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ANALYSIS OF FORCED SPATIAL VIBRATIONS OF A CENTRIFUGAL PUMP IMPELLER WITH AXIAL FORCES BALANCING DEVICE

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In this paper, a model of a pump impeller with annular seals and a balancing device, used as a combined support-seal assembly, is considered. The forced coupled radial, angular and axial vibrations of the rotor are determined with consideration of linearized inertial, damping, gyroscopic, positional and circulating forces and moments acting on the impeller from the side of the fluid flow in annular seals. The theoretical analysis is supplemented with a numerical example, the amplitude frequency characteristics are shown.

Key words: vibrations – radial, angular, axial, rotor, impeller, partial systems, self-centering, shaftless pump.

1. Introduction

In the rotary machines the rotor is the main assembly, which determines the reliability features. The common feature of centrifugal pump impellers is that they rotate in annular seals, which cause large pressure drops to occur. The resulting radial hydrodynamic forces and moments acting on the impeller have a decisive influence on its dynamics and, accordingly, on the dynamics of the whole machine.

The hydrostatic forces as well as inertial, damping, gyroscopic and circulation forces and moments generated in the clearances of the balancing device are known to contribute to changes in the natural and critical frequencies of the rotor and a loss in its dynamic stability, as described, for example, by Childs [1], San Andres [2] and Gosiewski [3].

In recent theoretical approaches (Cheng *et al*. [4]; Li *et al*. [5]; Faria and Miranda [6]), the transverse vibrations of the rotor are analyzed by applying a non-linear model of dynamic fluid forces generated in the annular seals proposed by Muszynska and Bently [7]. The results of the numerical calculations based on this complex non-linear model were represented in the form of dynamic trajectories of the impeller centre, Poincare maps, and bifurcation diagrams. A dynamic analysis of the impeller-clearance seals system based on linearized models is also essential as it provides us with analytical relationships to be used in engineering practice.

In their earlier works (Kundera and Martsynkovsky [8]; Martsynkovsky *et al*. [9]), the authors describe a single-stage pump with an impeller directly connected to a balancing device, in which the annular

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and face clearance seals act as a self-regulating coupled journal-and-thrust bearing. The papers present a design of the balancing device and analyze the effect of coupled radial and axial vibration of the impeller.

This paper is a follow-up theoretical dynamic analysis of an impeller-balancing device system, where the impeller is subjected to coupled forced radial, angular and axial vibrations. The dynamic analysis is based on linearized hydrodynamic forces and moments generated in the clearances of the seals of the impeller, described in the works by Martsynkovsky [10], Marcinkowski and Kundera [11], Korczak [12] and Jędral [13]. The results are analytical relationships that can be easily used as the first approximation at the design stage of new impeller-based systems.

2. Problem statement

The first impellers integrated with a balancing device were patented in the 1990s, for example, by Martsinkovsky *et al.* [14], Kubota [15] and Chiba et al. [16]. In the patent of Martsinkovsky *et al.* [14], the design of the single-stage pump impeller is characterized by the lack of a classic drive shaft connected to the rolling element bearings. The bearing node is replaced by longitudinal and lateral seal clearances of the impeller and the balancing device. The hydrodynamic forces and moments generated in these clearances position the impeller relative to the pump casing.

The diagram in Fig.1 shows a single-stage centrifugal pump with an impeller, 2, driven by a flexible shaft, 3, via a ball joint, 4. The longitudinal clearances 5 and 6, the sealing wear-ring clearances, and the lateral clearance, 10, with the annular chamber, 9, act as the lateral and longitudinal self-adjusting hydrostatic bearings of the impeller. In the pump casing, 1, behind the rear shroud of the impeller, there are radial vanes, 7, which suppress the rotation of the fluid (generated by the impeller) and prevent a loss of pressure in the centripetal direction.

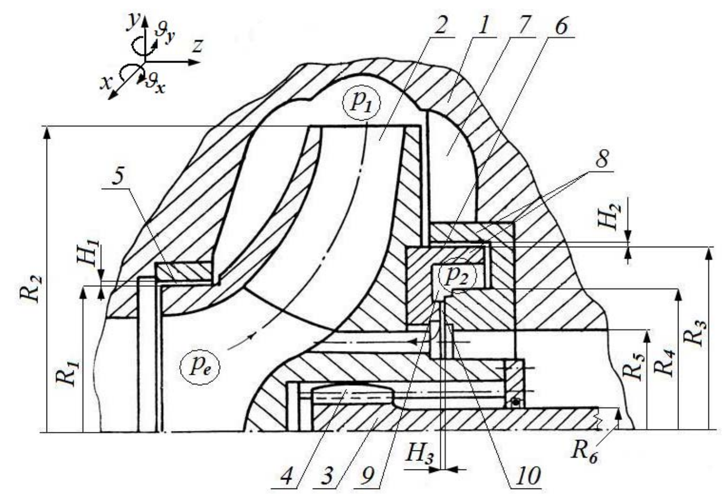


Fig.1. Geometry of the impeller with a balancing device: 1- pump casing, 2- impeller, 3- shaft; 4 – ball joint; 5, 6 – longitudinal seals of the impeller; 7 – radial vanes; 8- liner disks of balance device; 9- annular chamber; 10 – lateral (face) clearance.

The annular clearance (throttle) 6, the chamber 9 with variable pressure p_2, and the lateral clearance (throttle) 10, coupled with automatic axial force balancing systems control the axial position of the impeller. As small misalignments of the shaft relative to the axis of the annular seal have little effect on the flow rate, correlations between angular and axial vibrations can be neglected. However, angular and axial vibrations are dependent on radial vibrations.

The impeller is connected to a flexible drive shaft (torsion bar). The shaft diameter is chosen according to the strength under the influence of the engine torsional moment. Based on this, the design was called as *shaftless pump*. Since the shaft is flexible, the impeller is free to radial, angular and axial movements. Under the action of hydrodynamic forces and moments that occur in annular seals and the gyroscopic moment of the impeller during the operation process of the pump, the impeller self-centers and takes the most stable position against external disturbances.

3. Derivation of equations of system dynamics

The dynamics of the studied single-disc rotor model is described by five generalized coordinates $x, y, z, \vartheta_x, \vartheta_y$ (Fig.1). The balancing device adds a half of degree of freedom due to the condition of balance flow through its throttling channels. Since the hydrodynamic forces in annular seals depend on all generalized coordinates, the forced spatial vibrations of a single disc rotor (impeller) with a balancing device are described by the coupled system of nonhomogeneous differential equations of 11th order.

