DE GRUYTER OPEN

Int. J. of Applied Mechanics and Engineering, 2016, vol.21, No.2, pp.423-446
DOI: 10.1515/ijame-2016-0026

UNSTEADY TWO-LAYERED FLUID FLOW OF CONDUCTING FLUIDS IN A CHANNEL BETWEEN PARALLEL POROUS PLATES UNDER TRANSVERSE MAGNETIC FIELD IN A ROTATING SYSTEM

T. LINGA RAJU*
Department of Engineering Mathematics
AUCE(A), Andhra University
VISAKHAPATNAM, Pin code: 530 003, A.P, INDIA
E-mail: tlraju45@yahoo.com

B. NEELA RAO
Department of Mathematics
Aditya Institute of Technology and Management
TEKKALI, Pin code: 532 201, A.P, INDIA
E-mail: neelarao.b@gmail.com

An unsteady MHD two-layered fluid flow of electrically conducting fluids in a horizontal channel bounded by two parallel porous plates under the influence of a transversely applied uniform strong magnetic field in a rotating system is analyzed. The flow is driven by a common constant pressure gradient in a channel bounded by two parallel porous plates, one being stationary and the other oscillatory. The two fluids are assumed to be incompressible, electrically conducting with different viscosities and electrical conductivities. The governing partial differential equations are reduced to the linear ordinary differential equations using two-term series. The resulting equations are solved analytically to obtain exact solutions for the velocity distributions (primary and secondary) in the two regions respectively, by assuming their solutions as a combination of both the steady state and time dependent components of the solutions. Numerical values of the velocity distributions are computed for different sets of values of the governing parameters involved in the study and their corresponding profiles are also plotted. The details of the flow characteristics and their dependence on the governing parameters involved, such as the Hartmann number, Taylor number, porous parameter, ratio of the viscosities, electrical conductivities and heights are discussed. Also an observation is made how the velocity distributions vary with the rotating hydromagnetic interaction in the case of steady and unsteady flow motions. The primary velocity distributions in the two regions are seen to decrease with an increase in the Taylor number, but an increase in the Taylor number causes a rise in secondary velocity distributions. It is found that an increase in the porous parameter decreases both the primary and secondary velocity distributions in the two regions.

Key words: magnetohydrodynamics, two-layered fluids/immiscible fluids, rotating fluids, oscillating flow, unsteady flow, porous plates.

1. Introduction

The simultaneous influence of rotation and an external magnetic field on electrically conducting two-layered/two-phase fluid flow systems seems to be dynamically important and physically useful in many diversified fields mainly from geophysical applications to the atmosphere and oceans and to motions within the Earth’s core. It is apparent that the rotating flow of an electrically conducting fluid in the presence of a magnetic field is encountered in cosmological and geophysical fluid dynamics. It can provide an explanation for the observed maintenance and secular variation of the geomagnetic field (Hide and Roberts [1]), the

* To whom correspondence should be addressed

sunspot development, the solar cycle and the structure of rotating magnetic stars (Dieke [2]). Moreover, the presentation of an MHD two-layered/two-phase flow model with porous boundaries subject to normal suction is timely in view of the recent interest in the liquid metal flow in MHD power rotating generators, in proper design of cooling blankets in a magnetically confined thermo nuclear reactors. Also, this type of study would provide guidance for the investigation of boundary layer behavior along porous plates with fluid injection/suction. Since the review article of Squire [3], a great number of research work on MHD flows of viscous incompressible electrically conducting fluids in a rotating system under different conditions and of various aspects of the problem has been initiated by many researchers, namely, Gilmam and Benton [4], Benton and Loper [5], Nanda and Mohanty [6], Gupta [7], Debnath [8], Seth *et al*. [9], Seth *et al*. [10] and others. The oscillatory flows in rotating channels are important from a practical point of view, since fluid oscillations may be expected in many MHD devices and in natural phenomena where fluid flow is generated due to an oscillating pressure gradient or due to vibrating plates (or walls). The investigations on oscillatory flows of viscous incompressible electrically conducting fluids in a rotating system under different conditions and various aspects of the problem have been carried out by many authors, such as Mukherjee and Debnath [11], Seth and Jana [12], Singh [13], Ghosh [14], Gupta *et al.* [15], Ghosh and Pop [16], Hayat and Hutter [17], Guria and Jena [18] and many more.

The problems concerned with two immiscible fluids or multi-layered fluids flow situations which arise in the petroleum industry, geophysical fluid dynamics and in magnetohydrodynamics have been studied and reported in the literature by several authors. The stratified laminar flow of two immiscible liquids in a horizontal pipe was studied by Packham and Shail [19]. Hartmann flow of a conducting fluid in a channel between two horizontal insulating plates of infinite in extent with a layer of non-conducting fluid between upper channel wall and the conducting fluid was studied by Shail [20]. He observed that an increase of the order of *30*% can be obtained in the flow rate for suitable ratios of depths and viscosities of the two fluids with realistic values of the Hartmann number. In addition, there are numerous publications which are made available in the literature on experimental and theoretical aspects of magnetohydrodynamic two-phase/two-layered flow problems by several researchers, notably, Lielausis [21], Michiyoshi *et al.* [22], Chao *et al.* [23], Dunn [24], Gherson [25], Lohrasbi and Sahai [26], Serizawa *et al*. [27], Malashetty and Leela [28], Ramadan and Chamkha [29], Chamka [30], Raju and Murty [31] and others. All the above stated investigations were carried out by the authors for steady flow situations. However, a significant number of practical problems dealing with immiscible fluids are unsteady in nature. In many practical problems, it is advantageous to consider both immiscible fluids as electrically conducting, one of which is highly electrically conducting compared to the other. The fluid of low electrical conductivity compared to the other is helpful to reduce the power required to pump the fluid in MHD pumps and flow meters. In view of these facts, Healy and Young [32] studied a oscillating two-phase channel flows. Debnath and Basu [33] discussed the unsteady slip flow in an electrically conducting two-phase fluid under transverse magnetic fields. Chamkha [34] studied the unsteady MHD convective heat and mass transfer past a semi-infinite vertical permeable moving plate with heat absorption. Umavathi *et al*. [35] investigated an oscillatory Hartmann two-fluid flow and heat transfer in a horizontal channel, Tsuyoshi Inoue and Shu-ichiro Inutsuka [36] studied two-fluid magnetohydrodynamic simulations of converging Hi flows in the interstellar medium. Raju and Sreedhar [37] discussed unsteady two-fluid flow and heat transfer of conducting fluids in channels under a transverse magnetic field. Raju and Valli [38] discussed an unsteady two-layered fluid flow and heat transfer of conducting fluids in a channel between parallel porous plates under a transverse magnetic field. Subsequently, these authors [39] studied an MHD two-layered unsteady fluid flow and heat transfer through a horizontal channel between parallel plates in a rotating system.

Keeping in view the above mentioned wide range of applications, in this paper an unsteady magnetohydrodynamic (MHD) two-layered fluids flow in a horizontal channel between two parallel plates in the presence of an applied magnetic and electric field is investigated, when the whole system is rotated about an axis perpendicular to the flow. The flow is driven by a constant uniform pressure gradient in the channel bounded by two parallel insulating plates, when both fluids are considered as electrically conducting. Also, the two fluids are assumed to be incompressible with variable properties as different viscosities and electrical conductivities. The resulting governing partial differential equations are then reduced to linear ordinary differential equations by using two-term series. Analytical solutions for primary and secondary velocity distributions are obtained in both fluid regions of the channel. Profiles of these solutions are plotted to discuss the effect on the flow and their dependence on the governing parameters involved, such as the Hartmann number, Taylor number (rotation parameter), ratio of the viscosities, heights and electrical conductivities. Also, an examination is made how the velocity distributions vary through a hydromagnetic interaction in the case of steady and unsteady motions with rigid rotation.

The structure of the paper is as follows. Introduction of the problem is given in § 1. The formulation and mathematical analysis of the problem is given in § 2. Solutions of the problem are given in § 3. While, § 4 gives the results and discussion based on the velocity profiles, which are displayed in Figs 2 to 13. The conclusion is given in § 5, followed by Nomenclature and References.

2. Formulation and mathematical analysis of the governing equations of motion, energy, boundary and interface conditions

We consider an unsteady magnetohydrodynamics (MHD) two layered-fluids flow between two parallel plates extending along the x- and z- directions at $y = h_1$ and $y = -h_2$, when the fluids and plates are in a state of rigid rotation with uniform angular velocity $\overline{\Omega}$ about the y-axis normal to the plates. The length of the plates is much greater than the distance between them. The fluids in the upper and lower regions, i.e., $0 \le y \le h_1$ and $-h_2 \le y \le 0$ are designated as Region -I and Region-II respectively.

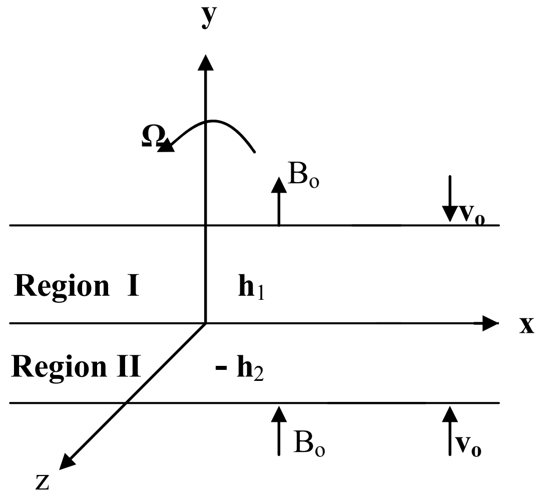

Fig.1 Physical regime and co-ordinate system.

Figure 1 represents the physical regime and co-ordinate system choosing the origin midway between the two plates. Fluid flow in both the upper and lower regions is generated due to a common constant pressure gradient $\left(-\dfrac{\partial p}{\partial x}\right)$ in a channel bounded by two parallel porous plates, one being stationary and the other oscillatory. A constant transverse magnetic field of strength B_0 is applied to the plates. A constant electric field E_0 is also applied in the z- direction. The induced magnetic field is being neglected by assuming that it is small when compared with the applied field. The two fluids are subjected to a constant suction v_0 applied normal to both the plates. If $(u_i, v_i, w_i), (i = 1,2)$, are the velocity components in the two fluids and $\overline{q}_i = (u_i, v_i, w_i)$, then the equation of continuity $\nabla.\overline{q_i} = 0$ gives $v_i = - v_0$ ($v_0 > 0$). Both regions are occupied by two immiscible electrically conducting, incompressible fluids with different densities ρ_1, ρ_2, viscosities

μ_1, μ_2 and electrical conductivities σ_1, σ_2, respectively. Under these physical conditions, the flow regime in the [x, y, z] coordinate system can be represented as in the analysis of Lohrasbi and Shahai [26] by the following simplified equations of motion, the corresponding boundary and interface conditions for both fluid regions in a rotating frame of reference as:

Region-I

$$\rho_1 \frac{\partial u_1}{\partial t} - \mu_1 \frac{\partial^2 u_1}{\partial y^2} + \rho_1 v_0 \frac{\partial u_1}{\partial y} + \frac{\partial p}{\partial x} + \sigma_1 u_1 B_0^2 + \sigma_1 E_0 B_0 = -2\rho_1 \Omega w_1, \tag{2.1}$$

$$\rho_1 \frac{\partial w_1}{\partial t} - \mu_1 \frac{\partial^2 w_1}{\partial y^2} + \rho_1 v_0 \frac{\partial w_1}{\partial y} + \sigma_1 u_1 B_0^2 = 2\rho_1 \Omega u_1 . \tag{2.2}$$

Region II

$$\rho_2 \frac{\partial u_2}{\partial t} - \mu_2 \frac{\partial^2 u_2}{\partial y^2} + \rho_2 v_0 \frac{\partial u_2}{\partial y} + \frac{\partial p}{\partial x} + \sigma_2 u_2 B_0^2 + \sigma_2 B_0 E_0 = -2\rho_2 \Omega w_2 , \tag{2.3}$$

$$\rho_2 \frac{\partial w_2}{\partial t} - \mu_2 \frac{\partial^2 w_2}{\partial y^2} + \rho_2 v_0 \frac{\partial w_2}{\partial y} + \sigma_2 w_2 B_0^2 = 2\rho_2 \Omega u_2 . \tag{2.4}$$

The subscripts 1 and 2 in the above equations represent the values for Region-I and Region-II respectively. The quantities u_1, u_2 and w_1, w_2 are the x- and z-components of fluid velocities known as the primary and secondary velocity distributions in the two regions, respectively. Ω is the angular velocity, where $\overline{\Omega}=(0,\Omega,0)$.; and '$t$' is the time. The boundary conditions on velocity are the no-slip condition at the lower plate and an oscillatory type at the upper plate. We also assume that the continuity of velocity, shear stress at the interface between the two fluid layers at $y = 0$.

