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RESPONSE OF THERMOELASTIC MICROPOLAR CUBIC CRYSTAL UNDER DYNAMIC LOAD AT AN INTERFACE

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The purpose of this paper is to study the two dimensional deformation in a thermoelastic micropolar solid with cubic symmetry. A mechanical force is applied along the interface of a thermoelastic micropolar solid with cubic symmetry (Medium I) and a thermoelastic solid with microtemperatures (Medium II). The normal mode analysis has been applied to obtain the exact expressions for components of normal displacement, temperature distribution, normal force stress and tangential coupled stress for a thermoelastic micropolar solid with cubic symmetry. The effects of anisotropy, micropolarity and thermoelasticity on the above components have been depicted graphically.

Key words: thermoelasticity, cubic symmetry, microtemperature, normal mode.

1. Introduction

A micropolar continum is a collection of inter-connected particles in the form of small rigid bodies. The deformation in such materials is characterized by both translational and rotational motion. In this motion, the force at a point of the surface element of the body is completely determined by the stress vector at that point. Micropolar solids may represent the materials that are made up of dipole atoms and are subjected to surface and body couples. Polymeric materials, rocks, wood and fibre glass are few examples of such materials. Eringen and Suhubi [1] and Suhubi and Eringen [2] developed a non linear theory of micro-elastic solids. Later Eringen [3-5] developed a theory for the special class of micro-elastic materials and called it the "linear theory of micropolar elasticity". Under this theory, solids can undergo macro-deformations and micro-rotations. Materials affected by micromotions and microdeformations are known as micromorphic materials. Thermoelasticity is the study of equilibrium of bodies, treated as thermodynamic

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systems, whose interactions with the surroundings are restricted to the mechanical work, heat exchange and external forces. The change of body temperature is caused not only by the external and internal heat sources but also by the process of deformation itself. The micropolar theory was extended to include thermal effects by Nowacki [6], Eringen [7], Tauchert *et al*. [8], Tauchert [9], Nowacki and Olszak [10]. One can refer to Dhaliwal and Singh [11-12] for a review on the micropolar thermoelasticity, as well as to Eringen and Kafadar [13] in "Continuum Physics" series in which the general theory of micromorphic media has been summed up.

In the cubic symmetry, the materials have nine planes of symmetry whose normals are on the three coordinate axes and on the coordinate planes making an angle $\pi / 4$ with the coordinate axes. With the chosen coordinate system along the crystalline directions, the mechanical behavior of a cubic crystal can be characterized by four independent elastic constants A_1, A_2, A_3 and A_4. A wide class of crystals such as Si, Cu, Ni, Fe, Au, Al etc., which are some frequently used substances, belong to cubic materials. Minagawa and Arakawa [14] discussed dispersion curves for waves in a cubic micropolar medium with reference to estimations of the material constants for diamond. Kumar and Ailawalia [15-18] and Ailawalia and Kumar [19] studied some source problems in a micropolar thermoelastic medium possessing cubic symmetry. Othman *et al*. [20] presented the effect of inclined load in a micropolar thermoelastic medium possessing cubic symmetry under three theories. Kumar and Partap [21] discussed the elastodynamic behavior of axisymmetric vibrations in a homogeneous isotropic micropolar thermoelastic cubic crystal plate. Lotfy and Yania [22] investigated the effect of the magnetic field and mode I crack in a micropolar thermoelastic cubic medium. Abbas *et al*. [23] studied the thermoelastic interaction in a thermally conducting cubic crystal subjected to ramp-type heating.

Grot [24] discussed a theory of thermodynamics of elastic bodies with microstructure whose microelements possess microtemperatures. Riha [25] studied heat conduction in materials with microtemperatures. Iesan and Quintanilla [26] studied a theory of thermoelasticity with microtemperatures. Iesan [27] proposed the theory of micromorphic elastic solids with microtemperatures. Exponential stability in thermoelasticity with microtemperatures was studied by Casas and Quintanilla [28]. Scalia and Svandze [29] gave the solutions of thermoelasticity with microtemperatures. Iesan [30] discussed thermoelasticity of bodies with microstructure and microtemperatures. Aouadi [31] discussed some theorems in the isotropic theory of microstretch thermoelasticity with microtemperatures. Quintanilla [32] discussed thermoelastic bodies with inner structure and microtemperatures. Scalia *et al*. [33] studied basic theorems in the equilibrium theory of thermoelasticity with microtemperatures. Quintanilla [34] discussed the growth and continuos dependence in thermoelasticity with microtemperatures. Steeb *et al*. [35] studied time harmonic waves in a thermoelastic material with microtemperatures. Chirita *et al*. [36] studied the theory of thermoelasticity with microtemperatures. Kumar and Kaur [37] studied the reflection and refraction of plane waves at the interface of an elastic solid and microstretch thermoelastic solid with microtemperatures. Ciarletta *et al*. [38] studied a homogeneous strongly elliptic thermoelastic medium with microstructures.

The present investigation is to determine the components of normal displacement, temperature distribution, normal force stress and tangential coupled stress in a thermoelastic micropolar solid with cubic symmetry due to mechanical source. The solution is obtained using normal mode analysis and effects of anisotropy, micropolarity and thermoelasticity on the above components are depicted graphically.

2. Formulation of the problem

We consider a normal force of magnitude P_1 acting along the interface of a micropolar thermoelastic cubic crystal (medium I) occupying the region $-\infty \le y \le 0$ and a thermoelastic medium with microtemperatures (medium II) in the region $0 \le y \le \infty$ as shown in Fig.1.

We restrict our analysis to the plane strain parallel to the xy plane with a displacement vector for micropolar thermoelastic solid with cubic symmetry (medium I) as $\boldsymbol{u}^I = \left(u_1^I, u_2^I, 0\right)$, microrotation vector as

$\phi = (0, 0, \phi_3)$ and displacement vector for a thermoelastic solid with microtemperatures (medium II) as $\boldsymbol{u}^{II} = \left(u_1^{II}, u_2^{II}, 0\right)$ and micro-temperature vector as $\boldsymbol{w}^{II} = \left(w_1^{II}, w_2^{II}, 0\right)$.

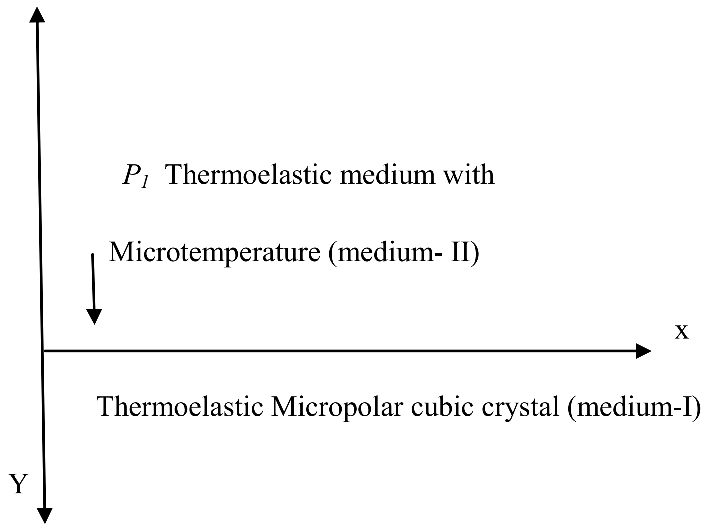

Fig.1. Geometry of the problem.