The derivation of equations of coupled radial, angular and axial vibrations of the impeller with a balancing device is based on the expression of the pressure p_2 in the annular chamber (9) (Fig.1), that depends both on the face seal clearance H_3, and on the radial rotor offsets $\varepsilon = e/H_2$.

The pressure in the annular chamber affects the value of the axial force, which is defined as a balancing device control action. This pressure is determined from the equation of continuity of flow through the system of seal clearances: longitudinal (6) and lateral (10) (Fig.1).

The dimensionless pressure ψ_2 can be written in the following operational form, by substitution $\left(p \equiv d/dt\right)$ (Martsynkovsky *et al*. [9]

$$\psi_2 = \frac{p_2}{p_n} = -\kappa_s \frac{M_2(p)}{D_2(p)} u_z + \frac{1}{D_2(p)}\left(k_1\psi_1 + k_2\psi_e + k_3\varepsilon\right) \tag{3.1}$$

where

$$D_2(p) = T_2 p + 1, \qquad M_2(p) = \tau_2 p + 1, \tag{3.2}$$

$$T_2 = \frac{2Vp_n}{EQ_0}\frac{\Delta\psi_2^0\Delta\psi_c^0}{\Delta\psi_s^0}, \qquad \tau_2 = \frac{2A_2 z_0}{3Q_0}, \qquad \kappa_s = \frac{3\Delta\psi_2^0\Delta\psi_c^0}{u^0\Delta\psi_s^0}, \tag{3.3}$$

$$k_1 = \frac{\Delta\psi_c^0}{\Delta\psi_s^0}, \qquad k_2 = \frac{\Delta\psi_2^0}{\Delta\psi_s^0}, \qquad k_3 = 2\varepsilon_c\frac{\Delta\psi_2^0\Delta\psi_c^0}{\Delta\psi_s^0}, \tag{3.4}$$

$$\psi_1 = \frac{p_1}{p_n},\ \psi_2 = \frac{p_2}{p_n},\ \psi_e = \frac{p_e}{p_n},\ \Delta\psi_s = \frac{p_1 - p_e}{p_n},\ \Delta\psi_2 = \frac{p_1 - p_2}{p_n},\ \Delta\psi_c = \frac{p_2 - p_e}{p_n}, \tag{3.5}$$

$$\Delta\psi_2^o = \psi_1^o - \psi_2^o, \quad \Delta\psi_s^o = \psi_1^o - \psi_e^o, \quad \Delta\psi_c^o = \psi_2^o - \psi_e^o,$$

$$Q^o = Q_2^o = Q_3^o = g_{2o}\sqrt{\Delta\psi_2^o} = g_{3o}\sqrt{\Delta\psi_s^o} . \tag{3.6}$$

The values that correspond to the steady state of the rotor are marked by the zero superscript.

The equation of forced axial vibrations is linearized with respect to to the pressure ψ_2 in operator form and is expressed as?

$$D_{zz}(p)u_z + d_{zr}u_r = -\bar{A}_1 N_1(p)\psi_1 - \bar{A}_e N_e(p)\psi_e . \tag{3.7}$$

In this equation the proper operator and deviation action operator are expressed by the equalities

$$D_{zz}(p) = c_0 p^3 + c_1 p^2 + c_2 p + c_3, \qquad d_{zr} = -\bar{A}_2 k_3 \bar{H} , \tag{3.8}$$

$$c_0 = T_1^2 T_2, \qquad c_1 = T_1^2 + 2\varsigma T_1 T_2, \qquad c_2 = 2\varsigma T_1 + \kappa_s \bar{A}_2 \tau_2, \qquad c_3 = \kappa_s \bar{A}_2 , \tag{3.9}$$

$$N_1(p) = T_2 p + 1 - k_1 A_2/A_1 , \qquad N_e(p) = T_2 p + 1 - k_2 A_2/A_e , \tag{3.10}$$

$$T_1^2 = \frac{mH_3}{A_A p_n}, \quad 2\varsigma T_1 = \frac{cH_3}{A_A p_n} , \tag{3.11}$$

$$A'_e = \pi\left(R_1^2 - R_5^2\right), \quad A_c = \pi\left(R_4^2 - R_5^2\right), \quad A'_2 = \pi\left(R_3^2 - R_4^2\right), \quad A_3 = \pi R_6^2 ,$$

$$A_1 = A_A - A_B, \quad A_e = A'_e + A_3 - 0.5 A_c, \quad A_2 = A'_2 + 0.5 A_c, \tag{3.12}$$

$$\bar{A}_1 = \frac{A_1}{A_1}, \qquad \bar{A}_2 = \frac{A_2}{A_A}, \qquad \bar{A}_e = \frac{A_e}{A_A},$$

$$u_z = \frac{z}{H_3}, \quad u_r = \frac{r}{H_3}, \quad \varepsilon = \frac{r}{H_2} = u_r \bar{H}, \quad \bar{H} = \frac{H_3}{H_2}. \tag{3.13}$$

The equations of forced radial and angular vibrations of the impeller in the annular seals were solved using expressions for the forces and moments are described in the above mentioned works. Using the linearized hydrodynamic forces and moments generated in the clearances of the impeller seals, we can write the equations of coupled forced radial and angular vibrations of the impeller as Martsynkovsky [10]

$$\begin{gathered}
a_1\ddot{u}_r + a_2\dot{u}_r + a_3 u_r - i\left(a'_4\dot{u}_r + a'_5 u_r\right)\omega - \left(\alpha'_2\dot{\theta} + \alpha'_3\theta\right)\omega + \\
-i\left(\alpha_4\dot{\theta} + \alpha_5\theta - \alpha_0\theta\right) = \omega^2 \mathrm{A} e^{i\omega t} , \\
b_1\ddot{\theta} + b_2\dot{\theta} + b_3\theta - i\left(b'_4\dot{\theta} + b'_5\theta\right)\omega + \left(\beta'_2\dot{u}_r - \beta'_3 u_r\right)\omega + \\
-i\left(\beta_4\dot{u}_r + \beta_5 u_r + \beta_0 u_r\right) = \omega^2 \Gamma e^{i\omega t} .
\end{gathered} \tag{3.14}$$

