'-$axis is along the plate in the direction of the flow and the $y'-$ axis normal to it. A uniform magnetic field is applied normal to the direction of the flow. It is assumed that the magnetic Reynolds number is much less than unity so that the induced magnetic field is neglected in comparison to the applied magnetic field. Further, all the fluid properties are assumed to be constant except that of the influence of density variation with temperature. Therefore, the basic flow in the medium is entirely due to the buoyancy force caused by a temperature difference between the wall and the medium. In a convective fluid the flow of mass is caused by a temperature difference, the thermal diffusion (Soret effect) cannot be neglected. Due to infinite plane surface assumption, the flow variables are the functions of y' and t' only. The governing equation for this investigation is based on the balances of mass, linear momentum, energy and species concentration. Taking into consideration the assumptions made above, these equations can be written in Cartesian frame of reference, as follows

$$\frac{\partial v'}{\partial y'}=0, \tag{2.1}$$

$$\frac{\partial u'}{\partial t'}+v'\frac{\partial u'}{\partial y'}=\nu\frac{\partial^2 u'}{\partial y'^2}+g\beta\left(T'-T'_\infty\right)+g\beta^*\left(C'-C'_\infty\right)-\frac{\sigma B_0^2}{\rho}u'-\frac{\nu}{K'_0}u', \tag{2.2}$$

$$\frac{\partial T'}{\partial t'}+v'\frac{\partial T'}{\partial y'}=\frac{k}{\rho C_p}\frac{\partial^2 T'}{\partial y'^2}+S'\left(T'-T'_\infty\right)+\frac{\nu}{C_p}\left(\frac{\partial u'}{\partial y'}\right)^2, \tag{2.3}$$

$$\frac{\partial C'}{\partial t'}+v'\frac{\partial C'}{\partial y'}=D_M\frac{\partial^2 C'}{\partial y'^2}+D_T\frac{\partial^2 T'}{\partial y'^2}. \tag{2.4}$$

The boundary conditions for the velocity, temperature and concentration fields in a dimensional form are given as follows

$$\begin{aligned} &u'=0, \quad T'=T'_w+\varepsilon\left(T'_w-T'_\infty\right)e^{in't'}, \quad C'=C'_w+\varepsilon\left(C'_w-C'_\infty\right)e^{in't'} \quad \text{at} \quad y'=0,\\ &u'\to 0, \quad T'\to T'_\infty, \quad C'\to C'_\infty \quad \text{as} \quad y'\to\infty. \end{aligned} \tag{2.5}$$

Now from Eq.(2.1) it can be seen that v' is either a constant or a function of time only. Hence, the suction velocity normal to the plate is assumed to be oscillatory about a non-zero constant mean, one can write

$$v'=-v_0\left(1+\varepsilon e^{in't'}\right) \tag{2.6}$$

where v_0 is the mean suction velocity and $v_0>0$, $\varepsilon<<1$ is a positive constant. The negative sign indicates that the suction velocity is normal to the plate and the permeability of the porous medium is considered to be

$$K_0\left(t'\right)=K'\left(1+\varepsilon e^{in't'}\right). \tag{2.7}$$

In order to write the governing equations and the boundary conditions in a dimensionless form, the following non-dimensional quantities are introduced.

$$\begin{aligned} &y=\frac{v_0 y'}{\nu}, \quad t=\frac{v_0^2 t'}{4\nu}, \quad n=\frac{4\nu n'}{v_0^2}, \quad u=\frac{u'}{v_0},\\ &\theta=\frac{T'-T'_\infty}{T'_w-T'_\infty}, \quad \phi=\frac{C'-C'_\infty}{C'_w-C'_\infty}, \quad K=\frac{K'v_0^2}{\nu^2}, \quad M=\frac{B_0}{v_0}\sqrt{\frac{\sigma\nu}{\rho}},\\ &\mathrm{S_c}=\frac{\nu}{D_M}, \quad \mathrm{P}_r=\frac{\mu C_p}{k}, \quad \mathrm{G_r}=\frac{g\beta\nu\left(T'_w-T'_\infty\right)}{v_0^3}, \quad \mathrm{G_m}=\frac{g\beta^*\nu\left(C'_w-C'_\infty\right)}{v_0^3},\\ &\mathrm{S_r}=\frac{D_T\left(T'_w-T'_\infty\right)}{\nu\left(C'_w-C'_\infty\right)}, \quad S=\frac{S'\nu}{v_0^2}, \quad \mathrm{E_c}=\frac{v_0^2}{C_p\left(T'_w-T'_\infty\right)}. \end{aligned} \tag{2.8}$$

On substitution of Eqs (2.6), (2.7) and (2.8) into Eqs (2.2), (2.3), (2.4) and (2.5), the following governing equations are obtained in a non-dimensional form

$$\frac{1}{4}\frac{\partial u}{\partial t}-\left(1+\varepsilon e^{int}\right)\frac{\partial u}{\partial y}=\mathrm{G_r}\theta+\mathrm{G_m}\phi+\frac{\partial^2 u}{\partial y^2}-\frac{u}{K\left(1+\varepsilon e^{int}\right)}-M^2 u, \tag{2.9}$$

$$\frac{1}{4}\frac{\partial\theta}{\partial t}-\left(1+\varepsilon e^{int}\right)\frac{\partial\theta}{\partial y}=\frac{1}{P_r}\frac{\partial^2\theta}{\partial y^2}+S\theta+E_c\left(\frac{\partial u}{\partial y}\right)^2, \tag{2.10}$$

$$\frac{1}{4}\frac{\partial\phi}{\partial t}-\left(1+\varepsilon e^{int}\right)\frac{\partial\phi}{\partial y}=\frac{1}{S_c}\frac{\partial^2\phi}{\partial y^2}+S_r\frac{\partial^2\theta}{\partial y^2}. \tag{2.11}$$

The corresponding boundary conditions for velocity, temperature and concentration fields in a non-dimensional form are

$$\begin{aligned} &u=0, \quad \theta=1+\varepsilon e^{int}, \quad \phi=1+\varepsilon e^{int} \quad \text{at} \quad y=0, \\ &u\to 0, \quad \theta\to 0, \quad C\to 0 \quad \text{as} \quad y\to\infty. \end{aligned} \tag{2.12}$$

The mathematical formulation of the problem is now completed. All the physical quantities have their usual meaning.

