DETERMINATION OF CONSTANT PARAMETERS OF COPPER AS POWER-LAW HARDENING MATERIAL AT DIFFERENT TEST CONDITIONS

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In this paper a technique has been developed to determine constant parameters of copper as a power-law hardening material by tensile test approach. A work-hardening process is used to describe the increase of the stress level necessary to continue plastic deformation. A computer program is used to show the variation of the stress-strain relation for different values of stress hardening exponent, n and power-law hardening constant, α. Due to its close tolerances, excellent corrosion resistance and high material strength, in this analysis copper (Cu) has been selected as the material. As a power-law hardening material, Cu has been used to compute stress hardening exponent, n and power-law hardening constant, α from tensile test experiment without heat treatment and after heat treatment. A wealth of information about mechanical behavior of a material can be determined by conducting a simple tensile test in which a cylindrical specimen of a uniform cross-section is pulled until it ruptures or fractures into separate pieces. The original cross sectional area and gauge length are measured prior to conducting the test and the applied load and gauge deformation are continuously measured throughout the test. Based on the initial geometry of the sample, the engineering stress-strain behavior (stress-strain curve) can be easily generated from which numerous mechanical properties, such as the yield strength and elastic modulus, can be determined. A universal testing machine is utilized to apply the load in a continuously increasing (ramp) manner according to ASTM specifications. Finally, theoretical results are compared with these obtained from experiments where the nature of curves is found similar to each other. It is observed that there is a significant change of the value of n obtained with and without heat treatment it means the value of n should be determined for the heat treated condition of copper material for their applications in engineering fields.

Key words: stress hardening exponent, power-law hardening constant, work-hardening process, power-law hardening material, heat treatment, tensile test, stress-strain curve.

1. Introduction

In elastic limit, strain is proportional to stress. So, within this limit, every level of stress causes some elastic deformation. On the other hand, a certain level of stress must be applied before plastic deformation starts. In the plastic deformation regime, the total strain is the sum of the elastic strain and the plastic strain. The contribution of elastic strain in total strain is much less than that of plastic strain. So, in plastic deformation range, the strain basically signifies plastic strain (Hosford, 2005; Rivello, 1960; Batdorf and Budiansky, 1949; Sanders, 1954; Perrone and Hodge, 1957).

The nonlinear elastic-plastic behavior is considered the same as that suggested by Ramberg and Osgood (1943) to model the tensile stress-strain relations

$$\varepsilon = \sigma + \alpha\sigma^{n} \tag{1.1}$$

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where σ is the true stress, α is the power-law hardening constant, ε is the true plastic strain and n is the stress hardening exponent. The early research work in this field can be found in Hutchinson (1967), Kucharski and Mróz (2007), Isakson *et al.* (1967), Xia and Tzuchiang (1994), Lin and Tzuchiang (1992), Denke (1956). Material parameters α and n are identified by Kucharski and Mróz (2007) from spherical indentation tests.

Stress hardening is the strengthening of the metal by plastic deformation. This strengthening occurs because of dislocation movements within the crystal structure of the material (Xia and Wang, 1993; Prager, 1956; Symonds and Prager, 1950; Hodge, 1959; Ramberg and Osgood, 1943; Naghdi, 1960).

Before stress hardening, the lattice of the material exhibits a regular, nearly defect-free pattern (almost no dislocations). The defect-free lattice can be created or restored at any time by annealing. As the material is work hardened it becomes increasingly saturated with new dislocations, and more dislocations are prevented from nucleating (a resistance to dislocation-formation develops). This resistance to dislocation-formation manifests itself as a resistance to plastic deformation; hence, the observed strengthening (Marin and Hue, 1953; Prager, 1955; Neal, 1950).

In metallic crystals, irreversible deformation is usually carried out on a microscopic scale by defects called dislocations, which are created by fluctuations in local stress fields within the material. This irreversible deformation gets culminated as the dislocation propagates through the lattice arrangement. At normal temperatures, the dislocations are not annihilated by annealing. Instead, the dislocations accumulate, interact with one another, and serve as pinning points or obstacles that significantly impede motion. This leads to an increase in the yield strength of the material and a subsequent decrease in ductility (Parker and Basset, 1964; Hue and Bratt, 1958).