The aggregate coefficients for the front and rear annular seals are as follows

$$\begin{gathered}
a_1 = 1 + a_{11} + a_{12}, \quad a_2 = a_{21} + a_{22}, \quad a_3 = \Omega_{u0}^2 + a_{31} + a_{32}, \\
a_4 = a'_4\omega, \quad a'_4 = a'_{41} + a'_{42}, \quad a_5 = a'_5\omega, \quad a'_5 = a'_{51} + a'_{52},
\end{gathered} \tag{3.15}$$

$$b_1 = 1 + b_{11} + b_{12}, \quad b_2 = b_{21} + b_{22}, \quad b_3 = \Omega_{\vartheta 0}^2 + b_{31} + b_{32},$$
$$b_4 = b_4'\omega, \quad b_4' = j_0 + b_{41}' + b_{42}', \quad b_5 = b_5'\omega, \quad b_5' = b_{51}' + b_{52}', \tag{3.16}$$

$$\alpha_2' = \alpha_{21}' + \alpha_{22}', \quad \alpha_3' = \alpha_{31}' + \alpha_{32}', \quad \alpha_4 = \alpha_{41} + \alpha_{42}, \quad \alpha_5 = \alpha_{51} + \alpha_{52}, \tag{3.17}$$

$$\beta_2' = \beta_{21}' + \beta_{22}', \quad \beta_3' = \beta_{31}' + \beta_{32}', \quad \beta_4 = \beta_{41} + \beta_{42}, \quad \beta_5 = \beta_{51} + \beta_{52} \tag{3.18}$$

where

$$u_r = u_x + iu_y, \qquad u_{x,y} = \frac{x,y}{H_3}, \qquad \theta = \theta_x + i\theta_y, \qquad \theta_{x,y} = \frac{\vartheta_{x,y} l_2}{2H_2}, \tag{3.19}$$

$$\Omega_{u0}^2 = \frac{k_{11}}{m}, \quad \Omega_{\vartheta 0}^2 = \frac{k_{22}}{I}, \quad \alpha_{0i} = \frac{2k_{12}}{ml_i}, \quad \beta_{0i} = \frac{k_{12} l_i}{2I}, \quad A = \frac{a}{H_2}, \quad \Gamma = (1 - j_0)\frac{\gamma l_2}{2H_2}. \tag{3.20}$$

The coefficients of stiffness for the shaftless pump (Fig.1) model are $k_{11} = k_{22} = k_{12} = 0$. The products $\omega \cdot j_0 = \omega I_0 / I$ describe the gyroscopic moment of the disc.

The coefficients for the front (*i=1*) and rear (*i=2*) seals are

$$a_{1i} = k_{gi}, \qquad a_{2i} = k_{di} + k_{gi} K_{ii} \theta_{\oplus}, \tag{3.21}$$

$$a_{3i} = k_{pi}\left(\theta_{\oplus} + \chi_{mi}\right), \qquad a_{4i}' = 0.5 k_{gi} \kappa_{1i}, \qquad a_{5i}' = 0.5 k_{di} \kappa_{1i}, \tag{3.22}$$

$$b_{1i} = 0{,}5 a_{1i}\left(j_i + j_c\right), \quad b_{2i} = 0{,}5 a_{2i} j_c + k_{di} j_i, \quad b_{4i}' = 0{,}5 a_{4i}'\left(j_i + j_c\right),$$
$$b_{5i}' = 0{,}.a_{5i}'\left(j_i + j_c\right), \quad b_{3i} = 0.5 a_{3i}\left(j_c - j_i \cdot b_{3i*}\right), \quad b_{3i*} = \frac{10\chi_{mi}}{\theta_{\oplus} + \chi_{mi}}, \tag{3.23}$$

$$\alpha_{2i}' = \frac{1}{15} k_{gi} \kappa_{1i} \theta_{\oplus}, \quad \alpha_{3i}' = \frac{1}{5} k_{di} \kappa_{1i} \theta_{\oplus}, \quad \alpha_{4i} = \frac{2}{5} k_{di} \theta_{\oplus}, \quad \alpha_{5i} = k_{pi}\left(1 + 2\Delta\chi\right), \tag{3.24}$$

$$\beta_{2i}' = 7.5 \alpha_{2i}' j_i, \quad \beta_{3i}' = 7.5 \alpha_{3i}' j_i, \quad \beta_{4i} = 7.5 \alpha_{4i}' j_i, \quad \beta_{5i} = \alpha_{5i} j_i \frac{2.5 \Delta\chi_i}{1 + 2 \cdot \Delta\chi_i}, \tag{3.25}$$

$$K_{ii} = 0.1 \frac{\rho q_0}{\mu}, \quad q_{0i} = 10 \left(\frac{\Delta p_{0i} H_i^3}{\rho l_i} \right)^{0.5}, \quad \chi_{mi} = \frac{1}{1 + 0.02 l_i / H_i}, \tag{3.26}$$

$$j_i = \frac{m l_i^2}{60 I}, \quad j_{ci} = \frac{m l_{ci}^2}{I}, \quad j_0 = \frac{I_0}{I}, \quad I = m\left(\frac{R^2}{4} + \frac{b_e^2}{12} \right), \quad I_0 = m \frac{R^2}{2}. \tag{3.27}$$

The formulas for the hydrostatic stiffness coefficient, damping and inertia coefficients in the annular seals have the following forms (Marcinkowski and Kundera [11])

$$k_{p1} = \Delta p_1 \frac{\pi R_1 l_1}{2H_1 m}, \left[s^{-2}\right], \quad k_{d1} = \frac{\pi R_1 l_1^3}{120 H_1^2 m}\left(\frac{\rho \Delta p_1 H_1}{l_1}\right)^{0.5}, \left[s^{-1}\right], \quad k_{g1} = \rho \frac{\pi R_1 l_1^3}{12 H_1 m}, [1], \tag{3.28}$$

$$k_{p2} = \Delta p_2 \frac{\pi R_3 l_2}{2H_2 m}, \left[s^{-2}\right], \quad k_{d2} = \frac{\pi R_3 l_2^3}{120 H_2^2 m}\left(\frac{\rho \Delta p_2 H_2}{l_2}\right)^{0.5}, \left[s^{-1}\right], \quad k_{g2} = \rho \frac{\pi R_3 l_2^3}{12 H_2 m}, [1]. \tag{3.29}$$

The connection of axial, radial and angular vibrations is caused by the majority of the coefficients of hydrodynamic forces and the moments that depend on the throttling pressure fall on the rear annular seal; $\Delta p_2 = p_1 - p_2$. Coefficients of the straight lines potential a_3, b_3 and cross circulation α_5, β_5 forces and moments are proportional to the first extent of difference Δp_2 and coefficients of straight lines dissipation – a_2, b_2 and circulation a_5, b_5, as well as the cross potential α_3, β_3 and gyroscope α_4, β_4 are proportional $\sqrt{\Delta p_2}$.