3. Numerical solution process

By applying the Galerkin finite element method for Eq.(2.9) over the two nodded linear element $(e),\left(y_j \le y \le y_k\right)$ is

$$\int_{y_j}^{y_k}\psi^{(e)^T}\left[\frac{\partial^2 u^{(e)}}{\partial y^2}+J_1\frac{\partial u^{(e)}}{\partial y}-\frac{1}{4}\frac{\partial u^{(e)}}{\partial t}-J_2u^{(e)}+\left(G_r\theta+G_m\phi\right)\right]dy=0 \tag{3.1}$$

where $J_1=1+\varepsilon e^{int}$ and $J_2=M^2+\dfrac{1}{K\left(1+\varepsilon e^{int}\right)}$. Integrating the first term in Eq.(3.1) by parts one obtains

$$\begin{aligned} &\left\{\psi^{(e)^T}\frac{\partial u^{(e)}}{\partial y}\right\}_{y_k}^{y_j}+ \\ &-\int_{y_j}^{y_k}\left\{\frac{\partial\psi^{(e)^T}}{\partial y}\frac{\partial u^{(e)}}{\partial y}-\psi^{(e)^T}\left(J_1\frac{\partial u^{(e)}}{\partial y}+\frac{1}{4}\frac{\partial u^{(e)}}{\partial t}+J_2u^{(e)}-\left(G_r\theta+G_m\varphi\right)\right)\right\}dy=0. \end{aligned} \tag{3.2}$$

Since the derivative du/dy is not specified at either ends of the element $(e),\left(y_j \le y \le y_k\right)$, so that neglecting the first term in Eq.(3.2) one gets

$$\int_{y_j}^{y_k}\left\{\frac{\partial\psi^{(e)^T}}{\partial y}\frac{\partial u^{(e)}}{\partial y}-\psi^{(e)^T}\left(J_1\frac{\partial u^{(e)}}{\partial y}+\frac{1}{4}\frac{\partial u^{(e)}}{\partial t}+J_2u^{(e)}-\left(G_r\theta+G_m\varphi\right)\right)\right\}dy=0. \tag{3.3}$$

The finite element model may be obtained from Eq.(3.3) by substituting the finite element approximation over the two-nodded linear element $(e),\left(y_j \le y \le y_k\right)$ of the form

$$u^{(e)} = \psi^{(e)}\chi^{(e)}, \qquad \text{here} \qquad \psi^{(e)} = \left[\psi_j, \psi_k\right] \qquad \text{and} \qquad \chi^{(e)} = \left[u_j, u_k\right]^T \tag{3.4}$$

where u_j, u_k are the velocity components at j^{th} and k^{th} nodes of the typical element (*e*), $\left(y_j \le y \le y_k\right)$ and ψ_j, ψ_k are the basis functions defined as

$$\psi_j = \frac{y_k - y}{y_k - y_j} \qquad \text{and} \qquad \psi_k = \frac{y - y_j}{y_k - y_j}.$$

On substitution of Eq.(3.4) into Eq.(3.3), assembling the element equations for two consecutive elements $y_{i-1} \le y \le y_i$ and $y_i \le y \le y_{i+1}$, then putting the row corresponding to the node i equal to zero, we obtain the difference schemes with $h = y_k - y_j$ is the length of the element (*e*)

$$\begin{gathered} u^{\bullet}_{i-1} + 4u^{\bullet}_i + u^{\bullet}_{i+1} = \frac{4}{h^2}\left(6 - 3J_1 h - J_2 h^2\right)u_{i-j} - \frac{4}{h^2}\left(12 + 4J_2 h^2\right)u_i + \\ -\frac{4}{h^2}\left(6 + 3J_1 h - J_2 h^2\right)u_{i+j} + 24\left(\mathrm{G_r}\theta + \mathrm{G_m}\phi\right). \end{gathered} \tag{3.5}$$

Applying the trapezoidal rule to Eq.(3.5) and using the Crank-Nicholson method, we have

$$\begin{gathered} \left(1 - 12r + 6rJ_1 h + 2rJ_2 h^2\right)u_{i-1}^{j+1} + \left(4 + 24r + 8rJ_2 h^2\right)u_i^{j+1} + \\ +\left(1 - 12r - 6rJ_1 h + 2rJ_2 h^2\right)u_{i+1}^{j+1} = \left(1 + 12r - 6rJ_1 h - 2rJ_2 h^2\right)u_{i-1}^{j} + \\ +\left(4 - 24r - 8rJ_2 h^2\right)u_i^j + \left(1 + 12r + 6rJ_1 h - 2rJ_2 h^2\right)u_{i+1}^j + \\ +24k\left(\mathrm{G_r}\theta_i^j + \mathrm{G_m}\varphi_i^j\right). \end{gathered} \tag{3.6}$$

Here $r = k / h^2$ and h, k are the mesh sizes along $y-$ direction and time $t-$ direction respectively, and indices i and j refer to the space and time. Analogous equations can be obtained from Eqs (2.10) and (2.11). In the three obtained equations, taking $i = 1(1)n$ and using physical boundary conditions Eq.(2.12), we obtain the following systems of equations

$$A_i X_i = B_i, \qquad i = 1,2,3$$

where A_i 's are matrices of order n and X_i, B_i are column matrices having n components. The Gauss-Seidal iteration scheme is employed to solve the above matrix system of equations. Computations are carried out until the steady state is reached. The Galerkin finite element method is shown to be convergent and stable.

Skin friction, rate of heat and mass transfer

The skin-friction coefficient at the plate is given by $\tau = \left(\frac{\partial u}{\partial y}\right)_{y=0}$.

The rate of heat transfer coefficient, in terms of the Nusselt number, is given by $\mathrm{Nu} = -\left(\frac{\partial \theta}{\partial y}\right)_{y=0}$.

The rate of mass transfer coefficient, in terms of the Sherwood number, is given by $\mathrm{Sh} = -\left(\frac{\partial \phi}{\partial y}\right)_{y=0}$.

4. Numerical results and discussion

The formulation of the problem that accounts for the effects of thermal diffusion and viscous dissipation on an unsteady MHD free convection involving heat and mass transfer fluid flow past an infinite vertical porous plate embedded in a porous medium of time dependent permeability under oscillatory suction velocity in the presence of a heat absorbing sink was studied. It is considered that the influence of a uniform magnetic field acts normal to the flow and the permeability of the porous medium fluctuates with time. Numerical calculations have been carried out for dimensionless velocity u for cooling and heating of the porous plate, temperature θ and concentration ϕ, skin-friction, Nusselt number and Sherwood number for various values of material parameters. Numerical results are presented in figures and tables. These results show the effect of the material parameters on the quantities mentioned.