To the best of our knowledge, there is no research in the literature on the determination of parameters of power-law hardening material at different tensile test conditions. In this research, two distinct conditions are considered: without heat treatment and with heat treatment to show if there is any variation of the values of *n* and α of Cu material. In Kowser and Mahiuddin (2012) and (2013) the techniques are also presented for Cu material.

Tensile curves with $\alpha = 1$ corresponding to a usual engineering definition of yield are shown in Fig.1 for several values of the stress hardening exponent, *n* (Hutchinson, 1967).
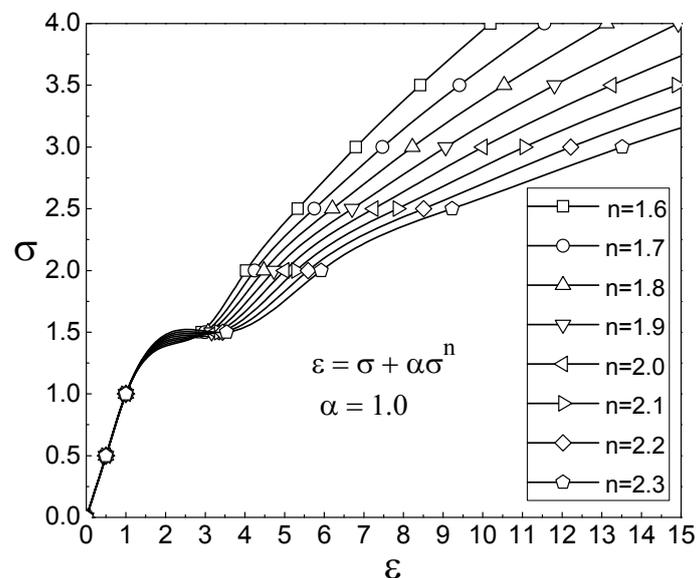
Fig.1. Stress-strain diagram for the variation of *n* at constant α .

Tensile curves with *n= 2.2* corresponding to a usual engineering definition of yield are shown in Fig.2 for several values of the power-law hardening constant α.
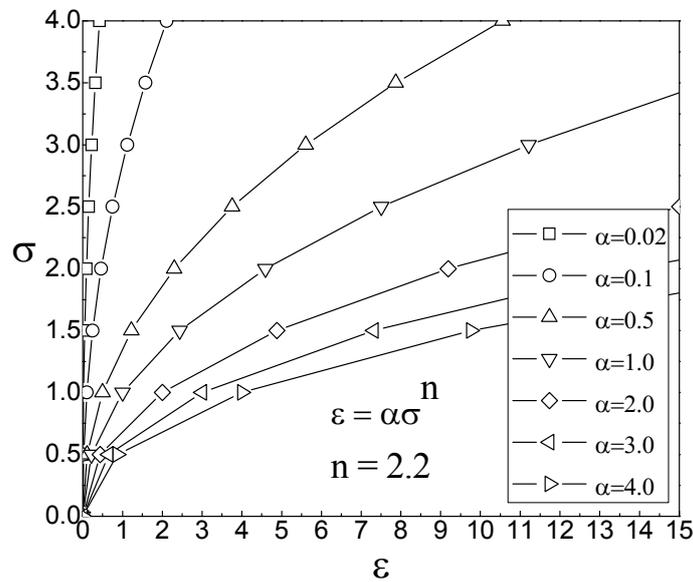
Fig.2. Stress-strain diagram for the variation of α at constant *n*.

The level of *n* is particularly significant in stretch forming because it indicates the ability of a metal to distribute the strain over a wide range.

Physical & Mechanical Properties: The physical and mechanical properties of the material are given in Tab.1a and Tab.1b, respectively.