The boundary and interface conditions on u_1, w_1 and u_2, w_2 for the two fluids are considered as

$$u_1(h_1) \quad \text{and} \quad w_1(h_1) = 0 \qquad \text{for} \qquad t \le 0,$$

$$= \text{Real}\left(\varepsilon e^{i\omega t}\right), \qquad \text{for} \qquad t > 0, \tag{2.5}$$

$$u_2(-h_2) = 0, \qquad w_2(-h_2) = 0, \tag{2.6}$$

$$u_1(0) = u_2(0), \qquad w_1(0) = w_2(0), \tag{2.7}$$

$$\mu_1 \frac{du_1}{dy} = \mu_2 \frac{du_2}{dy} \qquad \text{and} \qquad \mu_1 \frac{dw_1}{dy} = \mu_2 \frac{dw_2}{dy} \quad \text{at} \quad y = 0 \tag{2.8}$$

where ε (amplitude) is a small constant quantity such that $\varepsilon << 1$ and ω is the frequency of oscillation at the plate, and the perturbed fields initially are zero, since the system is at rest for $t \le 0$.

The following non-dimensional quantities are introduced to make the governing equations and boundary/interface conditions dimensionless

$$u^{\bullet}{}_{1}=\frac{u_{1}}{u_{p}}, \qquad u^{\bullet}{}_{2}=\frac{u_{2}}{u_{p}}, \qquad w^{\bullet}{}_{1}=\frac{w_{1}}{u_{p}}, \qquad w^{\bullet}{}_{2}=\frac{w_{2}}{u_{p}},$$

$$y^{\bullet}{}_{i}=\left(\frac{y_{i}}{h_{i}}\right)(i=1,2), \qquad u_{p}=\left(-\frac{\partial p}{\partial x}\right)\frac{h_{1}^{2}}{\mu_{1}}, \qquad t^{*}=\frac{\upsilon_{1}t}{h_{1}^{2}}, \qquad \omega^{*}=\frac{\omega h_{1}^{2}\rho_{1}}{\mu_{1}},$$

M^2 (the Hartmann number) $= B_0^2 h_1^2\left(\dfrac{\sigma_1}{\mu_1}\right)$,

T^2 (the Taylor number or rotation parameter) $= h_1^2\dfrac{\Omega}{\upsilon_1}$, (this is the reciprocal of the Ekman number),

λ (the porous parameter) = $h_1\rho_1 v_0 \big/ \mu_1$, α (the ratio of viscosities) $=\dfrac{\mu_1}{\mu_2}$, (2.9)

h (the ratio of heights) = $\dfrac{h_2}{h_1}$,

σ (the ratio of electrical conductivities) = $\dfrac{\sigma_1}{\sigma_2}$,

R_e (the electric load parameter) $= E_0 \big/ B_0 u_p$.

The non-dimensional forms of Eqs (2.1) to (2.4) in both the fluid regions after the use of transformations (2.9) and for simplicity neglecting the asterisks are obtained as:

Region-I

$$\frac{du_1}{dt}-\frac{d^2u_1}{dy^2}+\lambda\frac{du_1}{dy}+M^2\left(R_e+u_1\right)-1=-2T^2w_1, \tag{2.10}$$

$$\frac{dw_1}{dt}-\frac{d^2w_1}{dy^2}+\lambda\frac{dw_1}{dy}+M^2w_1=2T^2u_1. \tag{2.11}$$

Region-II

$$\frac{du_2}{dt}-\frac{d^2u_2}{dy^2}+\lambda\frac{du_2}{dy}+M^2h^2\alpha\sigma\left(R_e+u_2\right)-\alpha h^2=-2\rho\alpha h^2T^2w_2, \tag{2.12}$$

$$\frac{dw_2}{dt}-\frac{d^2w_2}{dy^2}+\lambda\frac{dw_2}{dy}+M^2h^2\alpha\sigma w_2=2\rho\alpha h^2T^2u_2. \tag{2.13}$$

With the use of the above transformations (2.9), the non-dimensional forms of boundary and interface conditions are given by

$$u_1(+1) \quad \text{and} \quad w_1(+1)=0 \quad \text{for} \quad t \le 0,$$

$$= \mathrm{Re}\left(\varepsilon e^{i\omega t}\right), \quad \text{for} \quad t>0, \tag{2.14}$$

$$u_2(-1)=0, \qquad w_2(-1)=0, \tag{2.15}$$

$$u_1(0)=u_2(0), \qquad w_1(0)=w_2(0), \tag{2.16}$$

$$\frac{du_1}{dy}=(1/\alpha h)\frac{du_2}{dy} \quad \text{and} \quad \frac{dw_1}{dy}=(1/\alpha h)\frac{dw_2}{dy} \quad \text{at} \quad y=0. \tag{2.17}$$

Conditions (2.15) denote no-slip conditions at the lower wall and conditions (2.14) are due to oscillation of the upper wall for any time *t*. Also, conditions (2.16) and (2.17) represent the continuity of velocities and shear stress at the interface $y = 0$.

3. Solutions of the problem

The governing momentum Eqs (2.10), (2.11) and (2.12), (2.13) are to be solved subject to the boundary and interface conditions (2.14) - (2.17) for the velocity distributions in both regions. Actually these equations are coupled partial differential equations, which cannot be solved in a closed form. But, they can be reduced to linear ordinary differential equations by assuming the following two term series

$$u_1(y,t)=u_{01}(y)+\varepsilon\cos\omega t.u_{11}(y), \tag{3.1}$$

$$w_1(y,t)=w_{01}(y)+\varepsilon\cos\omega t.w_{11}(y), \tag{3.2}$$

$$u_2(y,t)=u_{02}(y)+\varepsilon\cos\omega t.u_{12}(y), \tag{3.3}$$

$$w_2(y,t)=w_{02}(y)+\varepsilon\cos\omega t.w_{12}(y), \tag{3.4}$$

in which the terms $u_{01}(y), u_{02}(y)$ are velocities in basic steady state case in the two regions, while, $u_{11}(y)$, $u_{12}(y)$ are the corresponding time dependent components of the solutions, which are the factors of Real $\left(\varepsilon e^{i\omega t}\right)$ to be determined with the help of Eqs (2.10) to (2.13).

Making use of the expressions given in Eqs (3.1) - (3.4) into Eqs (2.10) - (2.13) and separating the steady-state and transient time dependent parts, the following differential equations for $u_{01}(y), u_{02}(y)$; also, $u_{11}(y), u_{12}(y)$ in terms of the complex notations: $q_{01}=u_{01}+iw_{01}, q_{11}=u_{11}+iw_{11}$, $q_{02}=u_{02}+iw_{02}$, $q_{12}=u_{12}+iw_{12}$ are obtained in both fluid regions as:

Region-I

For steady-state part

$$\frac{d^2 q_{01}}{dy^2} - \lambda \frac{dq_{01}}{dy} - a_1 q_{01} = a_2 . \tag{3.5}$$

For transient time dependent part

$$\frac{d^2 q_{11}}{dy^2} - \lambda \frac{dq_{11}}{dy} - a_3 . q_{11} = 0 . \tag{3.6}$$

Region – II

For steady-state part

$$\frac{d^2 q_{02}}{dy^2} - \lambda \frac{dq_{02}}{dy} - a_4 q_{02} = a_5 . \tag{3.7}$$

For transient time dependent part

$$\frac{d^2 q_{12}}{dy^2} - \lambda \frac{dq_{12}}{dy} - a_6 q_{12} = 0 . \tag{3.8}$$

The corresponding boundary and interface conditions on velocity become:

For steady-state part

$$q_{01}(+1) = 0 , \tag{3.9}$$

$$q_{02}(-1) = 0 , \tag{3.10}$$

$$q_{01}(0) = q_{02}(0) , \tag{3.11}$$

$$\frac{dq_{01}}{dy} = \frac{1}{\alpha h} \frac{dq_{02}}{dy} \qquad \text{at} \qquad y = 0. \tag{3.12}$$

For transient time dependent part

$$q_{11}(+1) = 1 , \tag{3.13}$$

$$q_{12}(-1) = 0 , \tag{3.14}$$

$$q_{11}(0) = q_{12}(0) , \tag{3.15}$$

$$\frac{dq_{11}}{dy} = \frac{1}{\alpha h} \frac{dq_{12}}{dy} \qquad \text{at} \qquad y = 0. \tag{3.16}$$

The differential equations given in Eqs (3.5) - (3.6) along with the boundary and interface conditions from (26) to (33) represent a system of linear ordinary differential equations and conditions. These equations are solved in closed form for both the steady state and transient time dependent cases separately in two parts. Hence, the complete solutions for velocity distributions of the unsteady flow problem become:

Region-I

$$\begin{aligned} q_1(y,t) &= q_{01}(y) + \text{Real}\left(\varepsilon e^{i\omega t}\right) q_{11}(y), \\ &= c_1 e^{m_1 y} + c_2 e^{m_2 y} - \frac{a_2}{a_1} + \text{Re}\,al\left(\varepsilon\, e^{i\omega t}\right)\left(c_5 e^{m_5 y} + c_6 e^{m_6 y}\right). \end{aligned} \tag{3.17}$$

Region-II

$$\begin{aligned} q_2(y,t) &= q_{02}(y) + \text{Real}\left(\varepsilon e^{i\omega t}\right) q_{12}(y), \\ &= c_3 e^{m_3 y} + c_4 e^{m_4 y} - \frac{a_5}{a_4} + \text{Re}\,al\left(\varepsilon\, e^{i\omega t}\right)\left(c_7 e^{m_7 y} + c_8 e^{m_8 y}\right). \end{aligned} \tag{3.18}$$

The solutions of the non-periodic terms give the steady-state fluid flow solutions for both regions and without going into details, the steady-state velocity profiles are shown in Figs 2 to 13. The solutions of the periodic terms represent the transient velocity distribution in both regions of the channel. The solutions of the unsteady problem given in Eqs (3.17) to (3.18) are evaluated numerically for different non-dimensional governing flow parameters involved in the study. Also, these results are plotted as graphs and are shown in Figs 2 to 13. The value for ε is fixed at 0.5 and $\Pr = 1$ for all graphs, also the constants appearing in the above solutions are given in the Appendix.