Figure 1 shows the effect of the Prandtl number $\mathrm{P_r}$ on the temperature field. It is seen that a decrease in the temperature and temperature boundary layer thickness takes place as the Prandtl number $\mathrm{P_r}$ increases. An increase in the Prandtl number results in the lowering of the average temperature within the boundary layer. The reason is that smaller values of $\mathrm{P_r}$ are equivalent to an increase in the thermal conductivity of the fluid, so that for higher values of $\mathrm{P_r}$ heat is able to diffuse away from the heated surface more rapidly. Hence, in the case of smaller values of $\mathrm{P_r}$ the thermal boundary layer is thicker and heat transfer is reduced. Figure 2 depicts the effect of the viscous dissipation parameter $\mathrm{E_c}$ on the temperature field. It is clear that an increase in the Eckert number $\mathrm{E_c}$ increases temperature in the boundary layer. Figure 3 shows the effect of the sink strength parameter S on the temperature field. It is seen that an increase in S leads to an increase in the temperature field. The effects of the Schmidt number $\mathrm{S_c}$ on the concentration field presented in Fig.4 for $\mathrm{S_c} = 0.22, 0.60, 0.78$ and $\mathrm{S_c} = 1.00$ correspond to hydrogen, water-vapour, ammonia and propyl-benzene, respectively. It can be clearly seen that an increase in the Schmidt number leads to a decrease in the concentration field in the boundary layer. Figure 5 shows the effect of the Soret number $\mathrm{S_r}$ on the concentration field. It is seen that an increase in $\mathrm{S_r}$ leads to an increase in the concentration field.

Figures 6 and 7 show the effect of the Prandtl number $\mathrm{P_r}$ on the velocity field for cooling and heating of the porous plate, respectively. It can be clearly seen that an increase in the Prandtl number leads to decreases in the fluid velocity in case of cooling of the plate whereas in case of heating of the plate it increases the fluid velocity. The effect of the Schmidt number $\mathrm{S_c}$ on the velocity field for cooling and heating of the porous plate are presented in Figs 8 and 9, respectively. It is seen that an increase in the Schmidt number decreases the fluid velocity in the boundary layer for cooling and heating of the porous plate due to the decrease in the molecular diffusivity and it results in a decrease in the concentration boundary layer which also leads to a decrease in the velocity boundary layer in both the cases. Figures 10 and 11 shows the effect of the Eckert number $\mathrm{E_c}$ on the velocity field for cooling and heating of the porous plate, respectively. It can be seen that an increase in the Eckert number increases the fluid velocity in case of cooling of the plate whereas in case of heating of the plate the fluid velocity decreases. The effects of the magnetic parameter M on the velocity field for cooling and heating of the porous plate are presented in Figs 12 and 13, respectively. It is seen that an increase in the magnetic parameter leads to a decrease in the fluid velocity in the boundary layer in case of cooling and heating of the plate. The effects of the porosity parameter K on the velocity field for cooling and heating of the porous plate are presented in Figs 14 and 15, respectively. It is observed that an increase in the porosity parameter leads to an increase in the fluid velocity in the boundary layer in case of cooling and heating of the plate. The effects of the Soret number S_r on the velocity field for cooling and heating of the porous plate are presented in Figs 16 and 17, respectively. It is clear that an increase in the Soret number increases the fluid velocity in the boundary layer in case of cooling and heating of the plate. The effects of the sink-strength parameter S on the velocity field for cooling and heating of the porous plate are presented in Figs 18 and 19, respectively. It is observed that an increase in the sink-strength parameter increases the fluid velocity in case of cooling of the plate whereas in case of heating of the plate a reverse effect is observed. Figures 20 and 21 show the effects of the thermal Grashof number G_r on the velocity field for cooling and heating of the porous plate, respectively. It is observed that increasing values of the thermal Grashof number increase the fluid velocity in the boundary layer in case of cooling of the plate as well as in case of heating of the plate. Figures 22 and 23 show the effect of the solutal Grashof number G_m on the velocity field for cooling and heating of the porous plate, respectively. It is seen that an increase in the solutal Grashof number increases the fluid velocity in the boundary layer in case of cooling and heating of the plate.

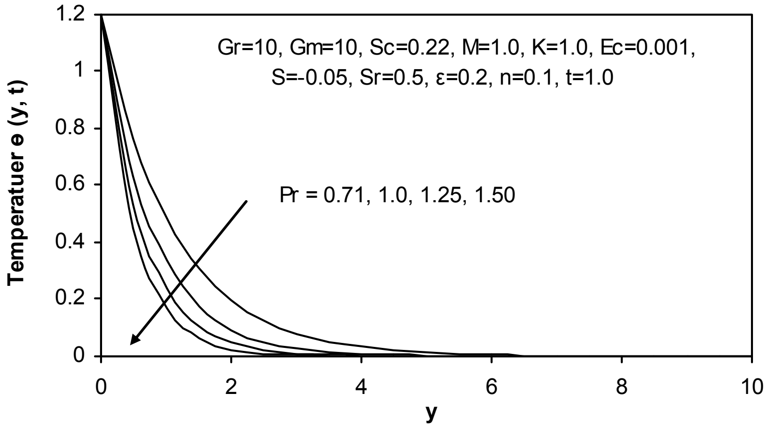

Fig.1. Effect of P_r on temperature profiles.

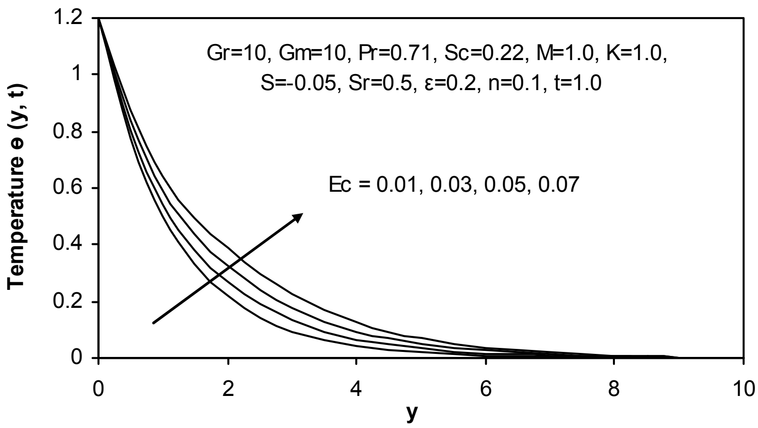

Fig.2. Effect of E_c on temperature profiles.