Table 1a. Physical properties of copper [25]

Parameter	Copper
Melting Temperature	*1084.62 °C*
Density	*8.96 gm/cm³ @ 20 °C*
Specific Gravity	*8.96*
Coefficient of Thermal Expansion	*16.5 μm·m⁻¹·K⁻¹ (25 °C)*
Thermal Conductivity	*401 W·m⁻¹·K⁻¹*
Electrical Resistivity (Annealed)	*16.78 nΩ·m (20 °C)*
Specific Heat	*24.440 J·mol⁻¹·K⁻¹*
Annealing Temperature	*371°C -649 °C*

Table 1b. Mechanical properties copper [25]

Parameter	Copper
Young's modulus	*110–128 GPa*
Shear modulus	*48 GPa*
Tensile strength	*240 MPa*
Elongation	*20 – 60 %*
Fatigue Strength	*75 MPa*
Yield strength	*70 MPa*

2. Experimental methods

The mechanical properties of a material are directly related to the response of the material when it is subjected to mechanical stresses. Since characteristic phenomena or behavior occur at discrete engineering stress and strain levels, the basic mechanical properties of a material are found by determining the stresses and corresponding strains for various critical occurrences. A wealth of information about mechanical behavior of a material can be determined by conducting a simple tensile test in which a cylindrical specimen of a uniform cross-section is pulled until it ruptures or fractures into separate pieces. The original cross sectional area and gauge length are measured prior to conducting the test and the applied load and gage displacement are continuously measured throughout the test. Based on the initial geometry of the sample, the engineering stress-strain behavior (stress-strain curve) can be easily generated from which numerous mechanical properties, such as yield strength and elastic modulus, can be determined. A universal testing machine shown in Fig.3 is utilized to apply the load in a continuously increasing (ramp) manner according to ASTM specifications.

Fig.3. Universal Testing Machine.

Tensile test specimens of Cu are presented in Fig.4 and Fig.5 respectively. Each specimen is of a specific shape and dimensions that prepared in accordance with ASTM (American Society for Testing and Materials) specifications for standardization. During tests, deformation is confined to the narrow center region which has a uniform cross section along its length.
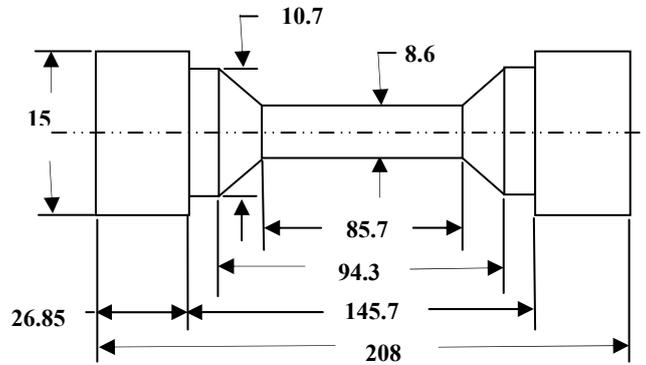
Fig.4. ASTM standard tensile test specimen.

Dimensions of tensile test specimens are given in Tab.2.

Table 2. Dimensions of tensile test specimens.

Mat.	Specimen No	Diameter (mm)	Gauge Length (mm)
Copper	1	*6.80*	*69.00*
	2	*6.85*	*74.30*
	3	*7.11*	*69.00*
	4	*6.95*	*74.70*
	5	*7.00*	*74.30*
	6	*6.90*	*74.80*

Fig.5. Tensile test specimen of Cu.

3. Experimentation

Experiments were carried out in two different conditions, without heat treatment and with heat treatment.

3.1. Heat treatment

Heat treatment is the controlled heating and cooling of metals to alter their physical and mechanical properties without changing the product shape.

Transformation hardening mechanisms resulting from quenching at high temperatures induce internal changes producing harder, stronger phases. Tempering such structures improves toughness and reduces hardness

The heat treatment process was carried out in an electric furnace. Processing of specimens and recorded temperatures are shown in Fig.6 and Fig.7.

Fig.6. Heat treatment process.

Fig.7. Heat treatment process showing recorded temperature.