4. Results and discussion

In the present study, we examine the influence of the Hartmann number (M), Taylor number/rotation parameter (T), porous parameter (λ), viscosity ratio (α), electrical conductivity ratio (σ) and height ratio (h) on the primary and secondary velocity distributions. The analytical solutions for velocity distributions, such as primary and secondary velocity distributions, namely u_1, u_2 and w_1, w_2 in the two fluid regions are given for small ε (the coefficient of exponent of periodic frequency parameter). These solutions are evaluated for various parametric conditions to plot their profiles. The numerical results are depicted graphically in Figs 2 – 13 for dimensionless primary and secondary velocity distributions: u_1, u_2 and w_1, w_2, respectively in both the regions (Region-I and Region-II) to elucidate the interesting features of the rotating magnetohydrodynamic steady and unsteady flow motions. The solid lines and the dash-dot lines show the profiles for the unsteady and for the steady flow motions, correspondingly. We note that, the analysis is in good agreement with the study of Malashetty and Leela [28] when the motion is in steady state condition, no rigid rotation and for non-porous plates (i.e., $T = 0$, $\lambda = 0$). Also, this study matches with the solutions of Raju and Valli [38] for the case of non-rigid rotation ($T = 0$). So the study leads some confidence, in addition the solutions satisfy all boundary and interface conditions.

Figures 2 and 3 show the effect of differing Hartmann number (M) on both primary and secondary velocity distributions in the two regions. From Fig.2, it is noticed that the primary and secondary velocity distributions in both regions increase as α increases for fixed values of the remaining parameters. From

Fig.3, it is seen that the secondary velocity distribution enhances as M increases up to its value equal to 2 and thereafter it diminishes in both the regions. Also, the maximum primary velocity in the channel tends to move above the channel centre line towards the upper fluid region as M increases, when all the remaining governing parameters are fixed. But, the maximum secondary velocity distribution in the channel tends to move above the channel centre line towards Region-I (i.e., in the upper fluid region) up to M = *3*, there after it tends to move below the channel centre line towards Region-II (i.e., in the lower fluid region) as the Hartmann number increases.

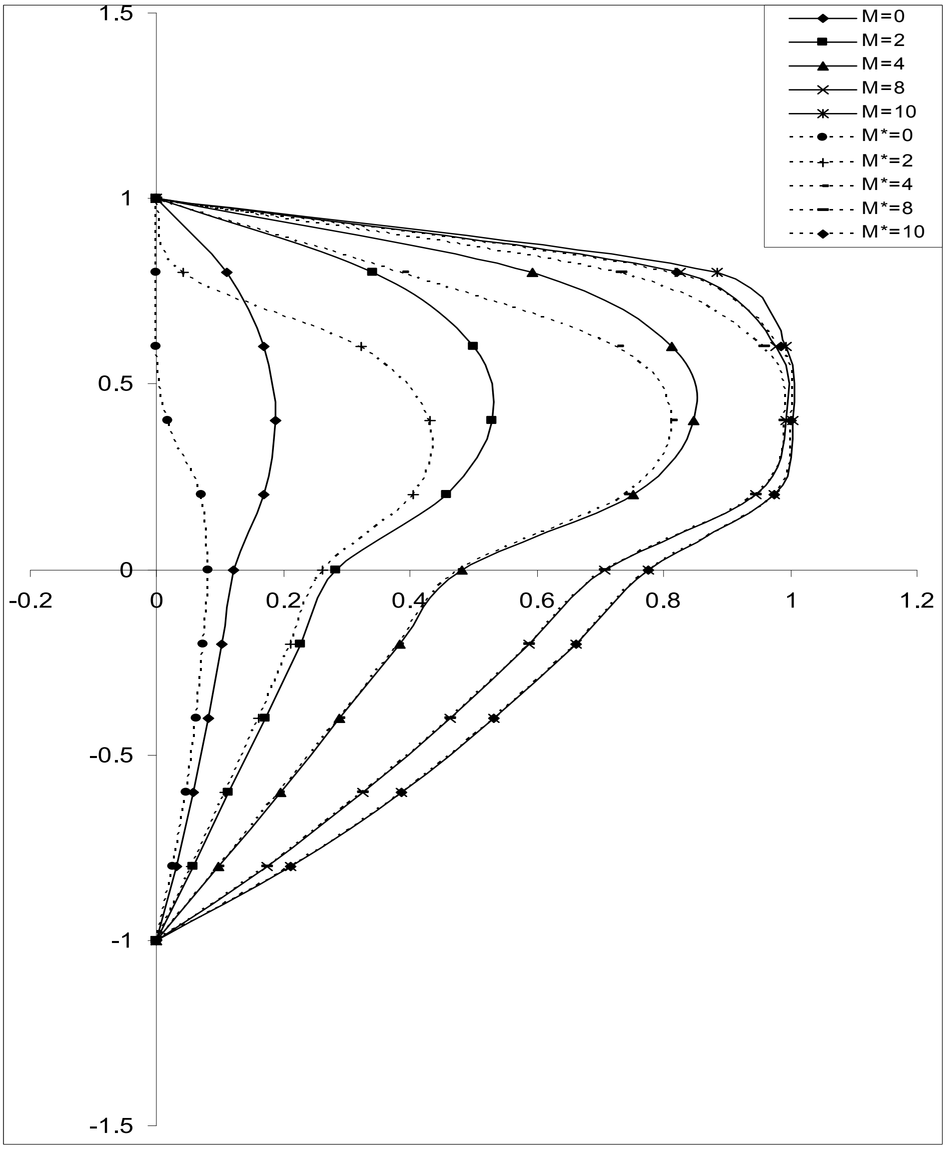

Fig.2. Primary velocity profiles u_1, u_2 (unsteady flow) u_1^*, u_2^* (steady flow) for different M and T=*1*, $\rho = 1.5$, $\alpha = 0.333$, $\sigma = 0.1$, *h*=*0.75*, Re=-*1*, $\lambda = 0.8$, $\varepsilon = 0.5$, $\omega = 1$, $t = \pi/\omega$.

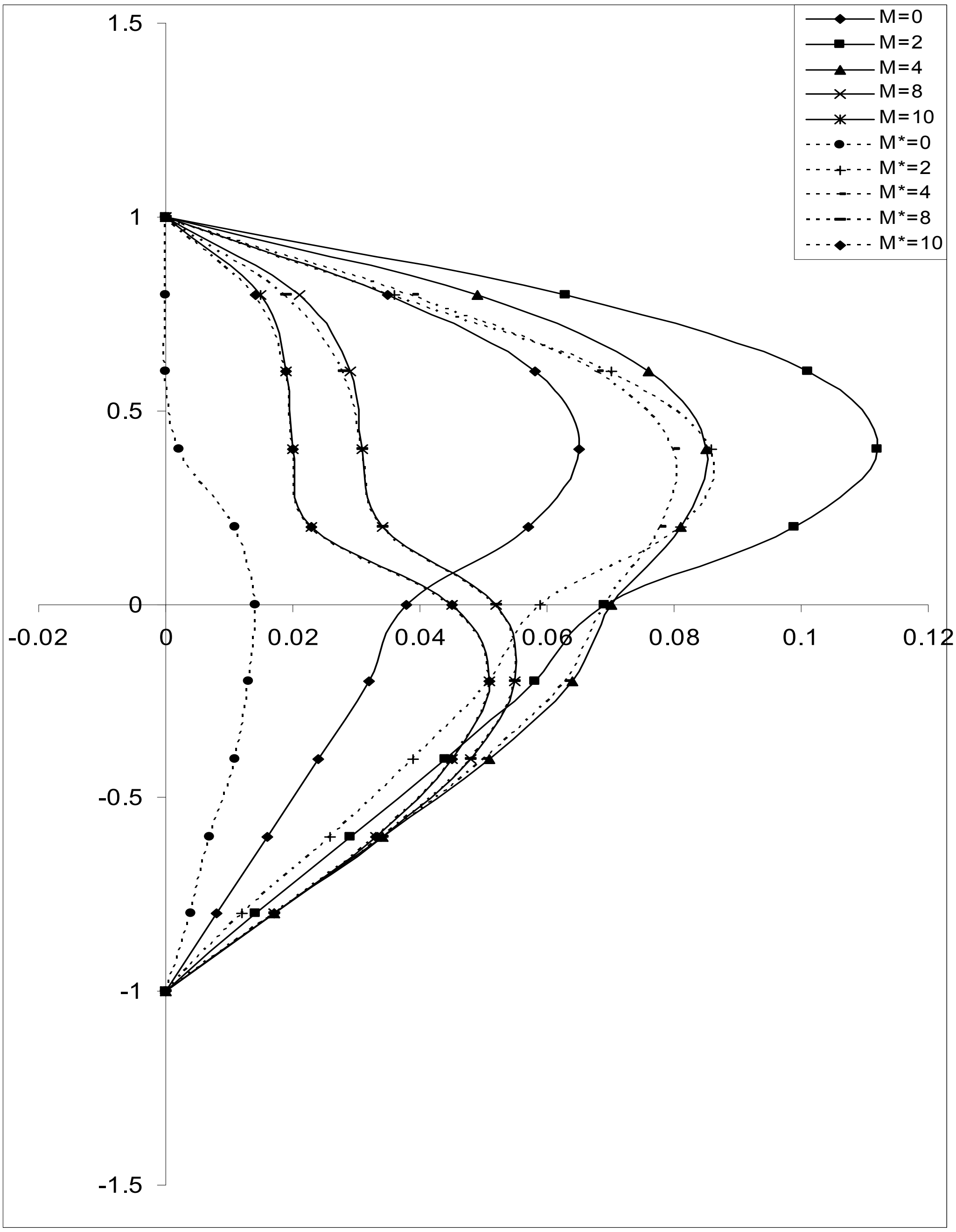

Fig.3. Secondary velocity profiles w_1, w_2 (unsteady flow) w_1^*, w_2^* (steady flow) for different M and T=*1*, $\rho = 1.5$, $\alpha = 0.333$, $\sigma = 0.1$, h=*0.75*, Re=*-1*, $\lambda = 0.8$, $\varepsilon = 0.5$, $\omega = 1$, $t = \pi/\omega$.

The effect of varying values of the Taylor number (the rotation parameter) T on both the primary and secondary velocity distributions in the two fluid regions is shown in Figs 4 and 5. From Fig.4, it is found that there is a reduction in the primary velocity distributions in region-I and region–II with an increase in the Taylor number (T). Probably, when the Coriolis force becomes stronger, there will be a heavy reduction in the primary velocity distributions due to the formation of thin boundary layers making the velocity distributions become very small for different values of T. From Fig.5, it is observed that an increase in T rises the secondary velocity distribution in both the regions and remains the same when T >

2. Of course, this is due to the tendency for instability at low values of T. Also, as T increases, the maximum primary and secondary velocity distributions in the channel tend to move above the channel center line towards Region-I.