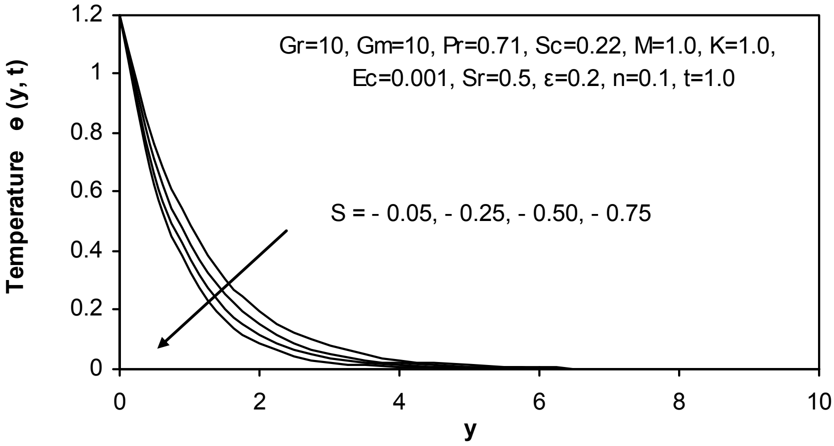

Fig.3. Effect of S on temperature profiles.

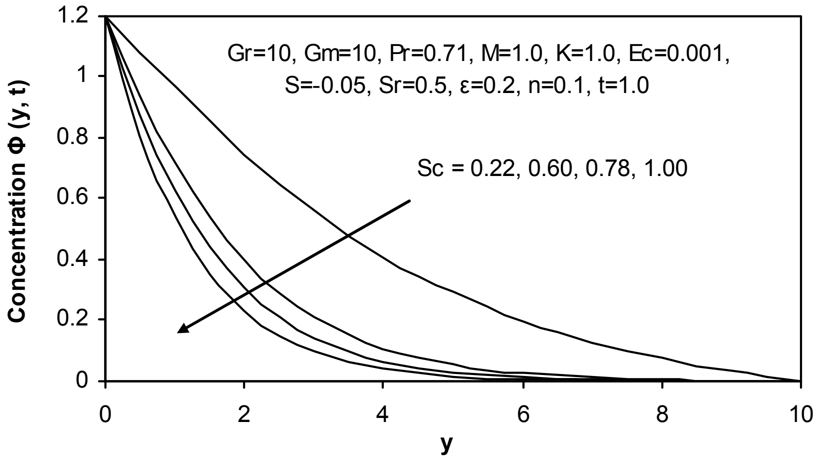

Fig.4. Effect of S_c on concentration profiles.

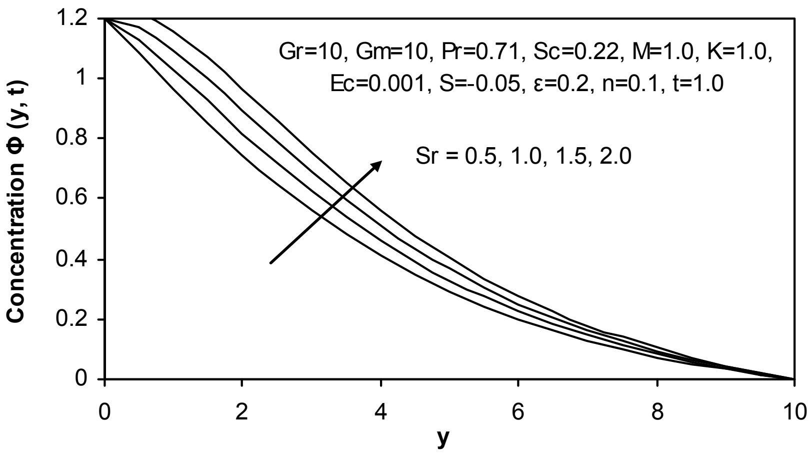

Fig.5. Effect of S_r on concentration profiles.

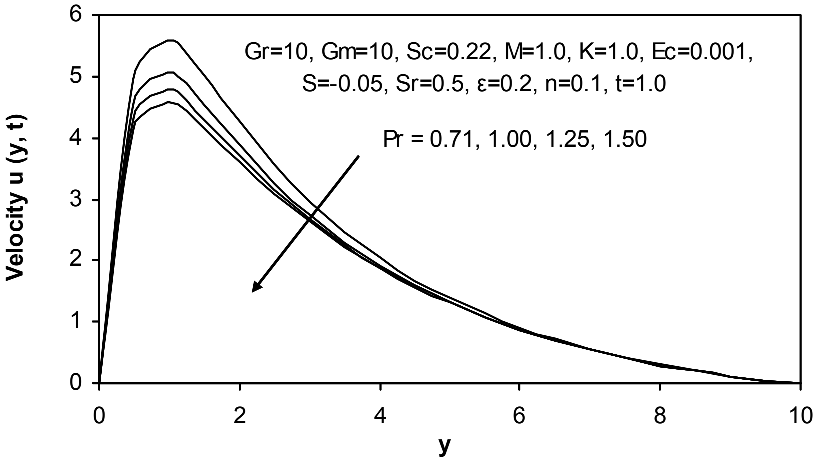

Fig.6. Effect of P_r on velocity profiles for cooling of the plate ($G_r > 0$).

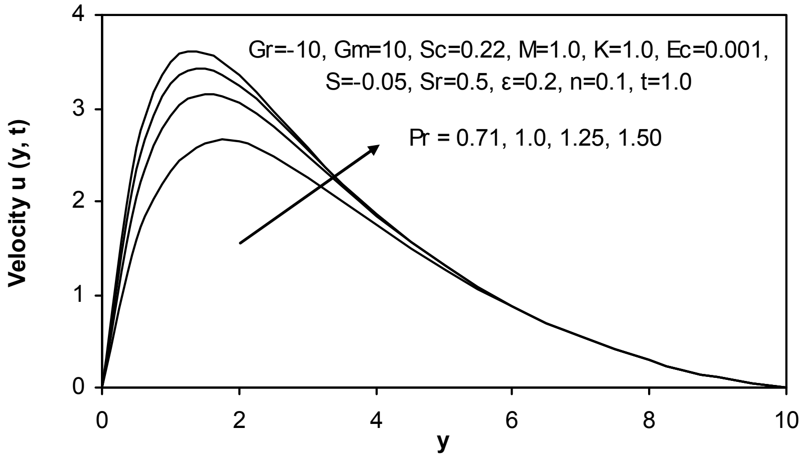

Fig.7. Effect of P_r on velocity profiles for heating of the plate ($G_r < 0$).