Cu specimens were heated to *829ᵒC* and *715ᵒC* respectively for *30* minutes and cooled to room temperature slowly. Time-temperature curve of heat treatment process is shown in Fig.8.
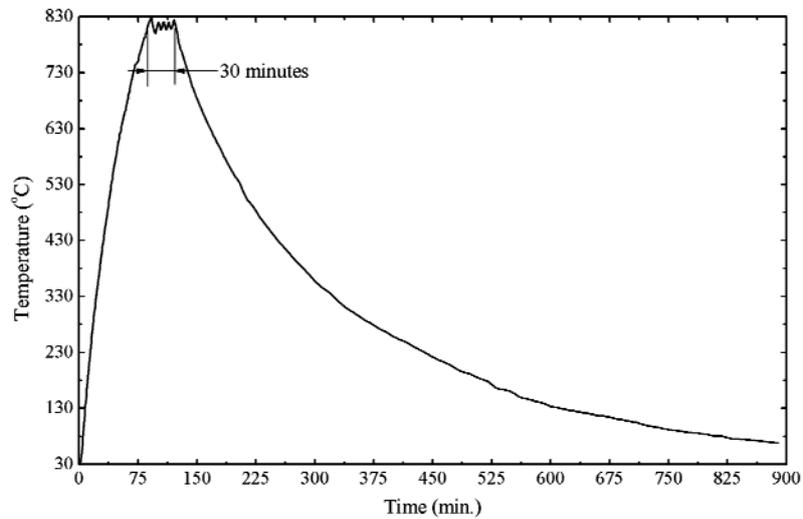
Fig.8. Time - Temperature curve of heat treatment process.

4. Test results

Engineering and true stress- strain curves for the tensile tests before heat treatment and after heat treatment are shown in Fig.9 to Fig.13.

The combined effect of engineering and true stress-strain curve before and after heat treatment for Cu is shown in Fig.9 and Fig.10.
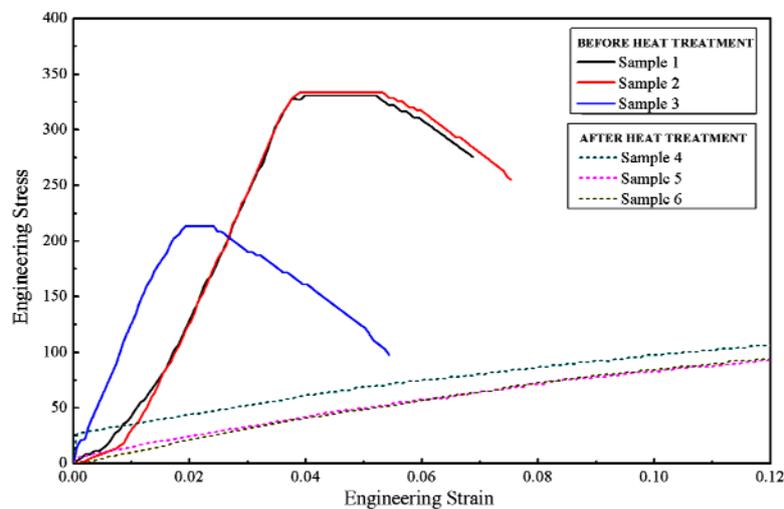
Fig.9. Engineering stress - strain diagram for Cu before and after heat treatment.
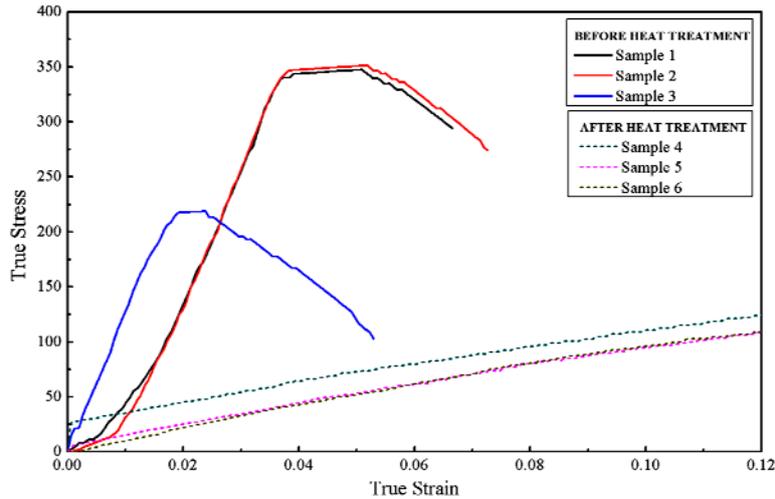
Fig.10. Combined true stress - strain diagram.