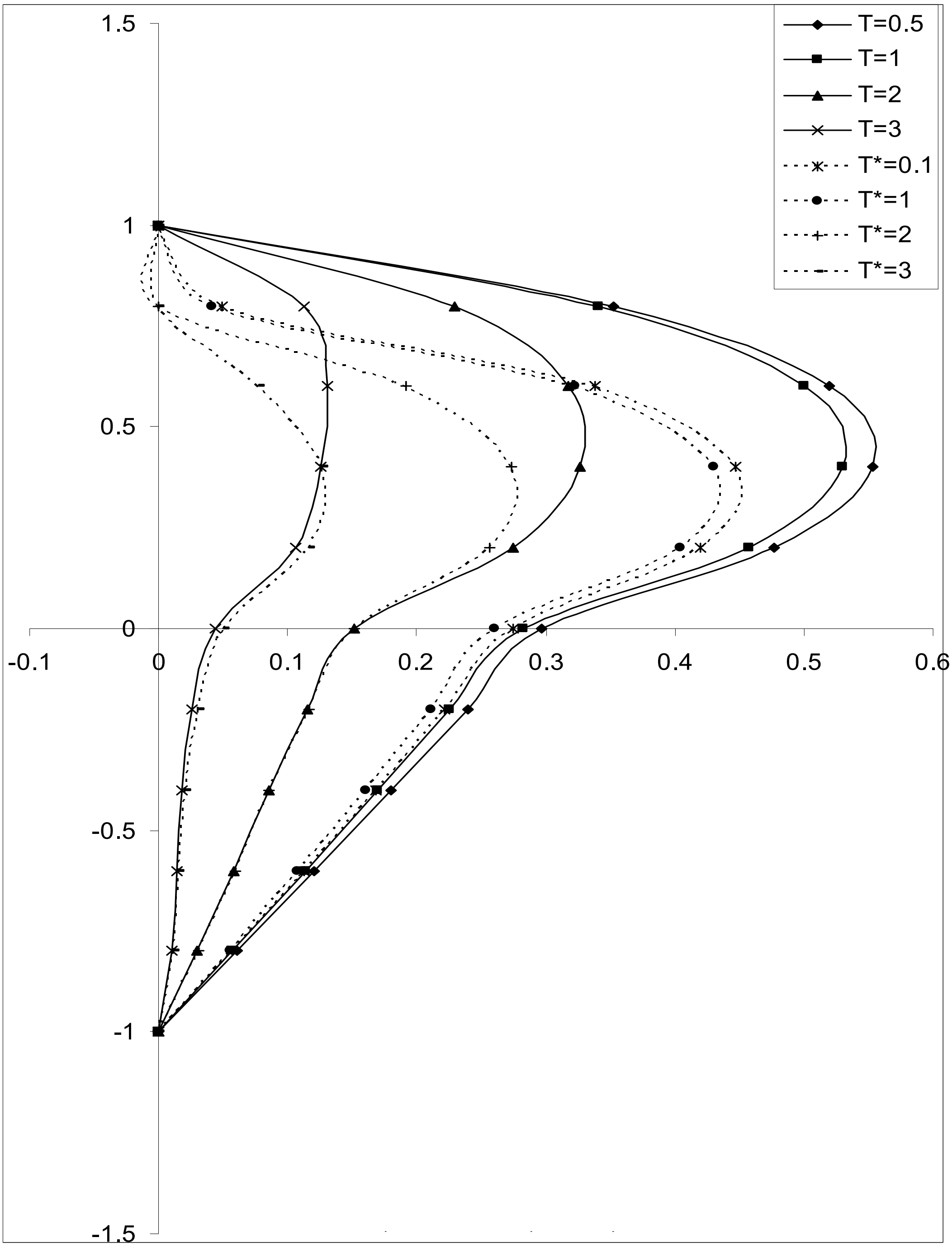

Fig.4. Primary velocity profiles u_1, u_2 (unsteady flow) u_1^*, u_2^* (steady flow) for different T and $\rho = 1.5$, $\lambda = 0.8$, M=*2*, $\alpha = 0.333$, $\sigma = 0.1$, Re=*-1*, *h=0.75*, $\varepsilon = 0.5$, $\omega = 1$, $t = \pi/\omega$.

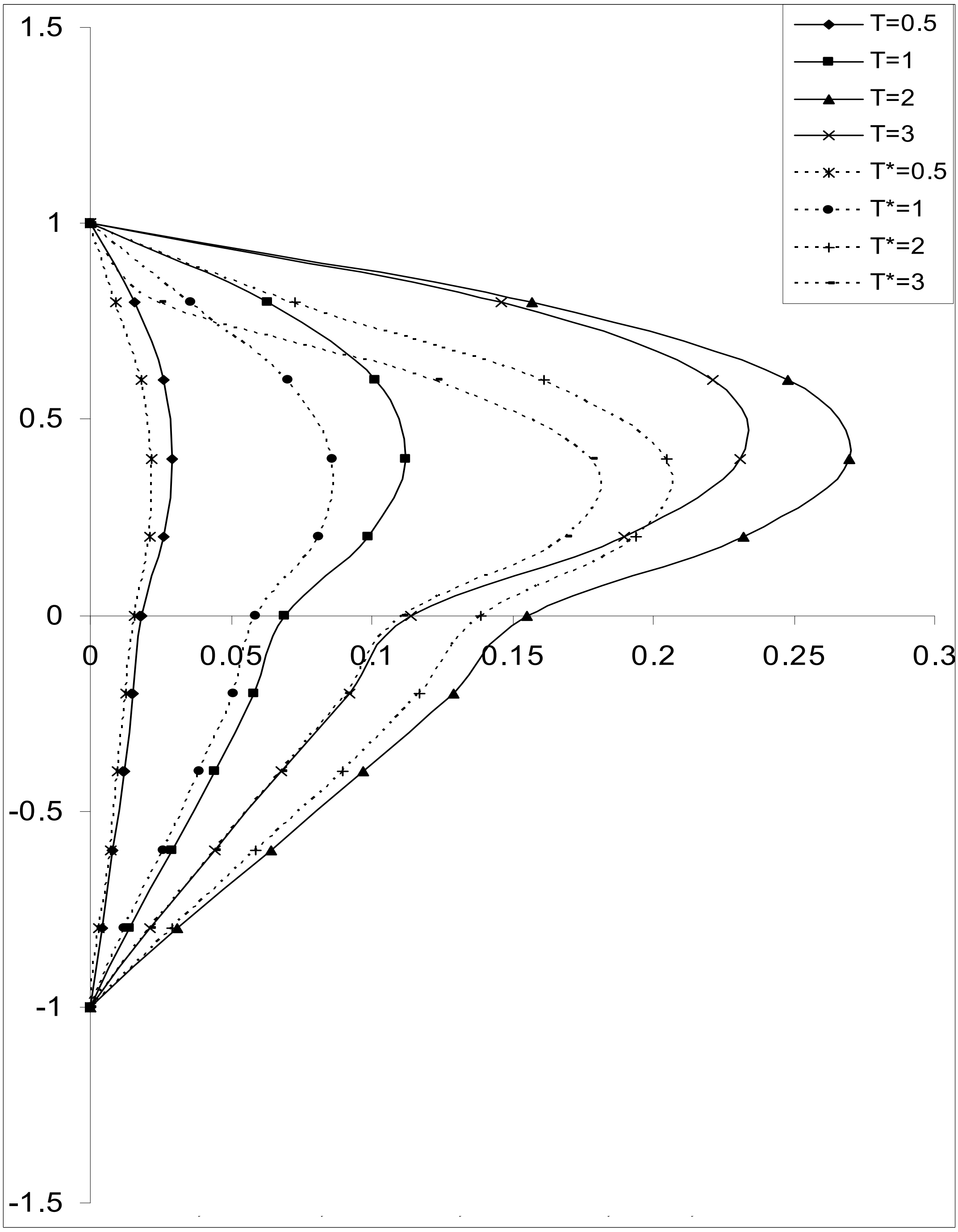

Fig.5. Secondary velocity profiles w_1, w_2 (unsteady flow) w_1^*, w_2^* (steady flow) for different T and $\rho = 1.5$, $\lambda = 0.8$, M=2, $\alpha = 0.333$, $\sigma = 0.1$, Re=-1, h=0.75, $\varepsilon = 0.5$, $\omega = 1$, $t = \pi/\omega$.

Figures 6 and 7 show the effect of varying values of the porous parameter (suction number) λ on both primary and secondary velocity distributions in the two fluid regions. From both the figures, it is found that there is a reduction in the velocity distributions with increasing values of the parameter λ. Also, the maximum primary and secondary velocity distributions in the channel tend to move above the channel centre line towards Region-I as λ increases.

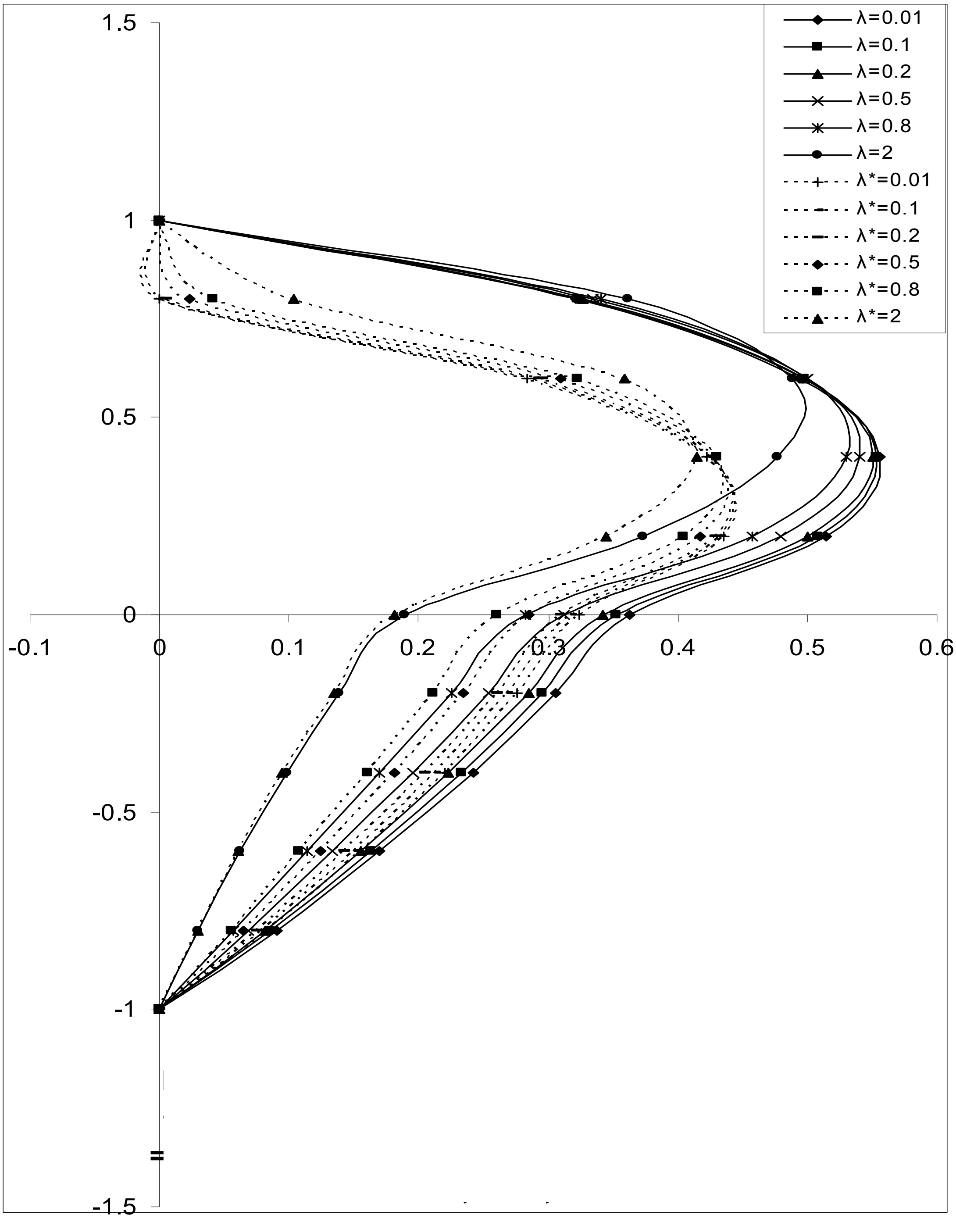

Fig.6. Primary velocity profiles u_1, u_2 (unsteady flow) u_1^*, u_2^* (steady flow) for different λ and $\rho = 1.5$, T=*1*, M=*2*, $\alpha = 0.333$, $\sigma = 0.1$, Re=*-1*, *h=0.75*, $\varepsilon = 0.5$, $\omega = 1$, $t = \pi/\omega$.