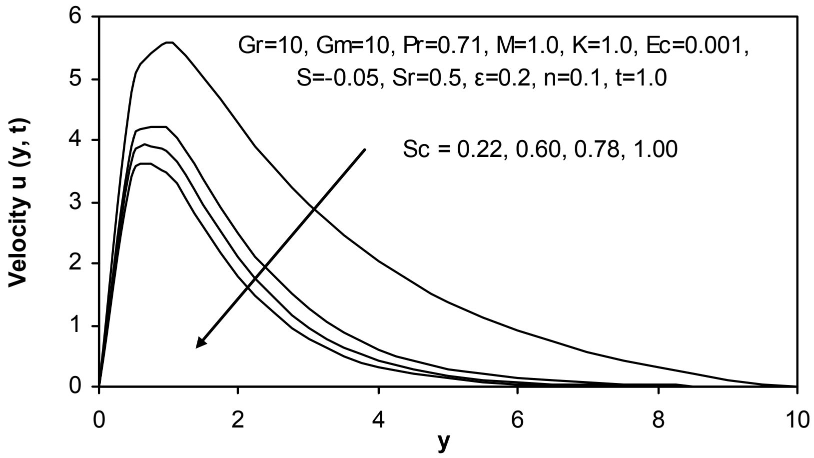

Fig.8. Effect of S_c on velocity profiles for cooling of the plate ($G_r > 0$).

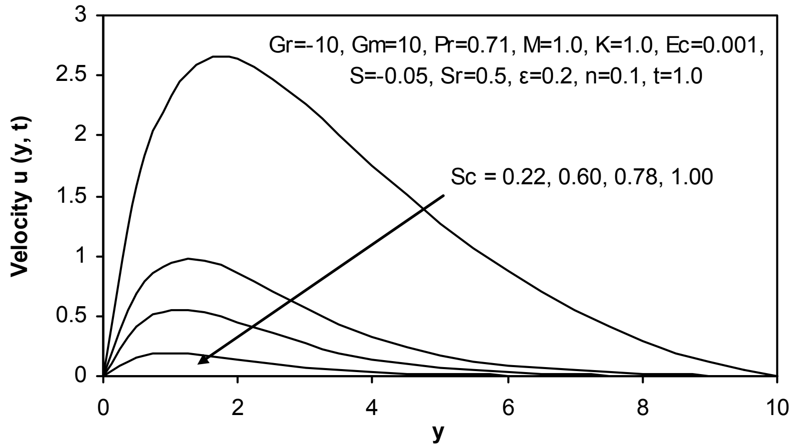

Fig.9. Effect of S_c on velocity profiles for heating of the plate $(G_r < 0)$.

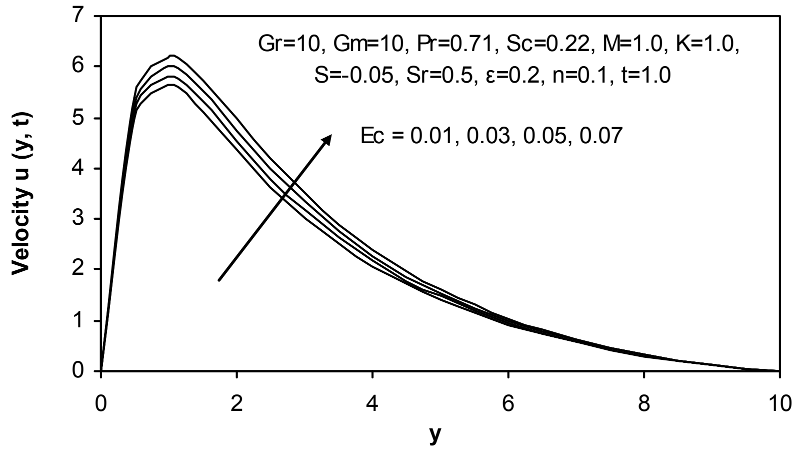

Fig.10. Effect of E_c on velocity profiles for cooling of the plate $(G_r < 0)$.

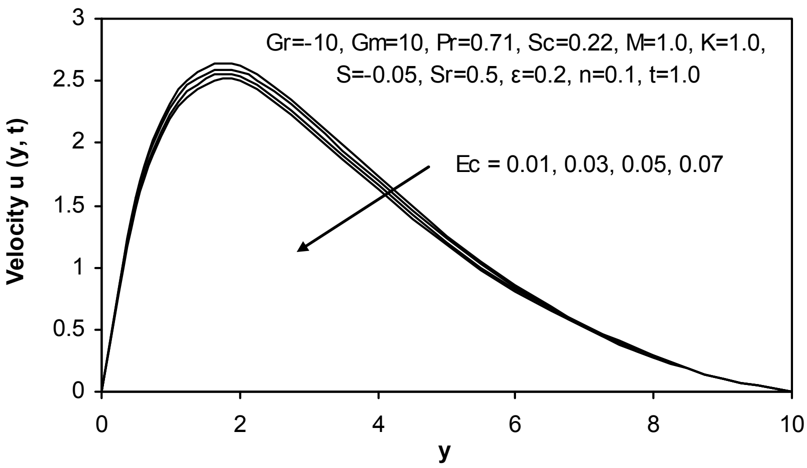

Fig.11. Effect of E_c on velocity profiles for heating of the plate $(G_r < 0)$.

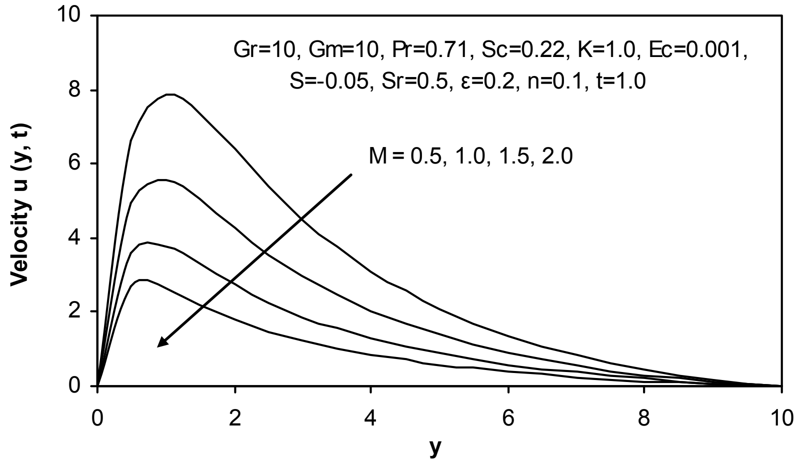

Fig.12. Effect of M on velocity profiles for cooling of the plate ($G_r < 0$).