The throttling pressure fall on the front annular seal is $\Delta p_1 = p_1 - p_e$. If the pressure pump and the inlet pressure are considered as the external influences, the linearized forces expressions for the front seal are obtained from the expressions for the rear seal through replacing p_2 by p_e.

4. Equations of coupled radial, angular and axial vibrations

We obtain the equations of coupled radial and angular vibrations having the expression for the pressure in the annular chamber (3.1) and the linearized coefficients of hydrodynamic forces and the moments in equations (3.14). Arranging the terms according to the order of the differential operator, we obtain the equations of the system in the following form

$$\begin{gathered}
\left(D_{rr}(p) - \frac{k_3 \bar{H}}{D_2(p)} C_{a2}\right) u_r + d_{r\theta}(p)\theta + \kappa_s \frac{M_2(p)}{D_2(p)} C_{a2} u_z = \\
= \omega^2 \mathrm{A} e^{i\omega t} - \psi_1 \left(C_{a1} + \left(1 - \frac{k_1}{D_2(p)}\right) C_{a2}\right) + \psi_e \left(C_{a1} + \frac{k_2}{D_2(p)} C_{a2}\right), \\
\left(d_{\theta r}(p) - \frac{k_3 \bar{H}}{D_2(p)} C_{b2}\right) u_r + D_{\theta\theta}(p)\theta + \kappa_s \frac{M_2(p)}{D_2(p)} C_{b2} u_z = \\
= \omega^2 \Gamma e^{i\omega t} - \psi_1 \left(C_{b1} + \left(1 - \frac{k_1}{D_2(p)}\right) C_{b2}\right) + \psi_e \left(C_{b1} + \frac{k_2}{D_2(p)} C_{b2}\right)
\end{gathered} \tag{4.1}$$

where

$$D_{rr}(p) = a_1^0 p^2 + a_2^0 p + a_3^0 - i\left(a_4'^0 p\omega + a_5'^0 \omega_n\right), \tag{4.2}$$

$$D_{\theta\theta}(p) = b_1^0 p^2 + b_2^0 p + b_3^0 - i\left(b_4'^0 p\omega + b_5'^0 \omega_n\right), \tag{4.3}$$

$$d_{r\theta}(p) = -\alpha_2'^0 p\omega - \omega_n \alpha_3'^0 - i\left(\alpha_4^0 p + \alpha_5^0 - \alpha_0\right), \tag{4.4}$$

$$d_{\theta r}(p) = \beta_2'^0 p\omega - \omega_n \beta_3'^0 - i\left(\beta_4^0 p + \beta_5^0 + \beta_0\right), \tag{4.5}$$

$$C_{a1} = \frac{2u_r^0 a_{31}^0 - \omega_n \theta^0 \alpha_{31}'^0 - i\left(\omega_n u_r^0 a_{51}'^0 + 2\theta^0 \alpha_{51}'^0\right)}{2\Delta\psi_1^0}, \tag{4.6}$$

$$C_{a2} = \frac{2u_r^0 a_{32}^0 - \omega_n \theta^0 \alpha_{32}'^0 - i\left(\omega_n u_r^0 a_{52}'^0 + 2\theta^0 \alpha_{52}'^0\right)}{2\Delta\psi_2^0}, \tag{4.7}$$

$$C_{b1} = \frac{2\theta^0 b_{31}^0 - \omega_n u_r^0 \beta_{31}^0 - i\left(\omega_n \theta^0 b_{51}'^0 + 2u_r^0 \beta_{51}'^0\right)}{2\Delta\psi_1^0}, \tag{4.8}$$

$$C_{b2} = \frac{2\theta^0 b_{32}^0 - \omega_n u_r^0 \beta_{32}'^0 - i\left(\omega_n \theta^0 b_{52}'^0 + 2u_r^0 \beta_{52}'^0\right)}{2\Delta\psi_2^0}. \tag{4.9}$$

The system of forced radial, angular and axial vibrations consist of Eqs (3.7) and (4.1)

$$R_r(p)u_r + R_\theta(p)\theta + R_z(p)u_z = M_{ra}(p)a + M_{r\psi 1}(p)\psi_1 + M_{r\psi e}(p)\psi_e, \tag{4.10}$$

$$\Theta_r(p)u_r + \Theta_\theta(p)\theta + \Theta_z(p)u_z = M_{\theta\gamma}(p)\gamma + M_{\theta\psi 1}(p)\psi_1 + M_{\theta\psi e}(p)\psi_e, \tag{4.11}$$

$$Z_r u_r + Z_z(p)u_z = M_{z\psi 1}(p)\psi_1 + M_{z\psi e}(p)\psi_e \tag{4.12}$$

where

$$\begin{gathered} R_r(p) = \left(D_{rr}(p)D_2(p) - k_3\bar{H}C_{a2}\right), \quad R_\theta(p) = \left(d_{r\theta}(p)D_2(p)\right), \\ R_z(p) = \kappa_s M_2(p)C_{a2}, \end{gathered} \tag{4.13}$$

$$\begin{gathered} \Theta_r(p) = \left(d_{\theta r}(p)D_2(p) - k_3\bar{H}C_{b2}\right), \quad \Theta_\theta(p) = D_{\theta\theta}(p)D_2(p), \\ \Theta_z(p) = \kappa_s M_2(p)C_{b2}, \end{gathered} \tag{4.14}$$

$$Z_z(p) = D_{zz}(p), \qquad Z_r = d_{zr}, \tag{4.15}$$

$$M_{ra}(p) = \frac{\omega^2}{H_1}D_2(p), \qquad M_{r\psi 1}(p) = -\frac{\omega^2}{\omega_n^2}\left(C_{a1}D_2(p) + \left(D_2(p) - k_1\right)C_{a2}\right), \tag{4.16}$$

$$M_{r\psi e}(p) = \left(C_{a1}D_2(p) + k_2 C_{a2}\right), \qquad M_{\theta\gamma}(p) = \omega^2 \frac{(1 - j_0)l_1}{2H_1}D_2(p), \tag{4.17}$$

$$M_{\theta\psi 1}(p) = -\frac{\omega^2}{{\omega_n}^2}\left(C_{b1}D_2(p) + \left(D_2(p) - k_1\right)C_{b2}\right), \tag{4.18}$$

$$M_{\theta\psi e}(p) = \left(C_{b1}D_2(p) + k_2 C_{b2}\right), \quad M_{z\psi 1}(p) = -\bar{A}_1 N_1(p), \quad M_{z\psi e}(p) = -\bar{A}_e N_e(p). \tag{4.19}$$