The field equations and constitutive relations in the absence of body forces, body couples and heat sources for medium I and medium II are given by:

For medium I, i.e., a micropolar thermoelastic medium with cubic symmetry, given by Kumar and Ailawalia [15] as

$$A_1 \frac{\partial^2 u_1^I}{\partial x^2} + \left(A_2 + A_4\right) \frac{\partial^2 u_2^I}{\partial x \partial y} + A_3 \frac{\partial^2 u_1^I}{\partial y^2} + \left(A_3 - A_4\right) \frac{\partial \phi_3}{\partial y} - \upsilon_1 \frac{\partial T_1}{\partial x} = \rho_1 \frac{\partial^2 u_1^I}{\partial t^2}, \tag{2.1}$$

$$A_3 \frac{\partial^2 u_2^I}{\partial x^2} + \left(A_2 + A_4\right) \frac{\partial^2 u_1^I}{\partial x \partial y} + A_1 \frac{\partial^2 u_2^I}{\partial y^2} - \left(A_3 - A_4\right) \frac{\partial \phi_3}{\partial x} - \upsilon_1 \frac{\partial T_1}{\partial y} = \rho_1 \frac{\partial^2 u_2^I}{\partial t^2}, \tag{2.2}$$

$$B_3 \nabla^2 \phi_3 + \left(A_3 - A_4\right)\left(\frac{\partial u_2^I}{\partial x} - \frac{\partial u_1^I}{\partial y}\right) - 2\left(A_3 - A_4\right)\phi_3 = \rho_1 j \frac{\partial^2 \phi_3}{\partial t^2}, \tag{2.3}$$

$$K_1^* \nabla^2 T_1 = \rho_1 c_1^* \frac{\partial T_1}{\partial t} + \upsilon_1 T_0 \frac{\partial}{\partial t}\left(\frac{\partial u_1^I}{\partial x} + \frac{\partial u_2^I}{\partial y}\right), \tag{2.4}$$

$$\sigma_{yy}^I = A_2 \frac{\partial u_1^I}{\partial x} + A_1 \frac{\partial u_2^I}{\partial y} - \upsilon_1 T_1, \tag{2.5}$$

$$\sigma^I_{yx} = A_4\left(\frac{\partial u^I_2}{\partial x} - \phi_3\right) + A_3\left(\frac{\partial u^I_1}{\partial y} + \phi_3\right), \tag{2.6}$$

$$m_{yz} = B_3 \frac{\partial \phi_3}{\partial y}. \tag{2.7}$$

For medium II, i.e., thermoelastic medium with microtemperatures, the equations are given by Steeb *et al.* [35] as

$$(\lambda_2 + 2\mu_2)\frac{\partial^2 u^{II}_1}{\partial x^2} + (\lambda_2 + \mu_2)\frac{\partial^2 u^{II}_2}{\partial x \partial y} + \mu_2 \frac{\partial^2 u^{II}_1}{\partial y^2} - \upsilon_2 \frac{\partial T_2}{\partial x} = \rho_2 \frac{\partial^2 u^{II}_1}{\partial t^2}, \tag{2.8}$$

$$(\lambda_2 + 2\mu_2)\frac{\partial^2 u^{II}_2}{\partial y^2} + (\lambda_2 + \mu_2)\frac{\partial^2 u^{II}_1}{\partial x \partial y} + \mu_2 \frac{\partial^2 u^{II}_2}{\partial x^2} - \upsilon_2 \frac{\partial T_2}{\partial y} = \rho_2 \frac{\partial^2 u^{II}_2}{\partial t^2}, \tag{2.9}$$

$$K^*_2 \nabla^2 T_2 - a_1 T_0 \frac{\partial T_2}{\partial t} - \upsilon_2 T_0 \left(\frac{\partial u^{II}_1}{\partial x} + \frac{\partial u^{II}_2}{\partial y}\right) + k_1\left(\frac{\partial w^{II}_1}{\partial x} + \frac{\partial w^{II}_2}{\partial y}\right) = 0, \tag{2.10}$$

$$(k_4 + k_5 + k_6)\frac{\partial^2 w^{II}_1}{\partial x^2} + (k_4 + k_5)\frac{\partial^2 w^{II}_2}{\partial x \partial y} + k_6 \frac{\partial^2 w^{II}_1}{\partial y^2} - b\frac{\partial w^{II}_1}{\partial t} - k_2 w^{II}_1 - k_3 \frac{\partial T_2}{\partial x} = 0, \tag{2.11}$$

$$k_6 \frac{\partial^2 w^{II}_2}{\partial x^2} + (k_4 + k_5)\frac{\partial^2 w^{II}_1}{\partial x \partial y} + (k_4 + k_5 + k_6)\frac{\partial^2 w^{II}_2}{\partial y^2} - b\frac{\partial w^{II}_2}{\partial t} - k_2 w^{II}_2 - k_3 \frac{\partial T_2}{\partial y} = 0, \tag{2.12}$$

$$\sigma^{II}_{yy} = \lambda_2 \frac{\partial u^{II}_1}{\partial x} + (\lambda_2 + 2\mu_2)\frac{\partial u^{II}_2}{\partial y} - \upsilon_2 T_2, \tag{2.13}$$

$$\sigma^{II}_{yx} = \mu_2\left(\frac{\partial u^{II}_1}{\partial y} + \frac{\partial u^{II}_2}{\partial x}\right), \tag{2.14}$$

$$q^{II}_{yy} = -k_4 \frac{\partial w^{II}_1}{\partial x} - (k_4 + k_5 + k_6)\frac{\partial w^{II}_2}{\partial y}, \tag{2.15}$$

$$q^{II}_{yx} = -k_5 \frac{\partial w^{II}_2}{\partial x} - k_6 \frac{\partial w^{II}_1}{\partial y}. \tag{2.16}$$

For convenience the following non-dimensional variables are used

$$x' = \frac{1}{L}x, \quad y' = \frac{1}{L}y, \quad u^{I'}_i = \frac{1}{L}u^I_i, \quad u^{II'}_i = \frac{1}{L}u^{II}_i, \quad w^{II'}_i = L w^{II}_i, \quad t' = \frac{c_1}{L}t,$$

$$\sigma_{ij}^{I'} = \frac{\sigma_{ij}^{I}}{\nu_2 T_0}, \quad \sigma_{ij}^{II'} = \frac{\sigma_{ij}^{II}}{\nu_2 T_0}, \quad \varphi_3' = \varphi_3, \quad m_{ij}' = \frac{m_{ij}}{L\nu_2 T_0}, \quad q_{ij}^{II'} = \frac{q_{ij}^{II}}{Lc_1\nu_2 T_0}, \quad T_i' = \frac{T_i}{T_0}$$

where L is the standard length and c_1 is the longitudinal wave velocity in medium II given by $c_1^2 = \dfrac{\lambda_2 + 2\mu_2}{\rho_2}$.