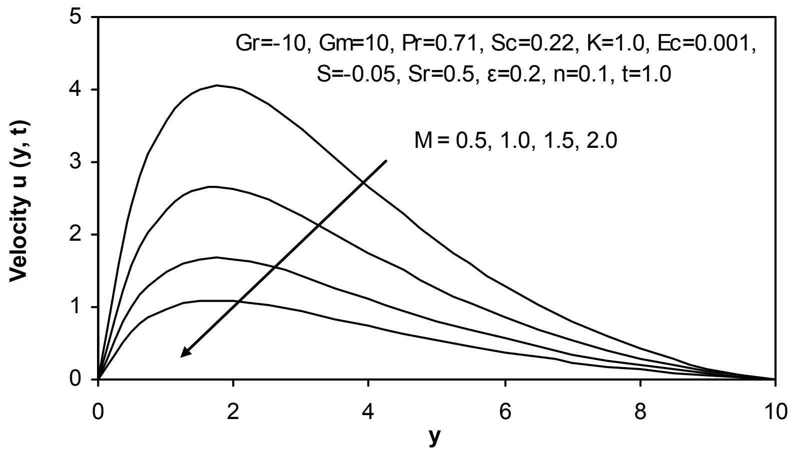

Fig.13. Effect of M on velocity profiles for heating of the plate ($G_r < 0$).

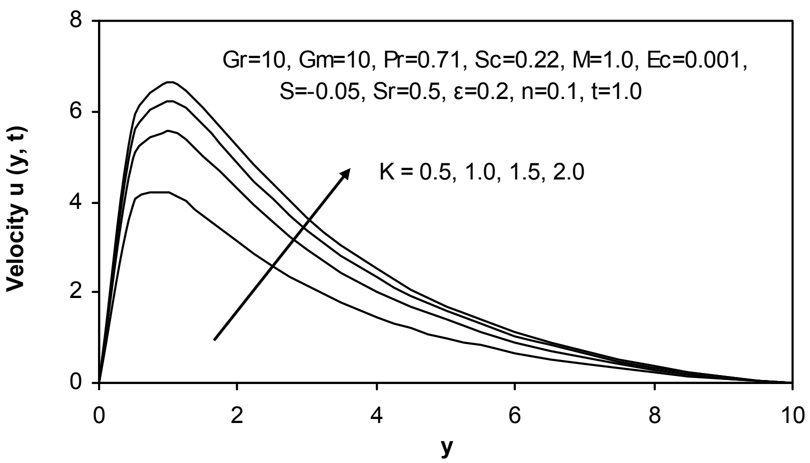

Fig.14. Effect of K on velocity profiles for cooling of the plate ($G_r < 0$).

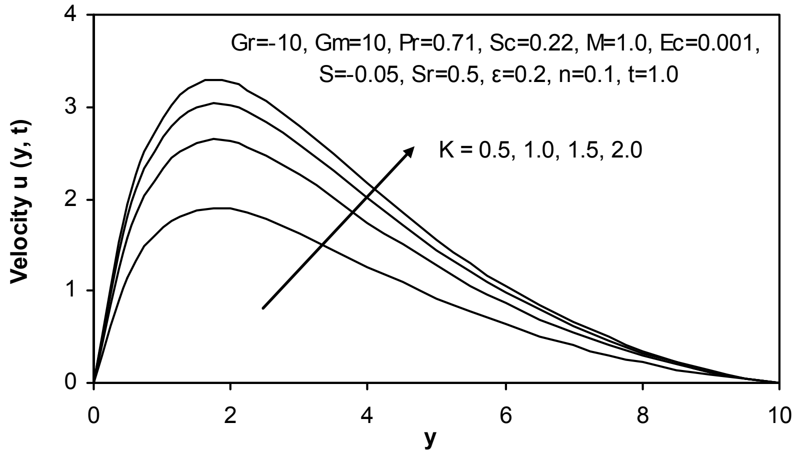

Fig.15. Effect of K on velocity profiles for heating of the plate $(G_r < 0)$.

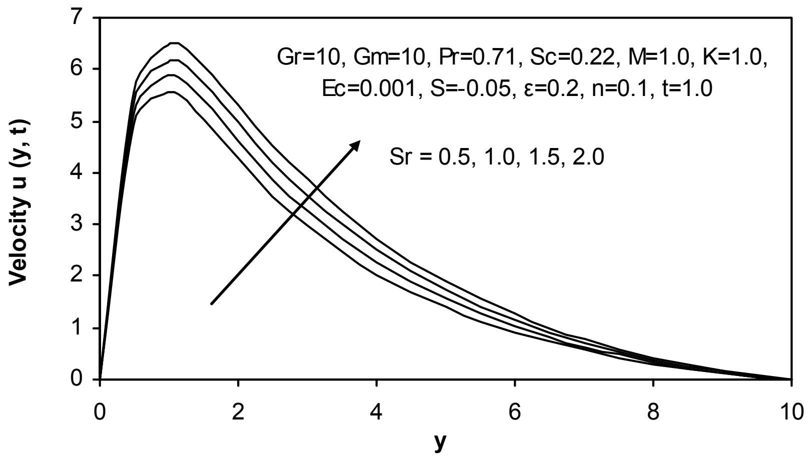

Fig.16. Effect of S_r on velocity profiles for cooling of the plate $(G_r < 0)$.

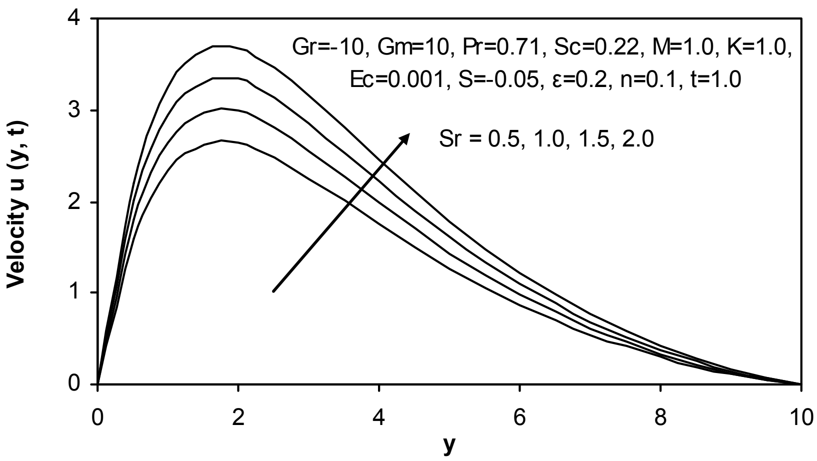

Fig.17. Effect of S_r on velocity profiles for heating of the plate $(G_r < 0)$.