The differential partial equations systems that perform independent radial, angular and axial vibrations can be obtained from the Eqs (4.13)-(4.15)

$$R_r(p)u_r = \Phi_r, \quad \Theta_\theta(p)\theta = \Phi_\theta, \quad Z_z(p)u_z = \Phi_z. \tag{4.20}$$

The system of Eqs (4.13)-(4.15) leads to an equation of joint radial and angular or radial and axial vibration, if we take $u_z = 0$ or $\theta = 0$

$$\begin{aligned} &R_r u_r + R_\theta \theta = \Phi_r, && R_r u_r + R_z u_z = \Phi_r, \\ &\Theta_r u_r + \Theta_\theta \theta = \Phi_\theta, && Z_r u_r + Z_z u_z = \Phi_z. \end{aligned} \tag{4.21}$$

Three components of the oscillations can be determined from Eqs (4.13)-(4.15) by Cramer`s rule. We considered at some length the radial vibration

$$u_r = \frac{1}{D_0}\begin{vmatrix} M_{ra}a e^{i\omega t} + M_{r\psi_1}\psi_1 + M_{r\psi_e}\psi_e & R_\theta & R_z \\ M_{\theta\gamma}\gamma e^{i\omega t} + M_{\theta\psi_1}\psi_1 + M_{\theta\psi_e}\psi_e & \Theta_\theta & \Theta_z \\ M_{z\psi_1}\psi_1 + M_{z\psi_e}\psi_e & 0 & Z_z \end{vmatrix} \tag{4.22}$$

where the proper system operator is

$$D_0 = \begin{vmatrix} R_r & R_\theta & R_z \\ \Theta_r & \Theta_\theta & \Theta_z \\ Z_r & 0 & Z_z \end{vmatrix}. \tag{4.23}$$

Expanding the determinant (4.22), grouping the terms with the same external influences and making the substitution, $p = i\omega$ we obtain

$$u_r(i\omega) = u_{ra}(i\omega) + u_{r\gamma}(i\omega) + u_{r\psi_1}(i\omega) + u_{r\psi_e}(i\omega), \tag{4.24}$$

$$u_{ra} = \frac{M_{ra}\Theta_\theta Z_z}{D_0} a e^{i\omega t} \frac{P_{ra}}{D_0} a e^{i\omega t}, \quad u_{r\gamma} = -\frac{M_{\theta\gamma}R_\theta Z_z}{D_0}\gamma e^{i\omega t} \frac{P_{r\gamma}}{D_0}\gamma e^{i\omega t},$$

$$u_{r\psi_1} = \frac{1}{D_0}\left[\left(M_{r\psi_1}\Theta_\theta - M_{\theta\psi_1}R_\theta\right)Z_z + M_{z\psi_1}\left(R_\theta\Theta_z - R_z\Theta_\theta\right)\right]\psi_{1na}e^{i\omega t} = \frac{P_{r\psi_1}}{D_0}\psi_{1na}e^{i\omega t}, \tag{4.25}$$

$$u_{r\psi_e} = \frac{1}{D_0}\Big[\Big(M_{r\psi_e}\Theta_\theta - M_{\theta\psi_e}R_\theta\Big)Z_z + M_{z\psi_e}\left(R_\theta\Theta_z - R_z\Theta_\theta\right)\Big]\psi_e = \frac{P_{r\psi_e}}{D_0}\psi_{ea}e^{i\omega t}.$$

In the expressions described above, for example, $P_{r\psi_1}$ is the operator of forced radial vibrations actuating pressure pump pulsations. Components of the angular and axial vibrations are calculated similarly.

External influences vary harmonically with the rotation frequency of the rotor ω, therefore the reactions of the linear system under consideration are harmonic functions with the same frequency

$$u_r = u_{ra}e^{i(\omega t+\phi_r)}, \qquad \theta = \theta_a e^{i(\omega t+\phi_\theta)}, \qquad u_z = u_{za}e^{i(\omega t+\phi_z)}. \tag{4.26}$$

The frequency transfer functions are equal to the ratio of reactions to harmonic influences. Substitute in differential operators $p = i\omega$. Then in view of Eqs (4.13)-(4.15), the frequency transfer functions of the radial vibrations under the influence of static unbalance

$$W_{ra} = \frac{u_{raa}e^{i(\omega t+\phi_{ra})}}{ae^{i\omega e}} = \frac{P_{ra}(i\omega)}{D_0(i\omega)} = U_{ra}(\omega) + i\omega V_{ra}(\omega) = A_{ra}(\omega)e^{i\phi_{ra}(\omega)}, \tag{4.27}$$

$$A_{ra}(\omega) = \left|W(i\omega)\right| = \frac{u_{raa}}{a} = \frac{r_{aa}}{aH_3}, \qquad \phi_{ra}(\omega)=\mathrm{arctg}\,\omega\frac{V_{ra}}{U_{ra}}. \tag{4.28}$$

$A_{ra}(\omega)$, $\phi_{ra}(\omega)$ are the amplitude and phase frequency properties. Similarly, the transfer functions are calculated by other external influences. The amplitude and phase frequency properties can be obtained as the modulus and argument of the transfer function.

Bearing in mind that $u_{ra} = r_a/H_2$, $\theta_a = \vartheta_a l_2/(2H_2)$, $u_{za} = z_a/H_3$ is accepted as a dimensionless displacement and for the absolute values of the amplitudes we obtain the formulas

$$\begin{gathered}
r_{aa} = H_3 aA_{ra}, \qquad r_{a\gamma} = H_3\gamma A_{r\gamma}, \qquad r_{a\psi_1} = H_3\psi_{1na}A_{r\psi_1}/\omega_n^2, \qquad r_{a\psi_e} = H_3\psi_{ea}A_{r\psi_e}, \\
\vartheta_{aa} = 2H_1 aA_{\theta a}/l_1, \qquad \vartheta_{a\gamma} = 2H_1\gamma A_{\theta\gamma}/l_1, \\
\vartheta_{a\psi_1} = 2H_1\psi_{1na}A_{\theta\psi_1}/l_1\omega_n^2, \qquad \vartheta_{a\psi_e} = 2H_1\psi_{ea}A_{\theta\psi_e}/l_1, \\
z_{aa} = H_3 aA_{za}, \qquad z_{a\gamma} = H_3\gamma A_{z\gamma}, \qquad z_{a\psi_1} = H_3\psi_{1na}A_{z\psi_1}/\omega_n^2, \qquad z_{a\psi_e} = H_3\psi_{ea}A_{z\psi_e}.
\end{gathered} \tag{4.29}$$

5. Numerical example

To analyze the coupled radial, angular and axial vibrations of the rotor it is necessary to use numerical methods.