Using the above non dimensional variables in Eqs (2.1)-(2.7), it may reduce these equations to (after dropping superscripts)

$$d_1 \frac{\partial^2 u_1^I}{\partial x^2} + d_2 \frac{\partial^2 u_2^I}{\partial x \partial y} + d_3 \frac{\partial^2 u_1^I}{\partial y^2} + d_4 \frac{\partial \phi_3}{\partial y} - d_5 \frac{\partial T_1}{\partial x} = \frac{\partial^2 u_1^I}{\partial t^2}, \tag{2.17}$$

$$d_3 \frac{\partial^2 u_2^I}{\partial x^2} + d_2 \frac{\partial^2 u_1^I}{\partial x \partial y} + d_1 \frac{\partial^2 u_2^I}{\partial y^2} - d_4 \frac{\partial \phi_3}{\partial x} - d_5 \frac{\partial T_1}{\partial y} = \frac{\partial^2 u_2^I}{\partial t^2}, \tag{2.18}$$

$$\left(\frac{\partial^2 \phi_3}{\partial x^2} + \frac{\partial^2 \phi_3}{\partial y^2} \right) + d_6 \left(\frac{\partial u_2^I}{\partial x} - \frac{\partial u_1^I}{\partial y} \right) - d_7 \phi_3 = d_8 \frac{\partial^2 \phi_3}{\partial t^2}, \tag{2.19}$$

$$\left(\frac{\partial^2 T_1}{\partial x^2} + \frac{\partial^2 T_1}{\partial y^2} \right) - d_9 \frac{\partial T_1}{\partial t} - d_{10} \frac{\partial}{\partial t} \left(\frac{\partial u_1^I}{\partial x} + \frac{\partial u_2^I}{\partial y} \right) = 0, \tag{2.20}$$

$$\sigma_{yy}^I = d_{11} \frac{\partial u_1^I}{\partial x} + d_{12} \frac{\partial u_2^I}{\partial y} - d_{13} T_1, \tag{2.21}$$

$$\sigma_{yx}^I = d_{14} \frac{\partial u_2^I}{\partial x} + d_{15} \frac{\partial u_1^I}{\partial y} + d_{16} \phi_3, \tag{2.22}$$

$$m_{yz} = d_{17} \frac{\partial \phi_3}{\partial y} \tag{2.23}$$

where

$$d_1 = \frac{A_1}{\rho_1 c_1^2}, \quad d_2 = \frac{A_2 + A_4}{\rho_1 c_1^2}, \quad d_3 = \frac{A_3}{\rho_1 c_1^2}, \quad d_4 = \frac{A_3 - A_4}{\rho_1 c_1^2}, \quad d_5 = \frac{\upsilon_1 T_0}{\rho_1 c_1^2},$$

$$d_6 = \frac{(A_3 - A_4)L}{B_3}, \quad d_7 = \frac{2(A_3 - A_4)L^2}{B_3}, \quad d_8 = \frac{\rho_1 j c_1^2}{B_3}, \quad d_9 = \frac{\rho_1 c_1^* L c_1}{K_1^*},$$

$$d_{10} = \frac{\upsilon_1 L c_1}{K_1^*}, \quad d_{11} = \frac{A_2}{\upsilon_2 T_0}, \quad d_{12} = \frac{A_1}{\upsilon_2 T_0}, \quad d_{13} = \frac{\upsilon_1}{\upsilon_2}, \quad d_{14} = \frac{A_4}{\upsilon_2 T_0},$$

$$d_{15}=\frac{A_3}{\upsilon_2 T_0},\qquad d_{16}=\frac{(A_3-A_4)}{\upsilon_2 T_0},\qquad d_{17}=\frac{B_3}{L^2\upsilon_2 T_0}.$$

3. Analytic solution

The solution of the physical variable under consideration can be decomposed in terms of normal modes and can be considered in the following form

$$\left(u_i^I,T_i,\varphi_3,\sigma_{ij}^I,m_{ij},u_i^{II},w_i^{II},\sigma_{ij}^{II},q_{ij}^{II}\right)(x,y,t)=\left(\overline{u}_i^I,\overline{T}_i,\overline{\varphi}_3,\overline{\sigma}_{ij}^I,\overline{m}_{ij},\overline{u}_i^{II},\overline{w}_i^{II},\overline{\sigma}_{ij}^{II},\overline{q}_{ij}^{II}\right)(y)e^{\omega t+iax}$$

where ω is the complex frequency, a is the wave number in the y-direction and $\overline{u}_i^I(y),\overline{T}_i(y),\overline{\varphi}_3(y)$, $\overline{\sigma}_{ij}^I(y),\overline{m}_{ij}(y)$, $\overline{u}_i^{II}(y)$, $\overline{w}_i^{II}(y)$, $\overline{\sigma}_{ij}^{II}(y)$, $\overline{q}_{ij}^{II}(y)$ are the amplitudes of field quantities.

Using normal modes in Eqs (2.17)-(2.20), we get

$$\left(d_3D^2-h_{42}\right)\overline{u}_1^I+h_{43}D\overline{u}_2^I+d_4D\overline{\phi}_3-h_{44}\overline{T}_1=0, \tag{3.1}$$

$$h_{43}D\overline{u}_1^I+\left(d_1D^2-h_{45}\right)\overline{u}_2^I-h_{46}\overline{\phi}_3-d_5D\overline{T}_1=0, \tag{3.2}$$

$$-d_6D\overline{u}_1^I+h_{47}\overline{u}_2^I+\left(D^2-h_{48}\right)\overline{\phi}_3=0, \tag{3.3}$$

$$-h_{49}\overline{u}_1^I-h_{50}D\overline{u}_2^I+\left(D^2-h_{51}\right)\overline{T}_1=0 \tag{3.4}$$

where

$$h_{42}=a^2d_1+\omega^2,\quad h_{43}=iad_2,\quad h_{44}=iad_5,\quad h_{45}=a^2d_3+\omega^2,\quad h_{46}=iad_4,$$

$$h_{47}=iad_6,\quad h_{48}=a^2+d_7+d_8\omega^2,\quad h_{49}=ia\omega d_{10},\quad h_{50}=\omega d_{10},\quad h_{51}=a^2+d_9\omega.$$

and constitutive relations (2.21)-(2.22) become

$$\overline{\sigma}_{yy}=iad_{11}\overline{u}_1^I+d_{12}D\overline{u}_2^I-d_{13}\overline{T}, \tag{3.5}$$

$$\overline{\sigma}_{yx}=iad_{14}\overline{u}_2^I+d_{15}D\overline{u}_1^I-d_{16}\overline{\phi}_3, \tag{3.6}$$

$$\overline{m}_{yz}=d_{17}D\overline{\phi}_3. \tag{3.7}$$

Eliminating $\overline{u}_2^I(y)$, $\overline{\phi}_3(y)$, $\overline{T}_1(y)$ between Eqs (3.1)-(3.2), we get the following eight order differential equation for $\overline{u}_1^I(y)$ as

$$\left(D^8 - PD^6 + QD^4 - RD^2 + S\right)\bar{u}_1^1(y) = 0 \tag{3.8}$$

where

$$P = \frac{1}{d_1 d_3}\left[d_1 d_3\left(h_{48} + h_{51}\right) + \left(d_3 h_{45} + d_1 h_{42}\right) + d_5 d_3 h_{50} + h_{43}^2 - d_1 d_4 d_6\right],$$