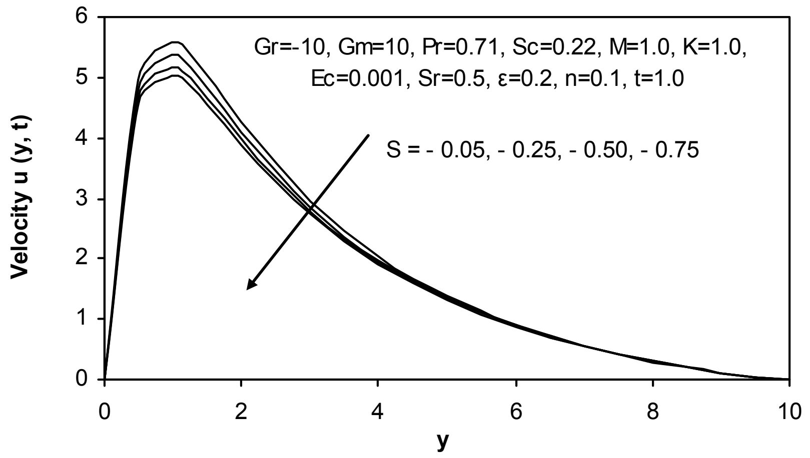

Fig.18. Effect of S on velocity profiles for cooling of the plate $(G_r < 0)$.

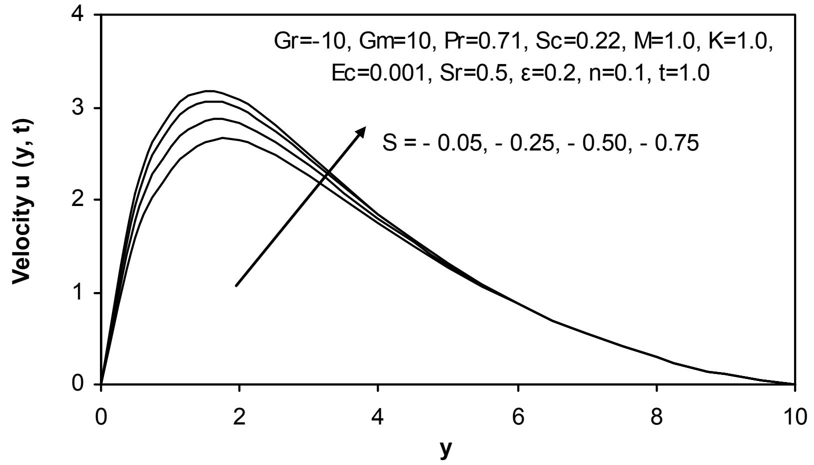

Fig.19. Effect of S on velocity profiles for heating of the plate $(G_r < 0)$.

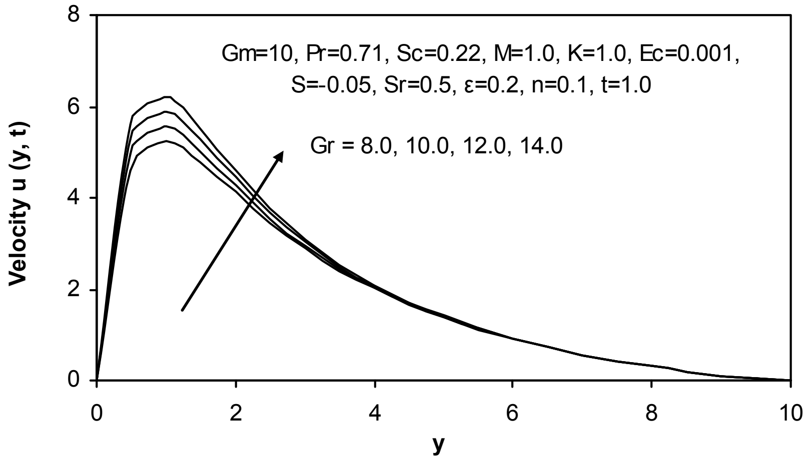

Fig.20. Effect of G_r on velocity profiles for cooling of the plate $(G_r < 0)$.

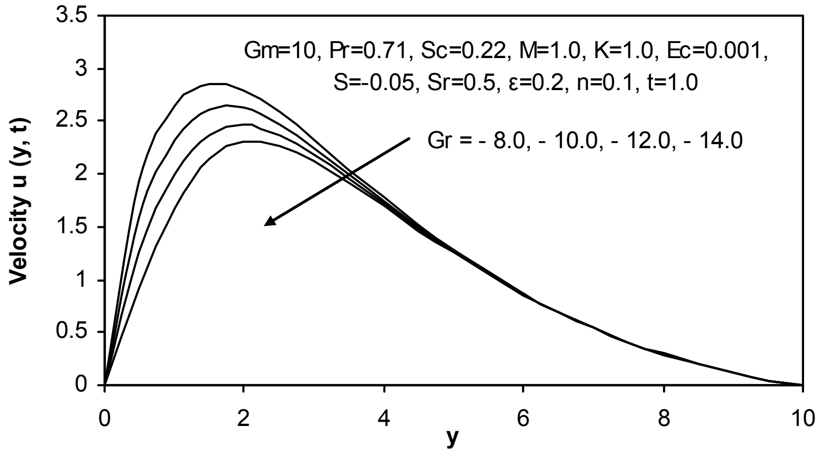

Fig.21. Effect of G_r on velocity profiles for heating of the plate ($G_r < 0$).

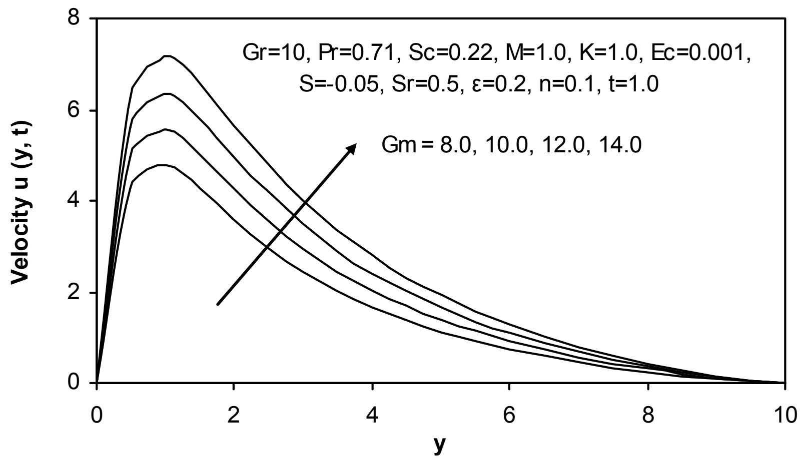

Fig.22. Effect of G_m on velocity profiles for cooling of the plate ($G_r < 0$).