5. Determination of α and n

Hollomon's equation is a power law relationship between the stress and the amount of plastic strain

$$\varepsilon = \alpha\sigma^n. \tag{5.1}$$

The constant α is structure dependent and is influenced by processing while n is a material property. Applying the power law at two levels of stress, ε_1 and ε_2 can be given as

$$\varepsilon_1 = \alpha\sigma_1^n, \tag{5.2}$$

$$\varepsilon_2 = \alpha\sigma_2^n \tag{5.3}$$

Taking the ratios equation yields

$$\frac{\varepsilon_1}{\varepsilon_2} = \left(\frac{\sigma_1}{\sigma_2}\right)^n. \tag{5.4}$$

Finally, taking the natural logs of both sides; the stress hardening exponent can be described by

$$n = \frac{ln\left(\dfrac{\varepsilon_1}{\varepsilon_2}\right)}{ln\left(\dfrac{\sigma_1}{\sigma_2}\right)}. \tag{5.5}$$

Equation (5.5) can be evaluated from the slope of a log (σ) - log (ε) plot. The true stress-strain relation plots as a straight line on log-log coordinates as shown in Fig.11. The stress hardening exponent n is the slope of the line. The value of n can be determined by Eq.(5.5).
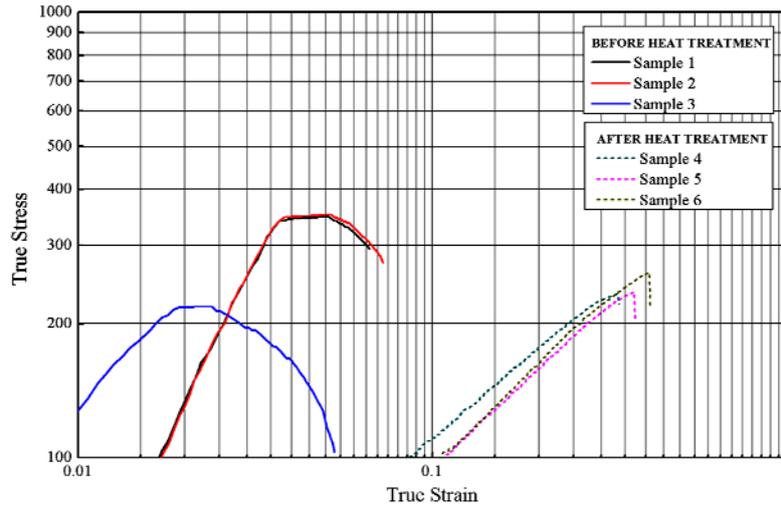
Fig.11. True stress - strain diagram on logarithmic scale.

The power-law hardening constant, α can be found by extrapolating to $\varepsilon = 1.0$, where α is the value of σ when $\varepsilon = 1.0$.

Experiments were carried out in two distinct conditions. One was in ambient condition and another was heat treated condition. Numerical results are presented in a tabular form in Tab.3, for Cu material.

Table 3. Experimental value of *n* and α .

Mat.	Specimen No	Value of *n*	Value of α	Avg. of *n*	Avg. of α
Copper	1	*1.51*	*5.30*		
	2	*1.49*	*4.21*	*1.52*	*4.54*
	3	*1.56*	*4.10*		
	4	*1.48*	*4.75*		
	5	*1.35*	*5.44*	*1.38*	*5.03*
	6	*1.30*	*4.91*		

From various sources including Hosford (2005) a comparison of the theoretical and experimental value of *n* and α was made and presented in Tab.4 and Tab.5, respectively.

Table 4. Comparison of theoretical and experimental value of *n* before heat treatment.