$$Q = \frac{1}{d_1 d_3}\left[d_1 d_3 h_{48} h_{51} + (d_3 h_{45} + d_1 h_{42})(h_{48} + h_{51}) + h_{42} h_{45} + h_{46} h_{47} d_3 + \right.$$
$$+ d_5 h_{50}(d_3 h_{48} - h_{42}) + h_{43}^2 (h_{48} + h_{51}) + h_{43} h_{46} d_6 + h_{43} h_{49} d_5 + h_{43} h_{47} d_4 +$$
$$\left. - d_4 d_6 (d_1 h_{51} + d_6 h_{45}) - d_4 d_5 d_6 h_{50} + h_{43} h_{44} h_{50} + h_{44} h_{49} d_1\right],$$

$$R = \frac{1}{d_1 d_3}\left[h_{48} h_{51}(d_3 h_{45} + d_1 h_{42}) + h_{42} h_{45}(h_{48} + h_{51}) + h_{46} h_{47}(d_3 h_{51} + h_{42}) + d_5 h_{42} h_{50} h_{48} + \right.$$
$$+ h_{43}^2 h_{48} h_{51} + h_{43} h_{51} h_{46} d_6 + h_{43} h_{49} h_{48} d_5 + h_{43} h_{47} h_{51} d_4 - h_{45} h_{51} d_4 d_6 +$$
$$\left. + h_{47} h_{49} d_4 d_5 + h_{43} h_{44} h_{48} h_{50} + h_{44} h_{49}(d_1 h_{48} + h_{45}) + h_{44} h_{46} h_{50} d_6\right],$$

$$S = \frac{1}{d_1 d_3}\left[h_{42} h_{51}\left(h_{45} h_{48} + h_{46} h_{47}\right) + h_{44} h_{45} h_{48} h_{49} - h_{44} h_{46} h_{47} h_{49}\right].$$

In a similar manner we can show that $\bar{u}_2^1(y)$, $\bar{\phi}_3(y)$, $\bar{T}_1(y)$ satisfies the equation

$$\left(D^8 - PD^6 + QD^4 - RD^2 + S\right)\left(\bar{u}_2^1(y)\,,\ \bar{\phi}_3(y)\,,\ \bar{T}_1(y)\right) = 0, \tag{3.9}$$

which can be factorized as follows

$$\left(D^2 - l_1^2\right)\left(D^2 - l_2^2\right)\left(D^2 - l_3^2\right)\left(D^2 - l_4^2\right)\bar{u}_1^1(y) = 0 \tag{3.10}$$

where l_n^2; $(n = 1,2,3,4)$ are roots of Eq.(3.10).

The series solution of Eq.(3.10) has the form

$$\bar{u}_1^1(y) = \sum_{n=1}^{4}\left[L_n(a,\omega) e^{-l_n y}\right], \tag{3.11}$$

$$\bar{u}_2^1(y) = \sum_{n=1}^{4}\left[L'_n(a,\omega) e^{-l_n y}\right], \tag{3.12}$$

$$\bar{T}_1(y) = \sum_{n=1}^{4}\left[L''_n(a,\omega) e^{-l_n y}\right], \tag{3.13}$$

$$\overline{\phi}_3(y)=\sum_{n=1}^{4}\left[L'''_n(a,\omega)e^{-l_n y}\right] \tag{3.14}$$

where $L_n(a,\omega)$, $L'_n(a,\omega)$, $L''_n(a,\omega)$ and $L'''_n(a,\omega)$ are specific functions depending upon a, ω.

Using Eqs (3.11)-(3.14) in Eqs (3.1)-(3.4), we get the following relations

$$L'_n(a,\omega)=R_{1n}L_n(a,\omega), \tag{3.15}$$

$$L''_n(a,\omega)=R_{2n}L_n(a,\omega), \tag{3.16}$$

$$L'''_n(a,\omega)=R_{3n}L_n(a,\omega). \tag{3.17}$$

Thus we have

$$\overline{u}_2^1(y)=\sum_{n=1}^{4}\left[R_{1n}L_n(a,\omega)e^{-l_n y}\right], \tag{3.18}$$

$$\overline{T}_1(y)=\sum_{n=1}^{4}\left[R_{2n}L_n(a,\omega)e^{-l_n y}\right], \tag{3.19}$$

$$\overline{\phi}_3(y)=\sum_{n=1}^{4}\left[R_{3n}L_n(a,\omega)e^{-l_n y}\right], \tag{3.20}$$

$$\overline{\sigma}_{yy}(y)=\sum_{n=1}^{4}\left[R_{4n}L_n(a,\omega)e^{-l_n y}\right], \tag{3.21}$$

$$\overline{\sigma}_{yx}(y)=\sum_{n=1}^{4}\left[R_{5n}L_n(a,\omega)e^{-l_n y}\right], \tag{3.22}$$

$$\overline{m}_{yz}(y)=\sum_{n=1}^{4}\left[R_{6n}L_n(a,\omega)e^{-l_n y}\right] \tag{3.23}$$

where

$$R_{1n}=-\frac{\left[-d_3d_5l_n^5+\left(d_5h_{42}+h_{43}h_{44}+h_{48}d_3d_5-d_4d_5d_6\right)l_n^3-\left\{h_{48}\left(d_5h_{42}+h_{43}h_{44}\right)-d_6h_{44}h_{46}\right\}l_n\right]}{\left[\left(d_5h_{43}-d_1h_{44}\right)l_n^4+\left\{h_{44}h_{45}-h_{48}\left(d_5h_{43}-d_1h_{44}\right)-h_{47}d_4d_5\right\}l_n^2-h_{44}\left(h_{45}h_{48}-h_{46}h_{47}\right)\right]},$$

$$R_{2n}=\frac{[h_{49}+h_{50}l_nR_{1n}]}{[l_n^2-h_{51}]},$$

$$R_{3n} = -\frac{[d_6 l_n + h_{47} R_{1n}]}{[l_n^2 - h_{48}]},$$

$$R_{4n} = [iad_{11} - l_n d_{12} R_{1n} - d_{13} R_{2n}],$$

$$R_{5n} = [iad_{14} R_{1n} - l_n d_{15} + d_{16} R_{3n}],$$

$$R_{6n} = [-l_n d_{17} R_{3n}].$$

Adopting the same methodology, the solutions for medium II (i.e., a thermoelastic solid with microtemperatures), are of the form

$$\bar{u}_1^{II}(y) = \sum_{m=1}^{5} \left[M_m(a,\omega) e^{-r_m y} \right], \tag{3.24}$$

$$\bar{u}_2^{II}(y) = \sum_{m=1}^{5} \left[M_m^{'}(a,\omega) e^{-r_m y} \right], \tag{3.25}$$

$$\bar{T}_2(y) = \sum_{m=1}^{5} \left[M_m^{''}(a,\omega) e^{-r_m y} \right], \tag{3.26}$$

$$\bar{w}_1^{II}(y) = \sum_{m=1}^{5} \left[M_m^{''}(a,\omega) e^{-r_m y} \right], \tag{3.27}$$

$$\bar{w}_2^{II}(y) = \sum_{m=1}^{5} \left[M_m^{'''}(a,\omega) e^{-r_m y} \right] \tag{3.28}$$