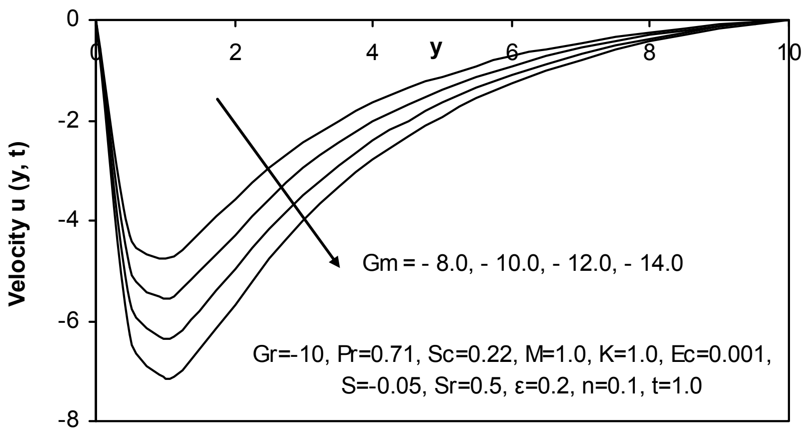

Fig.23. Effect of G_m on velocity profiles for heating of the plate ($G_r < 0$).

The numerical data for the skin-friction coefficient (τ) for cooling of the porous plate for variations in the material parameters are presented in Tab.1. It is seen that an increase in P_r, S_c and M leads to a decrease in the skin-friction coefficient whereas an increase in K, S_r, E_c, S, G_r and G_m leads to an increase in the skin-friction coefficient.

The numerical data for the skin-friction coefficient (τ) for heating of the porous plate for variations in the material parameters are presented in Tab.2. It is clear that an increase in S_c, M, E_c and S decreases the skin-friction coefficient whereas an increase in P_r, K, S_r, G_r and G_m leads to an increase in the skin-friction coefficient.

The numerical data for the Nusselt number (Nu) are presented in Tab.3 for variations in P_r, E_c and S respectively. It is clear that an increase in P_r increases the Nusselt number whereas an increase in E_c and S decreases the Nusselt number.

The numerical data for the Sherwood number (Sh) are presented in Tab.4 for variations in S_c and S_r, respectively. It is seen that an increase in S_c increases the Sherwood number whereas an increase in S_r decreases the Sherwood number.

Table 1. Numerical data for the skin-friction coefficient (τ) for cooling of the plate $(G_r > 0)$.

P_r	S_c	M	K	E_c	S	S_r	G_r	G_m	τ
0.71	0.22	0.5	1.0	0.001	- 0.05	0.5	10.0	10.0	9.892066
1.00	0.22	0.5	1.0	0.001	- 0.05	0.5	10.0	10.0	9.111524
0.71	0.60	0.5	1.0	0.001	- 0.05	0.5	10.0	10.0	8.096730
0.71	0.22	1.0	1.0	0.001	- 0.05	0.5	10.0	10.0	7.188526
0.71	0.22	0.5	1.5	0.001	- 0.05	0.5	10.0	10.0	10.877356
0.71	0.22	0.5	1.0	0.010	- 0.05	0.5	10.0	10.0	9.990562
0.71	0.22	0.5	1.0	0.001	- 0.25	0.5	10.0	10.0	9.575842
0.71	0.22	0.5	1.0	0.001	- 0.05	1.5	10.0	10.0	10.312084
0.71	0.22	0.5	1.0	0.001	- 0.05	0.5	12.0	10.0	10.568040
0.71	0.22	0.5	1.0	0.001	- 0.05	0.5	10.0	12.0	11.199374

Table 2. Numerical data for the skin-friction coefficient (τ) for heating of the plate $(G_r < 0)$.

P_r	S_c	M	K	E_c	S	S_r	G_r	G_m	τ
0.71	0.22	0.5	1.0	0.001	- 0.05	0.5	- 10.0	10.0	3.162220
1.00	0.22	0.5	1.0	0.001	- 0.05	0.5	- 10.0	10.0	4.095924
0.71	0.60	0.5	1.0	0.001	- 0.05	0.5	- 10.0	10.0	1.371208
0.71	0.22	1.0	1.0	0.001	- 0.05	0.5	- 10.0	10.0	2.012100
0.71	0.22	0.5	1.5	0.001	- 0.05	0.5	- 10.0	10.0	3.622610
0.71	0.22	0.5	1.0	0.010	- 0.05	0.5	- 10.0	10.0	3.132926
0.71	0.22	0.5	1.0	0.001	- 0.25	0.5	- 10.0	10.0	3.541200
0.71	0.22	0.5	1.0	0.001	- 0.05	1.5	- 10.0	10.0	3.581466
0.71	0.22	0.5	1.0	0.001	- 0.05	0.5	- 12.0	10.0	2.489982
0.71	0.22	0.5	1.0	0.001	- 0.05	0.5	- 10.0	12.0	4.465230

Table 3. Numerical data for the Nusselt number (Nu).

P_r	E_c	S	Nu
0.71	0.001	- 0.05	0.873726
1.00	0.001	- 0.05	1.132198
0.71	0.010	- 0.05	0.849068
0.71	0.001	- 0.25	0.973902

Table 4. Numerical data for the Sherwood number (Sh).

S_c	S_r	Sh
0.22	0.5	0.230964
0.60	0.5	0.523782
0.22	1.0	0.144702

5. Conclusions

In this paper, an attempt was made to investigate the effects of thermal diffusion and viscous dissipation on an unsteady MHD free convection involving heat and mass transfer flow past an infinite vertical porous plate with permeability variation under oscillatory suction velocity in the presence of a heat absorbing sink. The resulting system of partial differential equations was solved numerically by using the Galerkin finite element method. Numerical calculations were performed. It is concluded from the results obtained that an increase in the Prandtl number decreases the temperature and velocity of the fluid in case of cooling of the plate whereas in case of heating of the plate it increases the fluid velocity. An increase in the Schmidt number decreases the concentration, velocity of the fluid in case of cooling and heating of the plate. As the Soret number increases concentration and velocity of the fluid in case of cooling and heating of the porous plate increases. These results are in very good agreement with those in the literature.

Nomenclature

B_0 – magnetic induction
C' – species concentration of the fluid
C_p – specific heat at constant pressure
C'_w – concentration of the fluid near the plate
C'_∞ – concentration of the fluid far away from the plate
D_M – mass diffusivity
D_T – thermal diffusivity
E_c – Eckert number
G_m – solutal Grashof number
G_r – thermal Grashof number
g – acceleration due to gravity
K – porosity parameter
K'_0 – permeability of the porous medium
k – thermal conductivity of the fluid
M – magnetic parameter

n' – frequency of oscillation
P_r – Prandtl number
S – heat sink parameter
S_c – Schmidt number
S_r – Soret number
T' – temperature of the fluid
T'_w – temperature of the fluid near the plate
T'_∞ – temperature of the fluid far away from the plate
t' – time
u' – velocity in x' – direction
v' – velocity in y' – direction
β – thermal expansion coefficient
β^* – concentration expansion coefficient
μ – viscosity of the fluid
ν – kinematic viscosity
ρ – fluid density
σ – electrical conductivity of the fluid

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