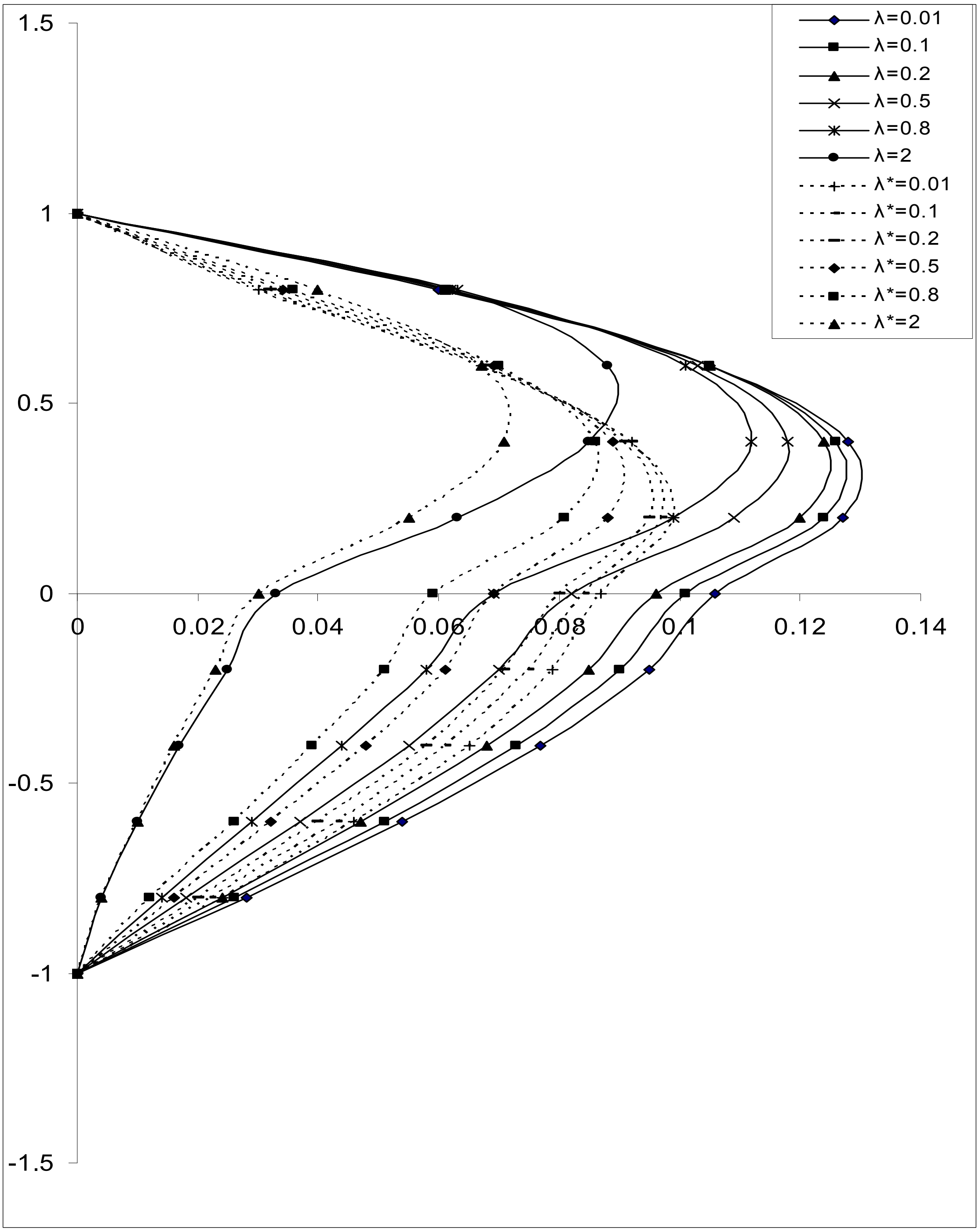

Fig.7. Primary velocity profiles w_1, w_2 (unsteady flow) w_1^*, w_2^* (steady flow) for different λ and $\rho = 1.5$, T=1, M=2, $\alpha = 0.333$, $\sigma = 0.1$, Re=-1, h=0.75, $\varepsilon = 0.5$, $\omega = 1$, $t = \pi/\omega$.

In Figs 8 and 9, we have illustrated the variation of the primary and secondary velocity distributions in the two regions with the electrical conductivity ratio σ. A substantial increase in the primary and secondary velocity distributions is observed as the electrical conductivity ratio σ increases. The maximum primary and secondary velocity distributions in the channel tend to move above the channel centre line towards Region-I as σ increases.

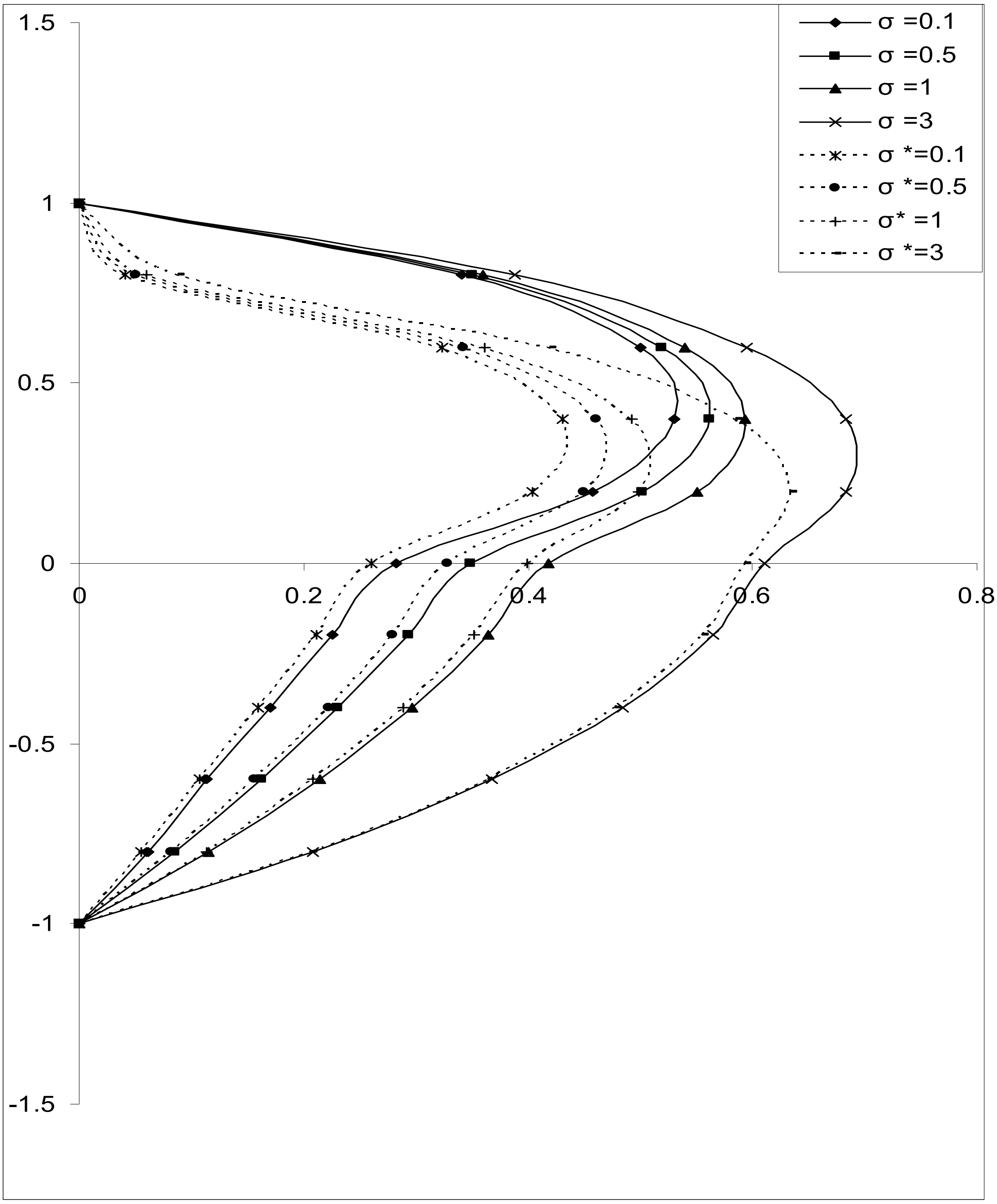

Fig.8. Primary velocity profiles u_1, u_2 (unsteady flow) u_1^*, u_2^* (steady flow) for different σ and $\rho = 1.5$, T=*1*, M=*2*, $\lambda = 0.8$, $\alpha = 0.333$, Re=*-1*, *h*=*0.75*, $\varepsilon = 0.5$, $\omega = 1$, $t = \pi/\omega$.

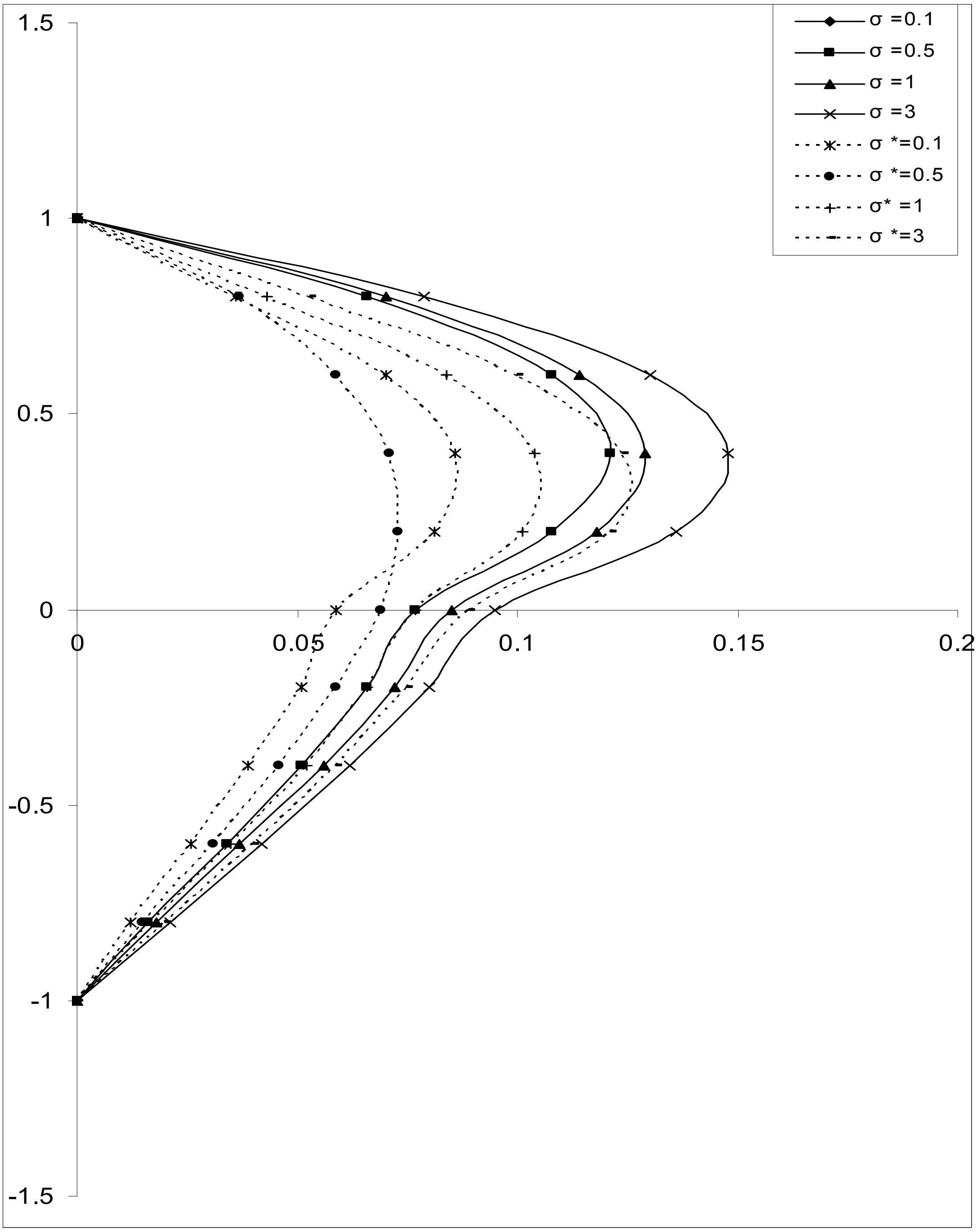

Fig.9. Primary velocity profiles w_1, w_2 (unsteady flow) w_1^*, w_2^* (steady flow) for different σ and $\rho = 1.5$, T=*1*, M=*2*, $\lambda = 0.8$, $\alpha = 0.333$, Re=*-1*, *h=0.75*, $\varepsilon = 0.5$, $\omega = 1$, $t = \pi/\omega$.