where r_m^2; $(m = 1,2,3,4,5)$ are the roots of the characteristic equation,

$$\left(D^{10} + AD^8 + BD^6 + CD^4 + ED^2 + F \right) \bar{u}_1^{II}(y) = 0 \tag{3.29}$$

and

$$A = \frac{1}{h_{13}^2 h_{18} h_{20}} \Big[-h_{13} \Big\{ h_{13} \Big(h_{20} h_{35} + h_{18} h_{34} + h_{40}^2 + h_{18} h_{20} h_{33} - h_{17} h_{20} \Big) +$$
$$+ h_{18} h_{20} h_{32} + h_{14} h_{16} h_{20} h_{18} \Big\} - h_{18} h_{20} h_{13} h_{31} + h_{20} h_{18} h_{36}^2 \Big],$$

$$B=\frac{1}{h_{13}^2h_{18}h_{20}}\Big[h_{13}\Big\{h_{13}\Big(h_{34}h_{35}+h_{20}h_{33}h_{35}+h_{18}h_{33}h_{34}+h_{33}h_{40}^2+h_{17}h_{40}h_{41}+$$
$$-h_{17}h_{34}-h_{18}h_{39}h_{41}-h_{39}h_{40}\Big)+h_{32}\Big(h_{20}h_{35}+h_{18}h_{34}+h_{40}^2+h_{18}h_{33}h_{20}-h_{17}h_{20}\Big)\Big\}+$$
$$+h_{13}\Big\{h_{14}h_{16}\Big(h_{20}h_{35}+h_{18}h_{34}+h_{40}^2\Big)\Big\}+h_{31}\Big\{h_{13}\Big(h_{20}h_{35}+h_{18}h_{34}+h_{40}^2+h_{18}h_{33}h_{20}-h_{17}h_{20}\Big)+$$
$$+h_{18}h_{20}h_{32}+h_{20}h_{14}h_{18}h_{16}\Big\}-\Big\{h_{14}h_{18}h_{20}h_{36}h_{38}+$$
$$-h_{36}^2\Big(h_{18}h_{34}+h_{35}h_{20}+h_{40}^2+h_{18}h_{33}h_{20}-h_{17}h_{20}\Big)\Big\}+h_{18}h_{20}h_{13}h_{37}h_{38}\Big],$$

$$C=\frac{1}{h_{13}^2h_{18}h_{20}}\Big[h_{13}\Big\{h_{13}h_{35}\big(h_{39}h_{41}-h_{33}h_{34}\big)-h_{32}\big(h_{35}h_{34}+h_{20}h_{33}h_{35}+h_{18}h_{33}h_{34}+$$
$$+h_{33}h_{40}^2+h_{17}h_{40}h_{41}-h_{17}h_{34}-h_{18}h_{39}h_{41}-h_{39}h_{40}\Big)-h_{14}h_{16}h_{34}h_{35}\Big\}+$$
$$-h_{31}\Big\{h_{13}\Big(h_{34}h_{35}+h_{20}h_{33}h_{35}+h_{18}h_{33}h_{34}+h_{33}h_{40}^2+h_{17}h_{40}h_{41}-h_{17}h_{34}+$$
$$-h_{18}h_{39}h_{41}-h_{39}h_{40}\big)+h_{32}\Big(h_{20}h_{35}+h_{18}h_{34}+h_{40}^2+h_{18}h_{33}h_{20}-h_{17}h_{20}\Big)\Big\}+$$
$$-h_{31}\Big\{h_{14}h_{16}\Big(h_{20}h_{35}+h_{18}h_{34}+h_{40}^2\Big)\Big\}-h_{36}^2\Big\{h_{35}h_{34}+h_{20}h_{33}h_{35}+h_{18}h_{33}h_{34}+h_{33}h_{40}^2+$$
$$-h_{39}h_{40}\big(h_{18}+1\big)+h_{17}\big(h_{40}h_{41}-h_{34}\big)\Big\}+h_{14}h_{36}h_{38}\Big(h_{20}h_{35}+h_{18}h_{34}+h_{40}^2\Big)+$$
$$-\Big\{h_{18}h_{20}h_{32}h_{37}h_{38}+h_{13}h_{37}h_{38}\Big(h_{20}h_{35}+h_{18}h_{34}+h_{40}^2\Big)-h_{18}h_{20}h_{16}h_{37}h_{36}\Big\}\Big],$$

$$E=\frac{1}{h_{13}^2h_{18}h_{20}}\Big[-h_{32}h_{13}h_{35}\big(h_{39}h_{41}-h_{33}h_{34}\big)-h_{31}\Big\{h_{35}h_{13}\big(h_{39}h_{41}-h_{33}h_{34}\big)+$$
$$-h_{32}\Big(h_{34}h_{35}+h_{20}h_{33}h_{35}+h_{18}h_{34}h_{33}+h_{33}h_{40}^2+h_{17}h_{40}h_{41}-h_{17}h_{34}+$$
$$-h_{18}h_{39}h_{41}-h_{39}h_{40}\big)-h_{14}h_{16}h_{34}h_{35}\Big\}-\Big\{h_{36}^2h_{35}\big(h_{39}h_{41}-h_{33}h_{34}\big)+$$
$$+h_{14}h_{36}h_{34}h_{35}h_{38}\Big\}-\Big\{h_{16}h_{36}h_{37}\Big(h_{20}h_{35}+h_{18}h_{34}+h_{40}^2\Big)+$$
$$-h_{13}h_{37}h_{34}h_{35}\,h_{38}-h_{32}h_{37}h_{38}\Big(h_{20}h_{35}+h_{18}h_{34}+h_{40}^2\Big)\Big\}\Big],$$

$$F=\frac{1}{h_{13}^2h_{18}h_{20}}\Big[h_{32}h_{31}h_{35}\big(h_{39}h_{41}-h_{33}h_{34}\big)-h_{34}h_{35}h_{37}\big(h_{32}h_{38}-h_{16}h_{36}\big)\Big],$$

$$h_{11}=\frac{(\lambda_2+2\mu_2)}{\rho_2c_1^2},\quad h_{12}=\frac{(\lambda_2+\mu_2)}{\rho_2c_1^2},\quad h_{13}=\frac{\mu_2}{\rho_2c_1^2},\quad h_{14}=\frac{\upsilon_2T_0}{\rho_2c_1^2},\quad h_{15}=\frac{a_1T_0c_1L}{K_2^*},$$

$$h_{16}=\frac{\upsilon_2L^2}{K_2^*},\quad h_{17}=\frac{k_1}{K_2^*T_0},\quad h_{18}=\frac{k_4+k_5+k_6}{L^2k_3T_0},\quad h_{19}=\frac{k_4+k_5}{L^2k_3T_0},\quad h_{20}=\frac{k_6}{L^2k_3T_0},$$

$$h_{21}=\frac{bc_1}{Lk_3T_0},\quad h_{22}=\frac{k_2}{k_3T_0},\quad h_{23}=\frac{(\lambda_2+2\mu_2)}{\upsilon_2T_0},\quad h_{24}=\frac{\lambda_2}{\upsilon_2T_0},\quad h_{25}=\frac{\mu_2}{\upsilon_2T_0},$$