Mat.	Theory[1] *n*	Experimental *n*	% Error
Copper	*2.0*	*1.52*	*24*

Table 5. Comparison of theoretical and experimental value of α before heat treatment.

Mat.	Theory[1] α	Experimental α	% Error
Copper	*4.50*	*4.54*	*0.88*

The comparison of theoretical results with experimentally obtained results at ambient conditions for Cu is also shown in Fig.12, where the best matched curve for one specimen is shown.
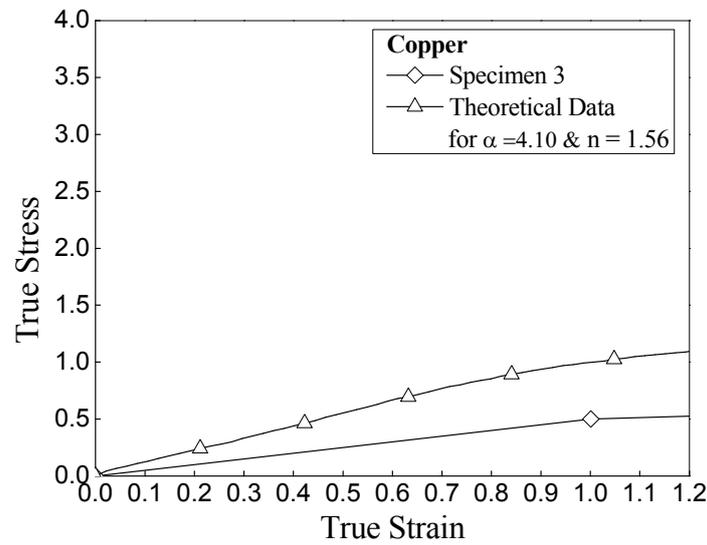
Fig.12. Comparison of theoretical and experimental results at ambient temperature.

The comparison of theoretical results with experimentally obtained results after heat treatment of Cu is shown in the Fig.13. Where the best matched curve for one specimen is shown for the simplicity of results.
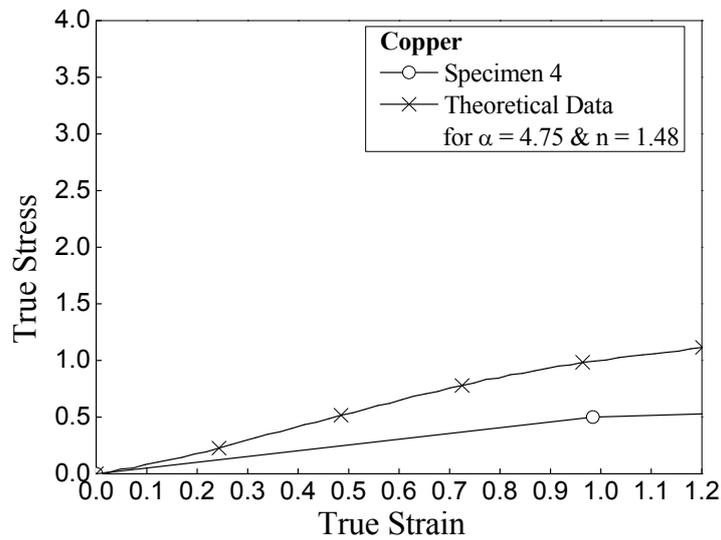
Fig.13. Comparison of theoretical and experimental results at ambient temperature.

6. Conclusion

From the tensile test data of ASTM standard specimens of Cu, a simple technique is presented to determine the power-law hardening parameters. From the tensile test data for two distinct conditions, i.e., without heat treatment and with heat treatment, the values of *n* and α are obtained for various specimens of Cu as a power law hardening material. Without heat treatment the obtained experimental results are compared with available theoretical results. There are significant reductions of the values of *n* for both the materials. There are remarkable changes in strain for the same stress at different *n* due to the non- linear behavior of the materials. The above results suggest that the value of *n* should be determined after heat treatment of Cu materials for their applications in engineering fields.

Nomenclature

Cu – copper
 n – stress hardening exponent
α – power-law hardening constant
ε – normalized strain
σ – normalized stress

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