Basic data and nominal parameters of the pump are as follows: $p_n = 3.5\,MPa$, $p_e = 0\,MPa$, $\omega_n = 1500\,s^{-1}$, $m=5$ kg, $\rho = 10^3\,kg/m^3$, $\mu = 10^{-3}\,N\cdot s/m^3$, $E = 2\cdot 10^9\,Pa$, $R_1 = 56$, $R_2=75$, $R_3=65$, $R_4=58$, $R_5=51$, $R_6=15$, $l_1=l_2=15$, $l_3=7$, $H_1 = H_2 = H_3 = 0{,}2$, $b = 9$, $l_c=12$ (all linear sizes are indicated in millimeters); $\varsigma = 0.05$, $\varepsilon_c = 0.15$. The parameter of the annular channel tapering is $\theta_0 = 0$. The steady values of the face clearance and the flow rate are: $u_0 \approx 1$, $u_0 \approx 1$, $Q_o \approx 0.01 m^3/s$.

The absolute value of the amplitudes of coupled radial, angular and axial vibrations of the rotor are defined by formulas (4.29), in the case when the amplitudes of the corresponding perturbation are set: $a, \gamma, \psi_{1a}, \psi_{ea}$. Their approximate estimation can be obtained from the following: the pump is high-speed $\left(\omega \approx 1500 s^{-1}\right)$, that is why according to the global standards 1940-73 for the balancing of the 4-th class of accuracy $a\omega \le 6.3\ mm/s$, where the eccentricity of the disc mass center is $a \approx 4.2$ μm. To calculate the value of the perturbation of dynamic unbalance, take up $\psi_{1a} \approx 0.05$.

For coupled radial, angular and axial vibrations the amplitude frequency characteristics of vibrations, which are given rise by static and dynamic unbalances and pump pressure pulsations, are made. The rotor reactions to the inlet pressure pulsation differ little from reactions to ψ_1, therefore not shown.

In Figs 2 - 4 the dimensional amplitudes for three parameters of tapering of annular seals are presented ($1 - \theta_\oplus = -0.3$, $2 - \theta_\oplus = 0$, $3 - \theta_\oplus = 0.3$). The resonances in the characteristics are absent due to large damping. The exceptions are radial and angular vibrations in diffuser seals at a low rotation frequency (about $4s^{-1}$). The amplitudes of the radial and axial vibrations in the frequency range are below 10 microns, that is not dangerous.

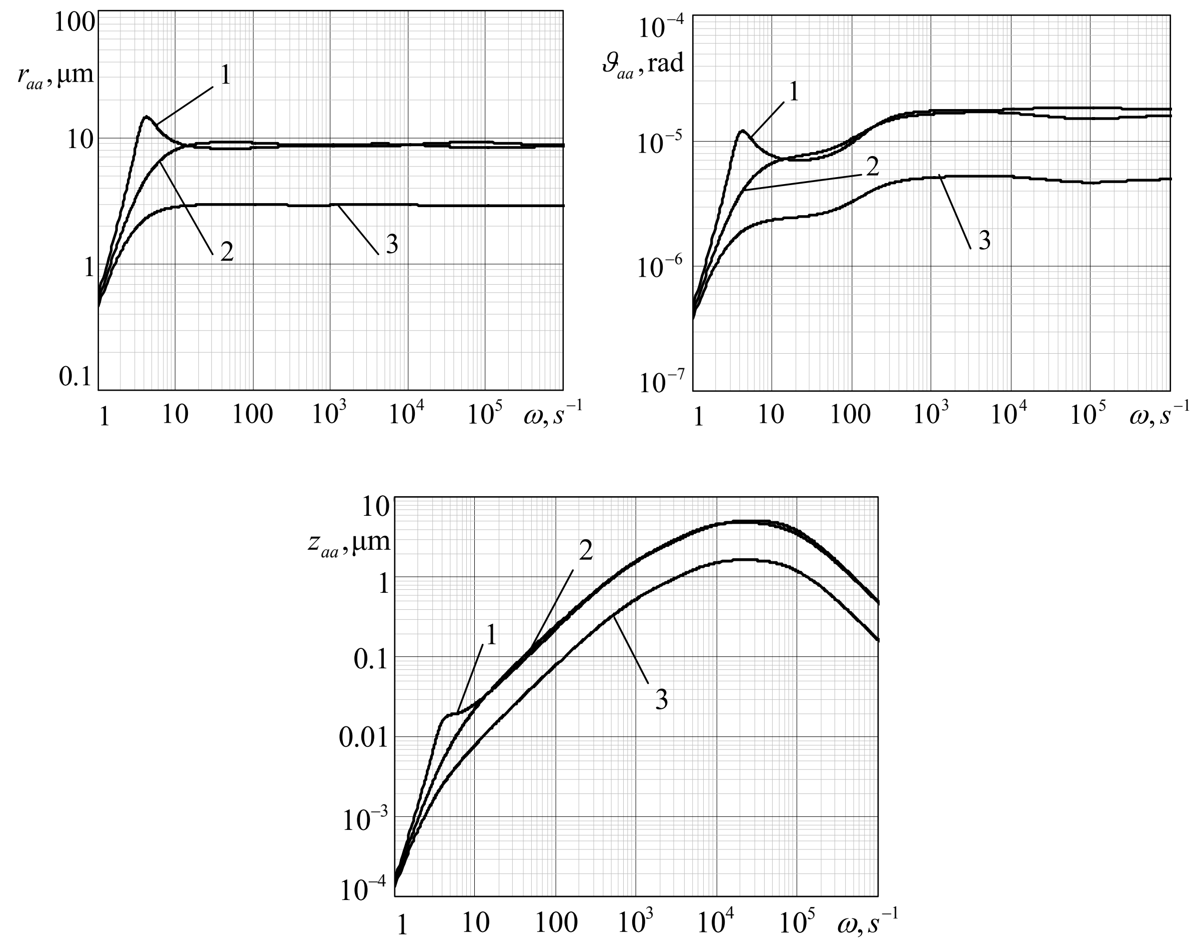


Fig.2. Amplitude-frequency response of radial vibrations, caused by the static unbalance a.