$$h_{26} = \frac{\mu_2}{\upsilon_2 T_0}, \quad h_{27} = \frac{k_4 + k_5 + k_6}{L^3 c_1 \upsilon_2 T_0}, \quad h_{28} = \frac{k_4}{L^3 c_1 \upsilon_2 T_0}, \quad h_{29} = \frac{k_5}{L^3 c_1 \upsilon_2 T_0}, \quad h_{30} = \frac{k_6}{L^3 c_1 \upsilon_2 T_0},$$

$$h_{31} = \omega^2 + a^2 h_{11}, \quad h_{32} = \omega^2 + a^2 h_{13}, \quad h_{33} = a^2 + \omega h_{15}, \quad h_{34} = a^2 h_{18} + \omega h_{21} + h_{22},$$

$$h_{35} = a^2 h_{20} + \omega h_{21} + h_{22}, \quad h_{36} = iah_{12}, \quad h_{37} = iah_{14}, \quad h_{38} = -iah_{16}, \quad h_{39} = iah_{17},$$

$$h_{40} = iah_{19}, \quad h_{41} = -ia.$$

Thus Eqs (3.24)-(3.28) and constitutive relations in medium II may be expressed in the form

$$\bar{u}_2^{II}(y) = \sum_{m=1}^{5} \left[H_{1m} M_m(a,\omega) e^{-r_m y} \right], \tag{3.30}$$

$$\bar{T}_2(y) = \sum_{m=1}^{5} \left[H_{2m} M_m(a,\omega) e^{-r_m y} \right], \tag{3.31}$$

$$\bar{w}_1^{II}(y) = \sum_{m=1}^{5} \left[H_{3m} M_m(a,\omega) e^{-r_m y} \right], \tag{3.32}$$

$$\bar{w}_2^{II}(y) = \sum_{m=1}^{5} \left[H_{4m} M_m(a,\omega) e^{-r_m y} \right], \tag{3.33}$$

$$\bar{\sigma}_{yy}^{II}(y) = \sum_{m=1}^{5} \left[H_{5m} M_m(a,\omega) e^{-r_m y} \right], \tag{3.34}$$

$$\bar{\sigma}_{yx}^{II}(y) = \sum_{m=1}^{5} \left[H_{6m} M_m(a,\omega) e^{-r_m y} \right], \tag{3.35}$$

$$\bar{q}_{yy}^{II}(y) = \sum_{m=1}^{5} \left[H_{7m} M_m(a,\omega) e^{-r_m y} \right], \tag{3.36}$$

$$\bar{q}_{yx}^{II}(y) = \sum_{m=1}^{5} \left[H_{8m} M_m(a,\omega) e^{-r_m y} \right] \tag{3.37}$$

where
$$H_{1m} = -\frac{[h_{13} h_{14} r_m^3 - (h_{14} h_{31} + h_{36} h_{37}) r_m]}{[(h_{14} h_{36} - h_{13} h_{37}) r_m^2 + h_{32} h_{37}]}$$

$$H_{2m} = \frac{\left(h_{13} r_m^2 - h_{31} - iah_{12} r_m H_{1m} \right)}{h_{37}},$$

$$H_{3m} = \frac{[h_{40}r_m^2 - (h_{33}h_{40} + h_{17}h_{41})]H_{2m} + h_{16}h_{40}r_mH_{1m} + h_{38}h_{40}}{[h_{17}h_{20}r_m^2 - (h_{39}h_{40} + h_{34}h_{37})]},$$

$$H_{4m} = \frac{[H_{2m} + h_{40}H_{3m}]r_m}{[h_{35} - h_{18}r_m^2]},$$

$$H_{5m} = [iah_{24} - r_mh_{23}H_{1m} - H_{2m}],$$

$$H_{6m} = [iah_{25}H_{1m} - r_mh_{26}],$$

$$H_{7m} = -iah_{28}H_{3m} + h_{27}r_mH_{4m},$$

$$H_{8m} = h_{30}r_mH_{3m} - iah_{29}H_{4m}.$$

4. Applications

In this section we determine the parameter L_n; $(n = 1,2,3,4)$ and M_m; $(m = 1,2,3,4,5)$. In the physical problem, we should suppress the positive exponential that are unbounded at infinity. Constants L_1, L_2, L_3, L_4 and M_1, M_2, M_3, M_4, M_5 have to be selected such that the boundary condition at the surface $y = 0$ takes the form

$$\sigma_{yy}^{I} = \sigma_{yy}^{II} - P_1 e^{\omega t + iax}; \qquad \sigma_{yx}^{I} = \sigma_{yx}^{II}; \qquad u_1^I = u_1^{II}; \qquad u_2^I = u_2^{II};$$

$$m_{yz} = 0; \qquad K_1^* \frac{\partial T_1}{\partial y} = K_2^* \frac{\partial T_2}{\partial y}; \qquad q_{yy}^{II} = 0; \qquad q_{yx}^{II} = 0; \qquad T_1 = T_2$$

where P_1 is the magnitude of mechanical force.

Using the expressions for σ_{yy}^I, $\sigma_{yy}^{II}, \sigma_{yx}^I$, σ_{yx}^{II}, $u_1^I, u_1^{II}, u_2^I, u_2^{II}, m_{yz}, T_1, T_2$ q_{yy}^{II}, q_{yx}^{II} from Eqs (3.18)-(3.23) and (3.30)-(3.37) in the above boundary conditions, we obtain the following non homogenous linear equations as

$$\sum_{n=1}^{4}[R_{4n}L_n] - \sum_{m=1}^{5}[H_{5m}M_m] = -P_1,$$

$$\sum_{n=1}^{4}[R_{5n}L_n] - \sum_{m=1}^{5}[H_{6m}M_m] = 0,$$

$$\sum_{n=1}^{4}[L_n] - \sum_{m=1}^{5}[M_m] = 0,$$

$$\sum_{n=1}^{4}[R_{1n}L_n] - \sum_{m=1}^{5}[H_{1m}M_m] = 0,$$

$$\sum_{n=1}^{4}\left[R_{6n}L_n\right]=0,$$

$$K_1^*\sum_{n=1}^{4}\left[l_nR_{2n}L_n\right]-K_2^*\sum_{m=1}^{5}\left[r_mH_{2m}M_m\right]=0,$$

$$\sum_{m=1}^{5}\left[H_{7m}M_m\right]=0,$$

$$\sum_{m=1}^{5}\left[H_{8m}M_m\right]=0,$$

$$\sum_{n=1}^{4}\left[R_{2n}L_n\right]-\sum_{m=1}^{5}\left[H_{2m}M_m\right]=0.$$

After solving the above system of nine equations, we get the values of constants $L_1,L_2,L_3,L_4,M_1,M_2,M_3,M_4$ and M_5 and hence obtain the components of normal displacement, temperature distribution, normal force stress and tangential couple stress for a thermoelastic micropolar solid with cubic symmetry at the interface of a thermoelastic solid with microtemperatures.

5. Special cases

1) Substituting $A_1=(\lambda_1+2\mu_1+k)$, $A_2=\lambda_1$, $A_3=(\mu_1+k)$, $A_4=\mu_1$, $B_3=\gamma$, we obtain the expression for the micropolar thermoelastic solid (MTS).
2) Neglecting the micropolarity effect, i.e., $B_3=j=0$ and $A_3=A_4$, the corrosponding expression are obtain in a thermoelastic solid with cubic symmetry (TCC).
3) Taking, $A_1=(\lambda_1+2\mu_1)$, $A_2=\lambda_1$, $A_3=\mu_1$, $A_4=\mu_1$, $B_3=0$ in the expression obtained in the previous step, the expressions for normal displacement, temperature distribution and normal force stress are obtained for thermoelastic solid(TS).