The influence of the viscosity ratio, α, on the primary and secondary velocity distributions in the two regions is depicted in Figs 10 and 11. It is noticed that the primary and secondary velocity distributions are increasing with an increasing values of α.

The maximum primary and secondary velocity distributions in the channel tend to move above the channel centre line towards region-I as α increases.

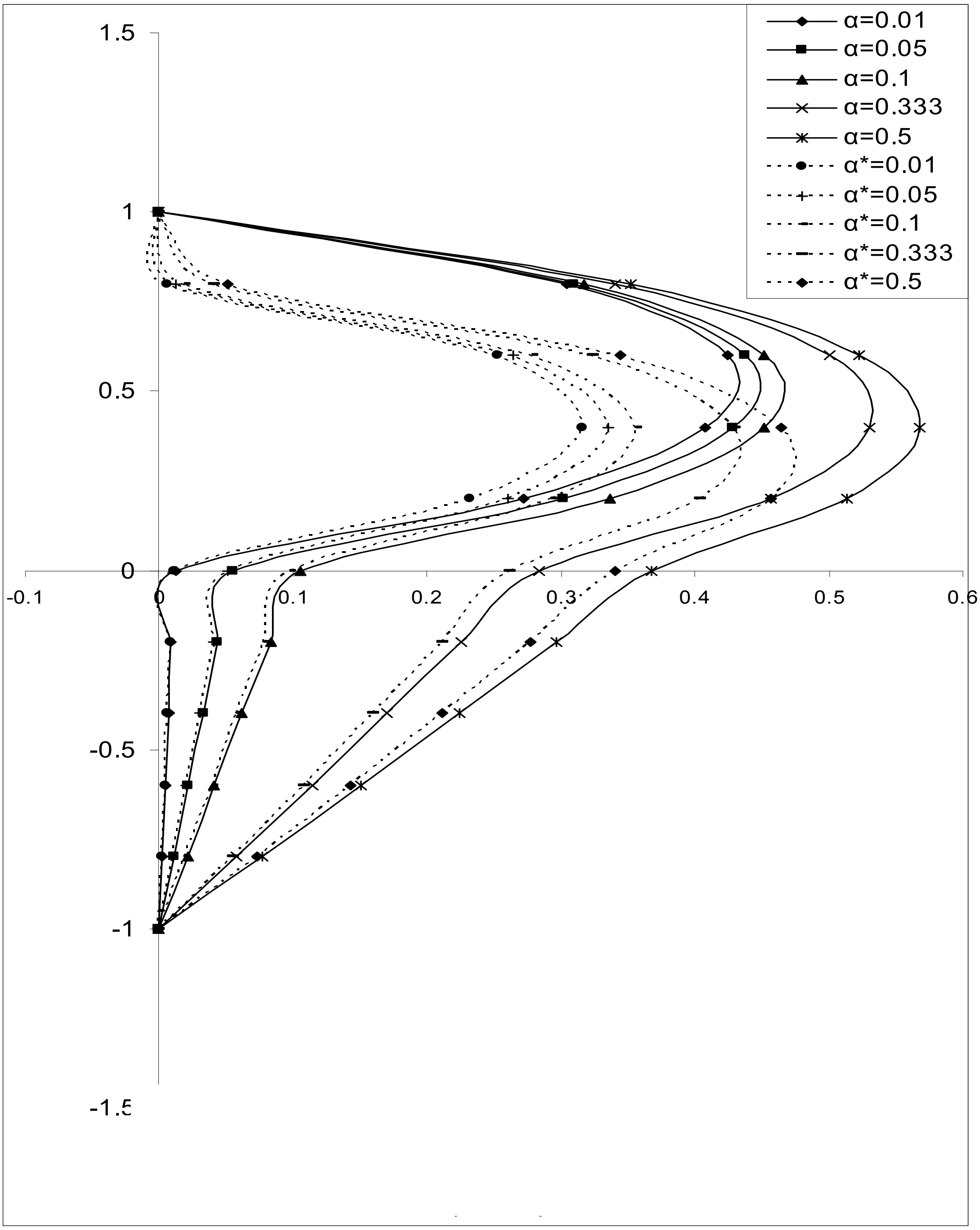

Fig.10. Primary velocity profiles u_1, u_2 (unsteady flow) u_1^*, u_2^* (steady flow) for different α and $\rho = 1.5$, T=*1*, M=*2*, $\lambda = 0.8$, $\sigma = 0.1$, Re=*-1*, *h=0.75*, $\varepsilon = 0.5$, $\omega = 1$, $t = \pi/\omega$.

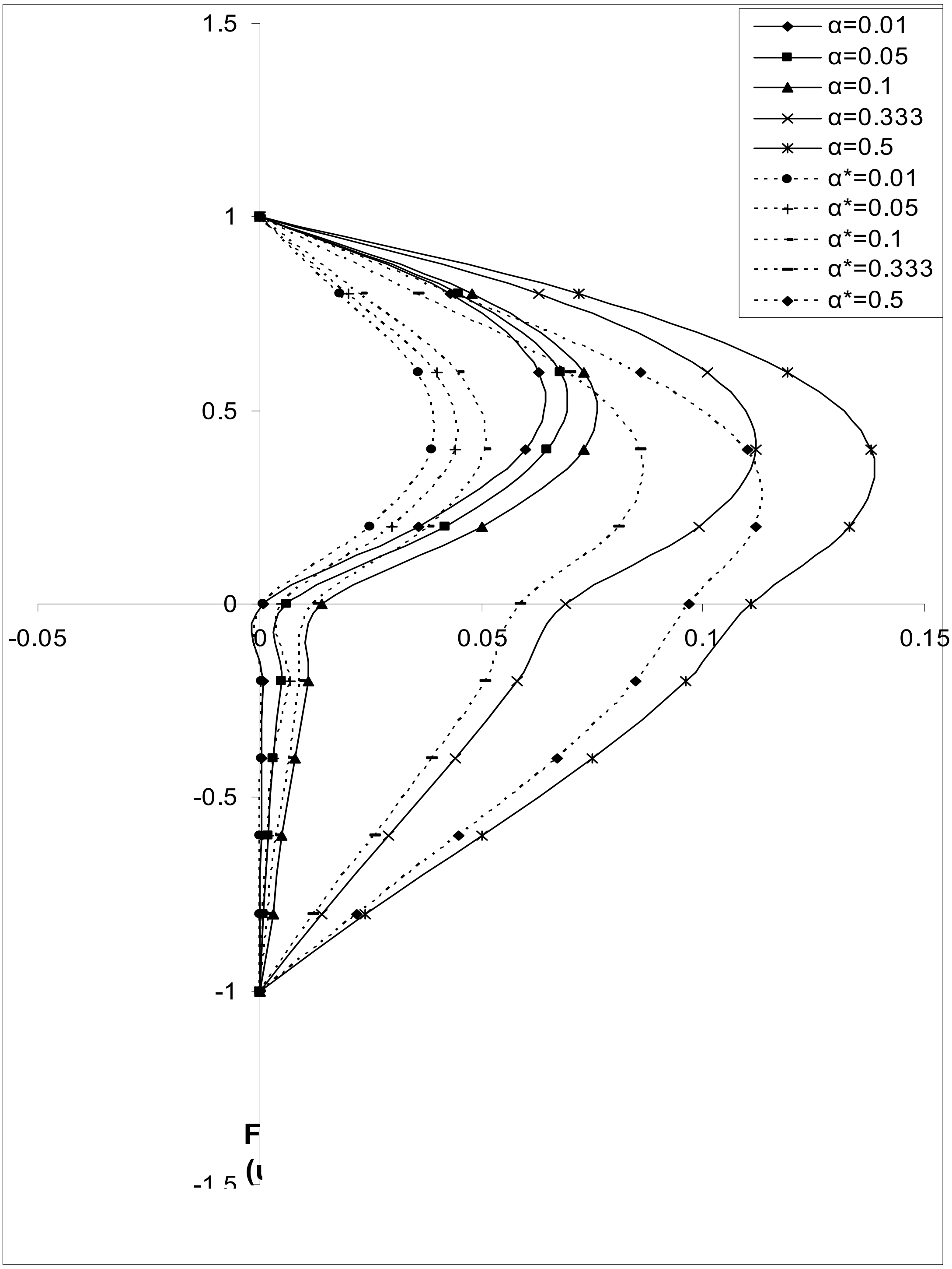

Fig.11. Primary velocity profiles w_1, w_2 (unsteady flow) w_1^*, w_2^* (steady flow) for different α and $\rho = 1.5$, T=*1*, M=*2*, $\lambda = 0.8$, $\sigma = 0.1$, Re=*-1*, *h=0.75*, $\varepsilon = 0.5$, $\omega = 1$, $t = \pi/\omega$.

The effect of varying values of the height ratio h on both the primary and secondary velocity distributions in the two regions is shown in Figs 12 and 13. It is seen that an increase in h increases both the primary and secondary velocity distributions in the two regions. Also as the parameter h increases, the maximum velocity in the channel tends to move above the channel centre line towards region-I.