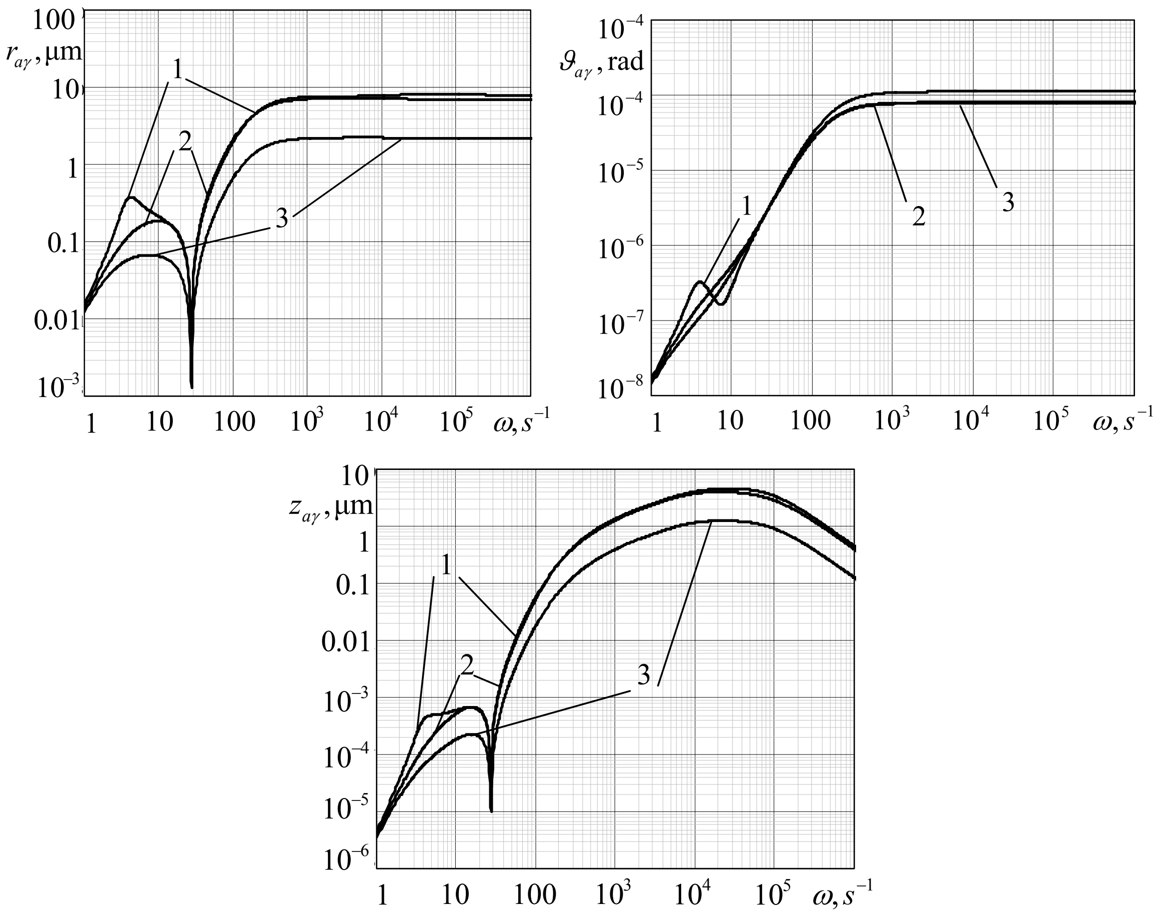
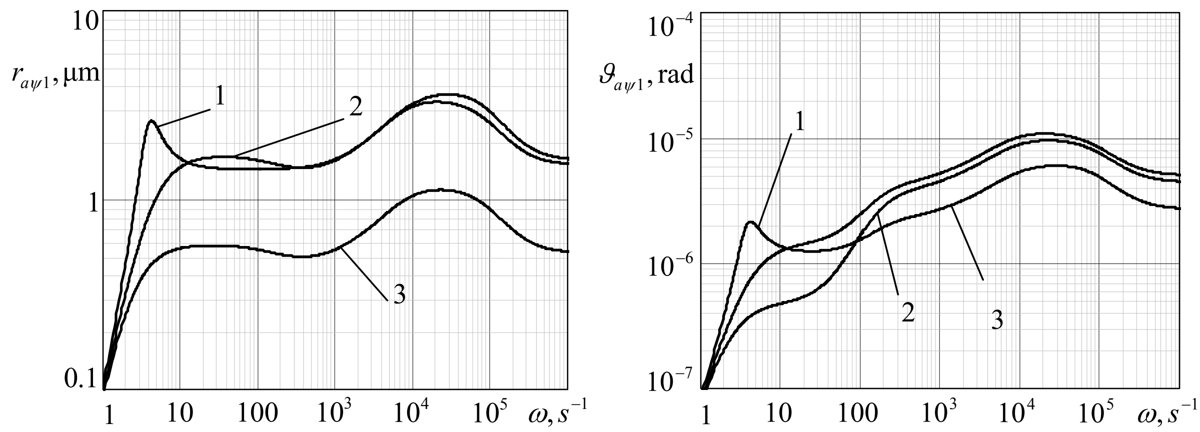


Fig.3. Amplitude-frequency response of radial vibrations, caused by the dynamic unbalance γ .



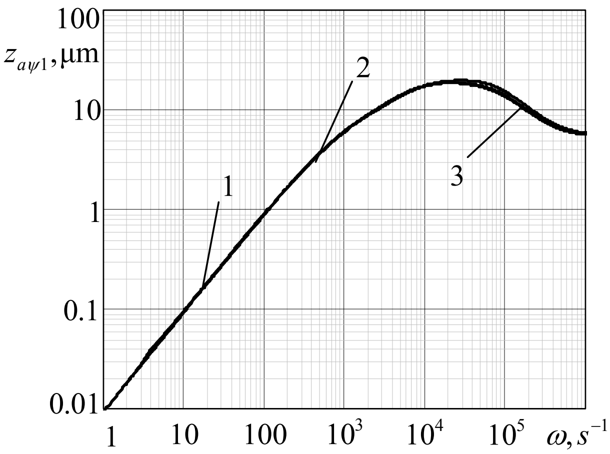


Fig.4. Amplitude-frequency response of vibrations, caused by pump pressure ψ_{1a}.