6. Numerical results and discussions

In order to illustrate the theoretical results obtained in the preceding section, we take the following values of parameters for the micropolar solid with cubic symmetry as [15]

$$A_1=19.6\times10^{10}\,N/m^2,\qquad A_2=5.6\times10^{10}\,N/m^2,\qquad A_3=11.7\times10^{10}\,N/m^2,$$
$$A_4=11.7\times10^{10}\,N/m^2,\qquad B_3=0.98\times10^{-9}\,N.$$

For micropolar thermoelastic solid , we take the following values of relevant parameters in case of Magnesium crystal like material as [15]

$$\lambda_1=9.4\times10^{10}\,N/m^2,\quad \mu_1=4.0\times10^{10}\,N/m^2,\quad \rho_1=1.74\times10^3\,kg/m^3,\quad k=10^{10}\,Nm^{-2},$$

$$\gamma = 0.779 \times 10^{-9} N, \qquad j = 0.0000002 \times 10^{-14} m^2, \qquad c_1^* = 0.104 \times 10^4 Nm / Kg / K,$$

$$T_0 = 298K, \qquad K_1^* = 1.7 \times 10^2 Ns^{-1}K^{-1}, \qquad \upsilon_1 = 0.0268 \times 10^8 N / m^2 K.$$

The values of relevant parameters for the thermoelastic solid with microtemperatures are [35]

$$\lambda_2 = 2.17 \times 10^{10} N / m^2, \quad \mu_2 = 3.278 \times 10^{10} N / m^2, \quad \rho_2 = 1.74 \times 10^3 kg / m^3,$$

$$b = 1.389 \times 10^{10} N, \quad K_2^* = 1.7 \times 10^2 Ns^{-1}K^{-1}, \quad a_1 T_0 = 1.8 \times 10^6 Jm^{-3}K^{-1},$$

$$\upsilon_2 = 0.0268 \times 10^8 Nm^{-2}K^{-1}., \quad k_1 = 2 \times 10^{10} Wm^{-1}, \quad k_2 = 0.1 \times 10^{10} Wm^{-1},$$

$$k_3 = 0.4 \times 10^{10} Wm^{-1}K^{-1}, \quad k_4 = 0.3 \times 10^{10} Wm^{-3}K^{-1}, \quad k_5 = 0.5 \times 10^{10} Wm^{-3}K^{-1},$$

$$k_6 = 0.7 \times 10^{10} Wm^{-3}, \quad L = 1.0 \times 10^{-10} m.$$

The computations are carried out for the value of non-dimensional time $t = 0.2$ in the range $0 \le x \le 10.0$ and on the surface $y = 1.0$. The numerical values for normal displacement, temperature distribution, normal force stress and tangential coupled stress are shown in Figs 2-5 for mechanical force with magnitude

$$P_1 = 1.0, \qquad \omega = \omega_0 + \iota\xi, \qquad \omega_0 = -0.3, \qquad \xi = 0.1 \qquad \text{and} \qquad a = 0.9 \qquad \text{for}$$

(a) Micropolar thermoelastic solid with cubic symmetry (MTCC) by solid line with centered symbol ♦.
(b) Thermoelastic solid with cubic symmetry (TCC) by solid line with centered symbol ■
(c) Micropolar thermoelastic solid (MTS) by dashed line with centered symbol ▲.
(d) Thermoelastic solid (MTS) by dashed line with centered symbol x .

7. Discussions

The variations of normal displacement for MTCC, TCC and TS are similar in nature. The variation for MTS are opposite in nature as observed from Fig.2. It is also observed that the variations of normal displacement for MTCC and MTS are mirror images of each other. The variations of temperature distribution are quite similar in nature for the thermoelastic medium with cubic symmetry (MTCC and TCC) as well as for the thermoelastic medium without cubic symmetry (MTS and TS). These variations of temperature distributions are shown in Fig.3.

It can be observed form Fig.4 that the variations of normal force stress are opposite in nature for the micropolar thermoelastic medium (MTCC and MTS). These values of normal force stress are less in magnitude for TCC. The values for all medium coincides at $x = 3.0$ and $x = 7.0$. The variations of tangential couple stress are exactly mirror images of each other as observed from Fig.5.

8. Conclusion

Anisotropy and micropolarity show a significant effect on all the quantities. The variations of temperature distribution are similar in nature for the anisotropic medium(MTCC and TCC) and isotropic medium(MTS and TS). Due to the anisotropic effect, the variations of normal force stress are opposite in nature for the micropolar thermoelastic medium (MTCC and MTS). The values of the quantities coincide for different media at $x = 7.0$.

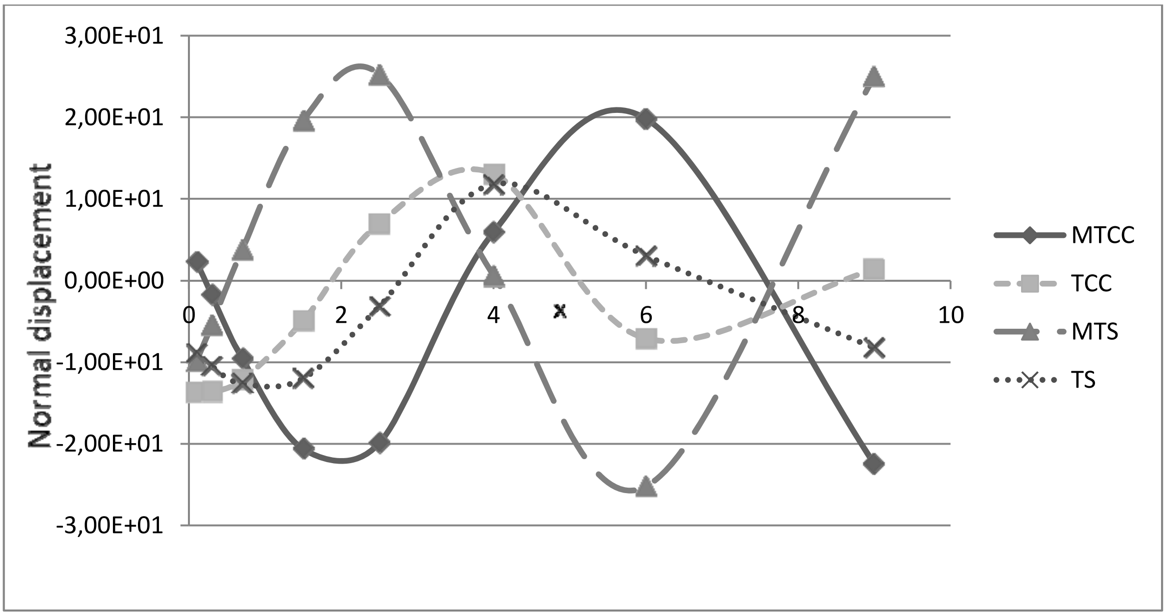

Fig.2. Variation of normal displacement with horizontal distance.