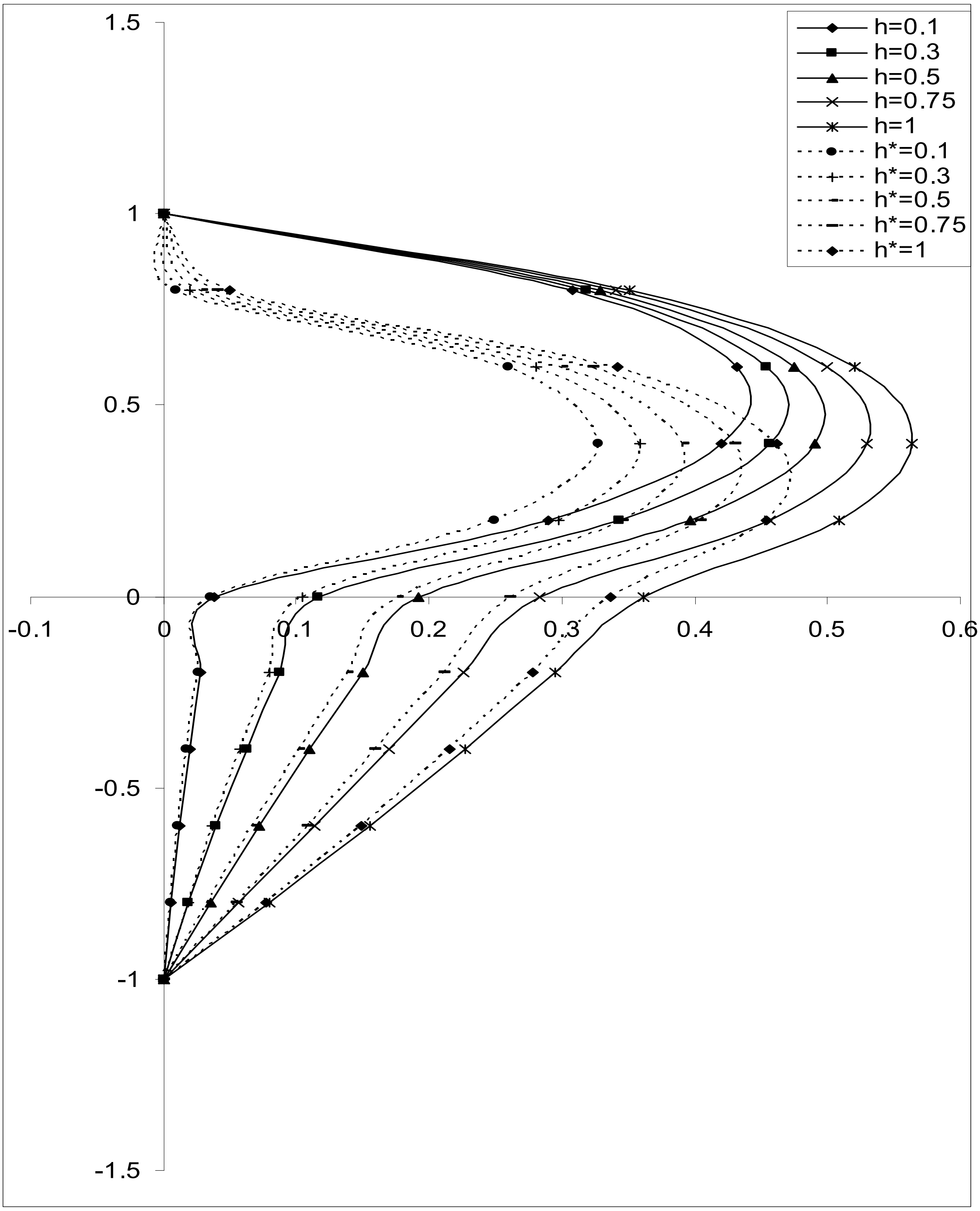

Fig.12. Primary velocity profiles u_1, u_2 (unsteady flow) u_1^*, u_2^* (steady flow) for different h and $\rho = 1.5$, T=1, M=2, $\lambda = 0.8$, $\alpha = 0.333$, Re=-1, $\sigma = 0.1$, $\varepsilon = 0.5$, $\omega = 1$, $t = \pi/\omega$.

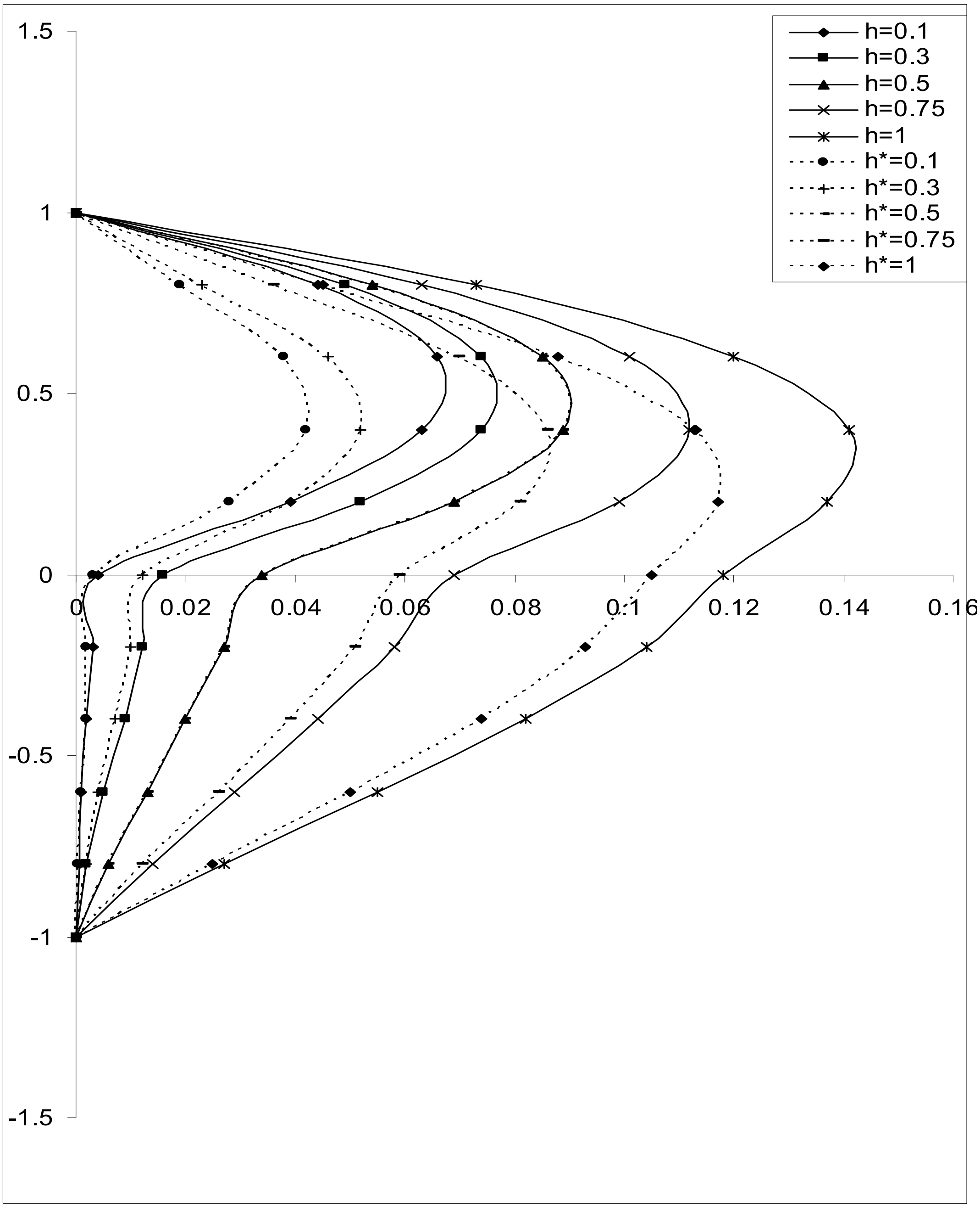

Fig.13. Primary velocity profiles w_1, w_2 (unsteady flow) w_1^*, w_2^* (steady flow) for different h and $\rho = 1.5$, T=1, M=2, $\lambda = 0.8$, $\alpha = 0.333$, Re=-1, $\sigma = 0.1$, $\varepsilon = 0.5$, $\omega = 1$, $t = \pi/\omega$.

5. Conclusion

The transient flow of electrically conducting two–layered fluids flow through a horizontal channel bounded by two parallel porous plates (one being stationary and the other oscillating) in a rotating system with an applied transverse magnetic field is studied analytically. The fluid is generated due to a common constant pressure gradient in a channel bounded by two parallel porous plates. The fluids in both regions are assumed to be immiscible, incompressible with different viscosities and electrical conductivities. The governing equations of motion are derived and are non-dimensionalised using an appropriate group of dimensionless variables controlled by physical parameters, namely, the Hartmann number (M), Taylor number (T), porous parameter (λ), ratio of the viscosities (α), electrical conductivities (σ) and heights (h). The resulting partial differential equations are transformed into a set of linear ordinary differential equations using two-term series as a combination of both steady state and transient time dependent parts, which in turn are solved in a closed form. The solutions are evaluated numerically to plot their graphs for the velocity distributions (primary and secondary) in both the regions. Comparisons with previously published theoretical studies are mode, achieving excellent agreement. A very good agreement is also obtained with the analytical solutions of Malashetty and Leela [28] when the motion is in steady state with non-porous plates and without rigid rotation. And only in the absence of rigid rotation, the results agree well with the solutions of Raju and Valli [38]. The primary velocity distributions in the two fluid regions are seen to increase with an increase in Hartmann number M, whereas the secondary velocity distribution increases and then remains the same when the value of M>*2* for selected values of the remaining parameters. It is found that an increase in T decreases the primary velocity distribution in the two regions, while an increase in the Taylor number T causes a rise in the secondary velocity distribution of the fluids in both regions for smaller values of T and remains in the same when the value of T > *2*. It is observed that an increase in the porous parameter decreases both the primary and secondary velocity distributions in the two regions. Hence, it is concluded that the velocities in the two regions can be enhanced with the suitable values of the ratios of viscosity, heights, electrical and thermal conductivities. However, it is hoped that the results reported herein will serve to motivate further experiments on this type of problems and are expected to be useful in verifying numerical algorithms used to solve more multifaceted or pragmatic problems of this type.

Nomenclature

$a_1, a_2, a_3, c_1, c_2, c_3, m_1, m_2, m_3$..etc.	– functions / real constants represented in the equations and solutions
B_0	– applied uniform transverse magnetic field
E_0	– constant electric field in the z-direction
h	– ratio of the heights of the two regions
h_1	– height of the channel in the upper region (Region-I)
h_2	– height of the channel in the lower region (Region-II)
M	– Hartmann number
Pr	– Prandtl number
p	– pressure
R_e	– electric load parameter
T	– Taylor number (rotation parameter)
t	– time
u_p	$=\left(-\frac{\partial p}{\partial x}\right)\frac{h_1^2}{\mu_1}$, characteristic velocity
$u_{01}(y), u_{02}(y)$	– primary velocity distributions in the basic steady state case in two regions
$u_{11}(y), u_{12}(y)$	– time dependent primary velocity components in the two regions
u_1, u_2	– x- component of velocity distributions in the two fluid regions, known as the primary velocity distributions
v_0	– constant suction velocity
w_1, w_2	– z- component of velocity distributions in the two fluid regions called the secondary velocity distributions
$w_{01}(y), w_{02}(y)$	– secondary velocity distributions in the basic steady state case in two regions
$w_{11}(y), w_{12}(y)$	– time dependent secondary velocity components in the two regions
(x, y, z)	– space co-ordinates in the rectangular Cartesian co-ordinate system

α – ratio of viscosities
β – ratio of thermal conductivities
λ – porous parameter (suction number)
σ – ratio of electrical conductivities
σ_1, σ_2 – electrical conductivities of the two fluids
ρ_1, ρ_2 – densities of the two fluids
μ_1, μ_2 – viscosities of the two fluids
ε – amplitude (a small constant quantity such that $\varepsilon << 1$)
ω – frequency of oscillation
Ω – angular velocity, where $\overline{\Omega} = (0, \Omega, 0)$

Subscripts

1, 2 – refers to the quantities in the upper and lower fluid, regions respectively

References

[1] Hide R. and Roberts P.H. (1961): *The origin of the mean geomagnetic field*. – In: Physics and Chemistry of the Earth. Pergamon Press, New York, vol.4, pp.27–98.

[2] Dieke R.H. (1970): *Internal rotation of the sun*. – In: L. Goldberg (eds.), Annual Reviews of Astronomy and Astrophysics, vol.8, Annual Reviews Inc., pp.297-328.

[3] Squire H.B. (1956): '*Surveys in mechanics*' Edited by Batchelor, G.K. and Davies, R.M. (Camb. Univ. Press, London).