The slight increase of the amplitude observed on most figures at $\omega = (20 \div 30) 10^3 s^{-1}$, indicate that radial, angular and axial vibrations are connected. The constitutive impact is done by the annular seals tapering. The negative tapering (diffusion airfoil) $\theta_{\oplus} = -0.3$ increases the amplitudes of vibrations on the resonant modes, their convergence $\theta_{\oplus} = 0.3$ reduces significantly.

Conclusions

The proposed method of dynamic calculation, based on a simple model of the rotor balancing system allows evaluation of the critical rotor speed and amplitudes of its forced vibrations. However, for more grounded conclusions a more comprehensive and deeper numerical analysis of the simplified model and the development and research of improved models is required in order to describe the dynamic properties of the rotor-balancing system. Useful analytical information can be obtained by considering the partial system.

Equations derived of the forced coupled radial, angular and axial vibrations include inertial, positional, damping, gyroscopic and circulation forces. The coefficients in the equations consider the forces and moments that occur in two annular seals. By matching the geometric parameters of annular and face seals the required dynamic characteristics of the system can be provided.

The numerical evaluation of the influence of annular seals conicity showed that the diffuser deteriorates vibration state of the rotor, unlike the confusor.

An analysis of amplitude properties shows the interrelation of radial, angular and axial oscillations. The taper of slot channel significantly affects the amplitude of vibrations. This imposes additional requirements to the stiffness of structural elements, because possible force deformations can occur in diffuser throttle channels.

The numerical analysis of dynamic properties of the shaftless pump shows that the annular seals have good damping properties. Therefore, the amplitude characteristics do not have clearly defined resonances and amplitudes in the working range are not more than 10 microns. Thus, the calculation confirms the working ability of the shaftless pump.

Prototypes of centrifugal pumps modernized in this way have been tested both under laboratory and factory conditions. The findings confirm their numerous advantages over conventional pumps, and these include:

- better vibroacoustic characteristics;
- higher reliability and a longer service life between overhauls;
- easier operation, assembly and transport.

It is possible to reduce the weight and size of this type of centrifugal pumps by eliminating the external bearing of the drive shaft.

Nomenclature

A – dimensionless static impeller unbalances Eq.(3.20)
A_A, A_B – outer surface areas of the impeller shrouds [m^2]
A'_e, A_c, A_3 – surface areas of the impeller [m^2]
$\bar{A}_1$, $\bar{A}_2$, $\bar{A}_e$ – dimensionless surface areas Eq.(3.12)
a – eccentricity of the impeller mass center [m]
a_{i1}, a_{i2} – radial forces coefficients, $i=(1\div5)$ Eqs (3.15)
b_{i1}, b_{i2} – gyroscopic moment coefficients, $i=(1\div5)$ Eqs (3.16)
C_a, C_b – constants (Eqs (4.6) - (4.9)) obtained by substituting equations of linearized forces
c_o, c_1, c_2, c_3 – parameters in Eq.(3.9)
$D_{rr}(p)$ – proper operator of independent radial vibrations Eq.(4.2)
$D_{zz}(p)$– proper operator of independent axial vibrations Eq.(3.8)
$D_{\theta\theta}(p)$ – proper operator of independent angular vibrations Eq.(4.3)
$d_{r\theta}(p)$, $d_{\theta r}(p)$ – operators characterizing the combination of radial and angular vibrations Eqs (4.4) and (4.5)
E – compression modulus [N/m^2]
e – radial displacement of the impeller axis
H_1, H_2 – widths of the longitudinal clearances [m]
H_3 – width of the lateral clearance [m]
I, I_0 – mass moments of inertia [kgm^2]
K_i – coefficient characterizing the local acceleration impact on the damping radial force [s^{-1}]
k_{11}, k_{12} – coefficients of radial and angular stiffness of the shaft
k_p, k_d, k_g – coefficients of hydrodynamic forces in annular seals, hydrostatic stiffness, damping and inertial forces, respectively, Eqs (3.28) and (3.29)
k_1, k_2, k_3, T_2, τ_2, κ_s – parameters of Eqs (3.3), (3.4)
l_c – distance from the impeller centre to the central seal
l_1, l_2 – lengths of longitudinal clearances [m]
m – reduced mass of the impeller [kg]
$N_1(p)$, $N_e(p)$ – perturbation operators, Eqs (3.10)
p_e, p_1 – inlet pressure and outlet pressure of the impeller [N/m^2]
p_2 – pressure in the annular chamber of the balancing device [N/m^2]
p_n – nominal pumping pressure [N/m^2]
p_{A^*}, p_{B^*} – pressures at the inlet to longitudinal clearances
Q – fluid flow rate at the static equilibrium [m^3/s]
q_0 –flow through the channel unit width [m^2/s]
R_1, R_2 – radii of the impeller [m]
R_3, R_4, R_5 – radii of the balancing device [m]
R_r – operator, Eq.(4.13)
T_1 – parameter, Eq.(3.11)
u_{r0} – initial dimensionless radial and angular displacement
u_z – dimensionless axial displacement of the impeller
V – volume of the annular chamber [m^3]
z, z_n – axial displacement of the impeller and its nominal value [m]
α_{i1}, β_{i2} – cross coefficients of radial and angular vibrations, $i=(1\div5)$ Eqs (3.18)
ε – relative radial displacement of the impeller axis
ς, ς_{11}, ς_{12}, ς_2 – coefficients of hydraulic losses
θ – relative angular coordinate
$\theta_\oplus$ – parameter of conicity of the annular clearance
$\vartheta_{x,y}$ – angular coordinate [rad]
κ – coefficients of swirl flow in the annular clearance

κ_1, κ_2 – coefficients of the geometry of the impeller in Eq.(3.24)
Λ_0 – modified frictional specific resistance
λ – frictional specific resistance
μ – coefficient of dynamic viscosity [*Pa s*]
ρ – fluid density [kg/m^3]
χ_m – relative coefficient of local resistance
ψ_1, ψ_e, ψ_2 – dimensionless pressures of Eq.(3.5)
ω – angular velocity of the impeller [*1/s*]
ω_n – nominal angular velocity[*1/s*]

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