[4] Gilman P.A. and Benton E.R. (1968): *Influence of an axial magnetic field on the steady linear Ekman boundary layer*. – Phys. Fluid, vol.11, pp.2397-.

[5] Benton E.R. and Loper D.E. (1969): *On the spin-up of an electrically conducting fluid part-1. The unsteady hydromagnetic Ekman-Hartmann boundary-layer problem*. – J. Fluid Mech., vol.39, pp.561-.

[6] Nanda R.S. and Mohanty H.K. (1971): *Hydromagnetic flow in a rotating channel*. – Applied Scientific Research, vol.24, pp.65.

[7] Gupta A.S. (1972): *Magnetohydrodynamic Ekmann layer*. – Acta Mechanica, vol.13, pp.155.

[8] Debnath L. (1972): *On unsteady magnetohydrodynamic boundary layers in a rotating system*. – ZAMM., vol.52, pp.623.

[9] Seth G.S., Jana R.N. and Maiti M.K. (1982): *Unsteady hydromagnetic Couette flow in a rotating system*. – International Journal of Engineering Science, vol.20, pp.989.

[10] Seth G.S., Nandkeolyar R., Mahto N. and Singh S.K. (2009): *MHD Couette flow in a rotating system in the presence of an inclined magnetic field*. – Applied Mathematical Sciences, vol.3, pp.2919.

[11] Mukherjee S. and Debnath L. (1977): *On unsteady rotating boundary layer flows between two porous plates*. ZAMM, vol.57, pp.188.

[12] Seth G.S. and Jana R.N (1980): *Unsteady hydromagnetic flow in a rotating channel with oscillating pressure gradient*. – Acta Mechanica, vol.37, p.29.

[13] Singh K.D. (2000): *An oscillatory hydromagnetic Couette flow in a rotating system*. – ZAMM, vol.80, pp.429.

[14] Ghosh S.K. (1993): *Unsteady hydromagnetic flow in a rotating channel with oscillating pressure gradient*. – Journal of the Physical Society of Japan, vol.62, pp.3893.

[15] Gupta A.S., Misra J.C., Reza M. and Soundalgekar V.M. (2003): *Flow in the Ekman layer on an oscillating porous plate*. – Acta Mechanica, vol.165, No.1, pp.1-16.

[16]Ghosh S.K. and Pop I. (2003). *Hall effects on unsteady hydromagnetic flow in a rotating system with oscillatory pressure gradient*. – Applied Mechanics and Engineering, vol.8, pp.43-.

[17]Hayat T. and Hutter K. (2004): *Rotating flow of a second-order fluid on a porous plate*. – International Journal of Non-Linear Mechanics, vol.39, pp.767.

[18]Guria M. and Jana R.N. (2007): *Hydromagnetic flow in the Ekman layer on an oscillating porous plate*. – Magnetohydrodynamics, vol.43, pp.3-11.

[19]Packham B.A. and Shail R. (1971): *Stratified laminar flow of two immiscible fluids*. – Proceedings of Cambridge Philosophical Society, vol.69, pp.443-448.

[20]Shail R. (1973): *On laminar tow-phase flow in magnetohydrodynamics*. – International Journal of Engineering Science, vol.11, pp.1103.

[21]Lielausis O. (1975): *Liquid metal magnetohydrodynamics*. – Atomic Energy Review, vol.13, pp.527.

[22]Michiyoshi, Funakawa, Kuramoto, C., Akita Y. and Takahashi O. (1977): *Instead of the helium-lithium annular-mist flow at high temperature, an air-mercury stratified flow in a horizontal rectangular duct in a vertical magnetic field.* – International Journal of Multiphase Flow, vol.3, pp.445.

[23]Chao J., Mikic B.B. and Todreas N.E. (1979): *Radiation streaming in power reactors*. – Proceedings of the Special Session, American Nuclear Society (ANS) Winter Meeting, Washington, D.C. Nuclear Technology, vol.42, pp.22-.

[24]Dunn P.F. (1980): *Single-phase and two-phase magnetohydrodynamic pipe flow*. – International Journal of Heat Mass Transfer, vol.23, pp.373.

[25]Gherson P. and Lykoudis P.S. (1984): *Local measurements in two-phase liquid-metal magneto-fluid mechanic flow*. – Journal of Fluid Mechanics, vol.147, pp.81-104.

[26]Lohrasbi J. and Sahai V. (1987): *Magnetohydrodynamic heat transfer in two phase flow between parallel plates*. – Journal of Applied Sciences Research, vol.45, pp.53-66.

[27]Serizawa A., Ida T., Takahashi O. and Michiyoshi I. (1990): *MHD effect on Nak-nitrogen two-phase flow and heat transfer in a vertical round tube*. – International Journal Multi-Phase Flow, vol.16, No.5, pp.761.

[28]Malashetty M.S. and Leela V. (1992): *Magnetohydrodynamic heat transfer in two phase flow*. – International Journal of Engineering Science, vol.30, pp.371-377.

[29]Ramadan H.M. and Chamkha A.J. (1999): *Two-phase free convection flow over an infinite permeable inclined plate with non-uniform particle-phase density*. – International Journal of Engineering Science, vol.37, pp.1351.

[30]Chamkha A.J. (2000): *Flow of two-immiscible fluids in porous and non-porous channels*. – ASME Journal of Fluids Engineering, 122, 117-124.

[31]Raju T.L. and Murti P.S.R. (2006): *Hydromagnetic two-phase flow and heat transfer through two parallel plates in a rotating system*. – J. Indian Academy of Mathematics, Indore, India, vol.28, No.2, pp.343-360.

[32]Healy J.V. and Young H.T. (1970): *Oscillating two- phase channel flows*. – Z. Angew. Math. Phys., vol.21, pp.454.

[33]Debnath L. and Basu U. (1975): *Unsteady slip flow in an electrically conducting two-phase fluid under transverse magnetic fields*. – NUOVO Cimento, vol.28B, pp.349-362 .

[34]Chamkha A.J. (2004): *Unsteady MHD convective heat and mass transfer past a semi-infinite vertical permeable moving plate with heat absorption*. – International Journal of Engineering Science, vol.42, pp.217-230.

[35]Umavathi J.C., Abdul Mateen, Chamkha A.J. and Al- Mudhaf A. (2006): *Oscillatory Hartmann two-fluid flow and heat transfer in a horizontal channel*. – International Journal of Applied Mechanics and Engineering, vol.11, No.1, pp.155-178.

[36]Tsuyoshi I. and Shu-Ichiro I. (2008): *Two-fluid magnetohydrodynamic simulation of converging Hi flows in the interstellar medium*. – The Astrophysical Journal, vol.687, No.1, pp.303-310.

[37]Raju T.L. and Sreedhar M. (2009): *Unsteady two-fluid flow and heat transfer of conducting fluids in channels under transverse magnetic field*. – International Journal of Applied Mechanics and Engineering, vol.14, No.4, pp.1093-1114.

[38]Raju T.L. and Nagavalli M. (2013): *Unsteady two-layered fluid flow and heat transfer of conducting fluids in a channel between parallel porous plates under transverse magnetic field.* – International Journal of Applied Mechanics and Engineering, vol.18, No.3, pp.699-726.

[39]Raju T.L. and Nagavalli M. (2014): *MHD two-layered unsteady fluid flow and heat transfer through a horizontal channel between parallel plates in a rotating system.* – International Journal of Applied Mechanics and Engineering, vol.19, No.1, pp.97-121.

[40]Holton J.R. (1965). *The influence of viscous boundary layers on transient motions in a stratified rotating fluid.* – International Journal of Atmospheric Science, vol.22, pp.402.

[41]Siegmann W.L. (1971): *The spin-down of rotating stratified fluids.* – Journal of Fluid Mechanics, vol.47, pp.689.

Received: August 19, 2015
Revised: March 16, 2016

Appendix

$$a_1 = M^2 + 2T^2 i\,, \qquad a_2 = M^2 R_e - 1\,, \qquad a_3 = a_1 - \omega \tan \omega t\,,$$

$$a_4 = M^2 h^2 \alpha\sigma + i2\rho\alpha h^2 T^2\,, \qquad a_5 = M^2 h^2 \alpha\sigma R_e - \alpha h^2\,, \qquad a_6 = a_4 - \omega \tan \omega t\,,$$

$$m_1 = \frac{\lambda + \sqrt{\lambda^2 + 4a_1}}{2}\,, \quad m_2 = \frac{\lambda - \sqrt{\lambda^2 + 4a_1}}{2}\,, \quad m_3 = \frac{\lambda + \sqrt{\lambda^2 + 4a_4}}{2}\,, \quad m_4 = \frac{\lambda - \sqrt{\lambda^2 + 4a_4}}{2}\,,$$

$$a_7 = e^{m_1 - m_2}\,, \qquad a_8 = \frac{a_2}{a_1} e^{-m_2}\,, \qquad a_9 = e^{-m_3 + m_4}\,, \qquad a_{10} = \frac{a_5}{a_4} e^{m_4}\,, \qquad a_{11} = \frac{-a_5}{a_4} + \frac{a_2}{a_1}\,,$$

$$a_{12} = 1 - a_7\,, \qquad a_{13} = -1 + a_9\,, \qquad a_{14} = a_{11} + a_{10} - a_8\,, \qquad a_{15} = m_1 - m_2 a_7\,,$$

$$a_{16} = \frac{1}{\alpha h}\left(m_3 - m_4 a_9\right), \qquad a_{17} = \frac{m_4 a_{10}}{\alpha h} - m_2 a_8\,, \qquad a_{18} = \frac{a_{14}a_{15} - a_{12}a_{17}}{a_{12}a_{15} + a_{12}a_{16}}\,,$$

$$a_{19} = \frac{a_{17} + a_{16}a_{18}}{a_{15}}\,, \qquad c_2 = -c_1 a_7 + a_8\,, \qquad c_1 = a_{19}\,, \qquad c_4 = -c_3 a_9 + a_{10}\,, \qquad c_3 = a_{18}\,,$$

$$m_5 = \frac{\lambda + \sqrt{\lambda^2 + 4a_3}}{2}\,, \quad m_6 = \frac{\lambda - \sqrt{\lambda^2 + 4a_3}}{2}\,, \quad m_7 = \frac{\lambda + \sqrt{\lambda^2 + 4a_6}}{2}\,, \quad m_8 = \frac{\lambda - \sqrt{\lambda^2 + 4a_6}}{2}\,,$$

$$a_{20} = e^{m_5 - m_6}\,, \qquad a_{21} = e^{-m_6}\,, \qquad a_{22} = e^{-m_8 + m_7}\,, \qquad a_{23} = 1 - a_{20}\,, \qquad a_{24} = 1 - a_{22}\,,$$

$$a_{25} = m_5 - m_6 a_{20}\,, \qquad a_{26} = \frac{1}{\alpha h}\left(-m_7 a_{22} + m_8\right), \qquad a_{27} = m_6 a_{21}\,, \qquad a_{28} = \frac{a_{23}a_{27} - a_{21}a_{25}}{a_{23}a_{26} - a_{24}a_{25}}\,,$$

$$a_{29} = \frac{-a_{27} + a_{28}a_{26}}{a_{25}}\,, \qquad c_5 = a_{29}\,, \qquad c_6 = -c_5 a_{20} + a_{21}\,, \qquad c_7 = -c_8 a_{22}\,, \qquad c_8 = a_{28}\,,$$

$$q_{11}(y) = c_5 e^{m_5 y} + c_6 e^{m_6 y}\,, \qquad q_{12}(y) = c_7 e^{m_7 y} + c_8 e^{m_8 y}\,.$$