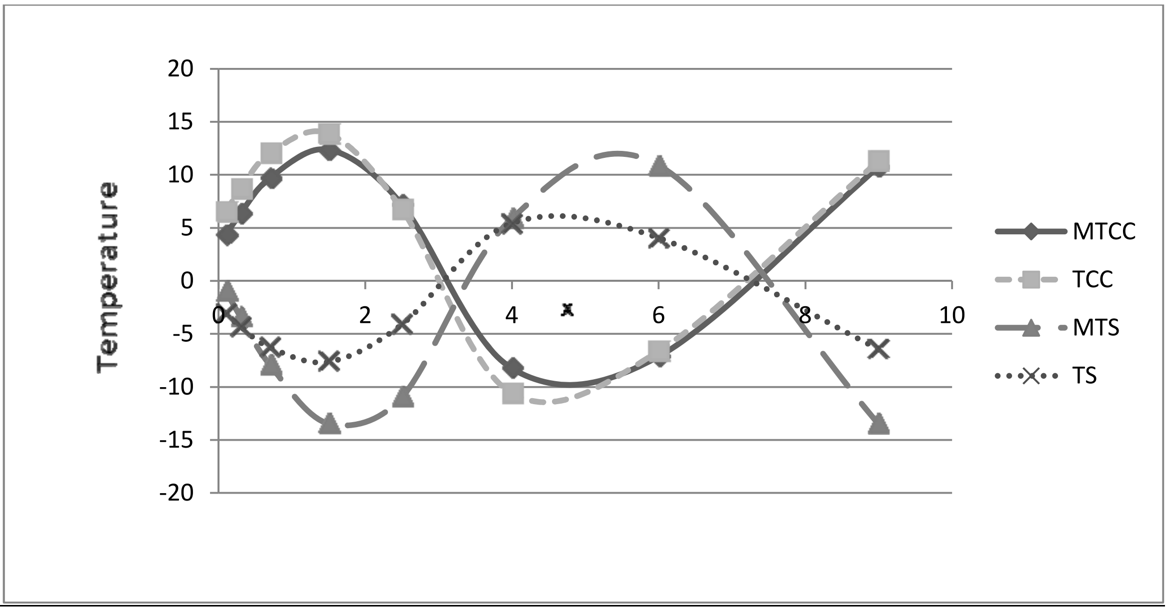

Fig.3. Variation of temperature distribution with horizontal distance.

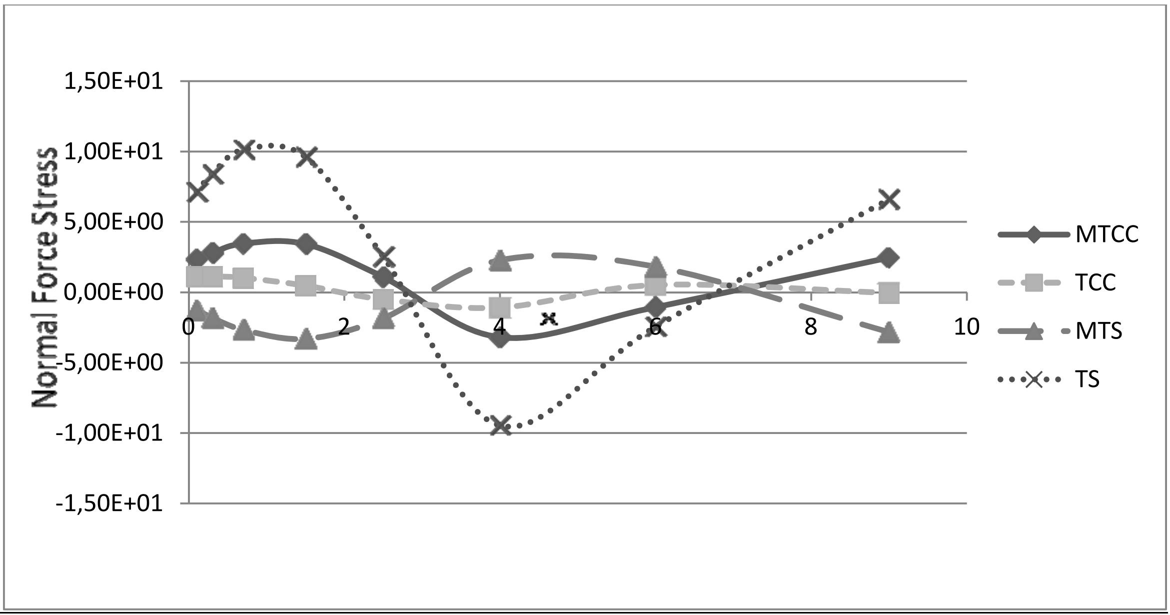

Fig.4. Variation of normal force stress with horizontal distance.

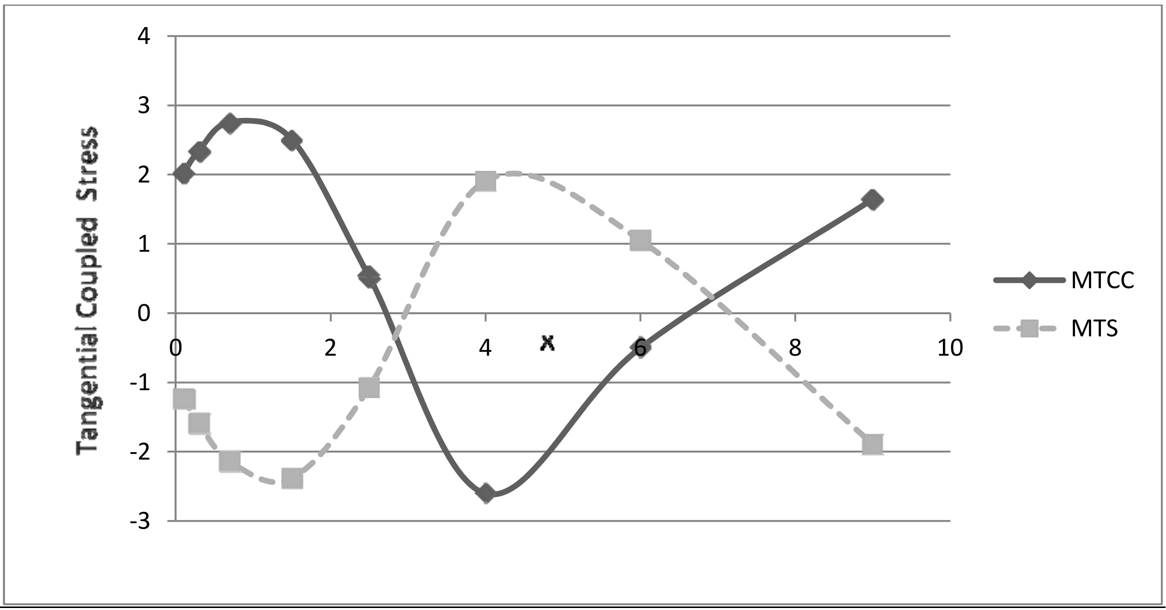

Fig.5. Variation of tangential couple stress with horizontal distance.

Nomenclature

Medium-I

A_1, A_2, A_3, A_4, B_3 – material constants

c_1^* – specific heat at constant strain

j – microinertia

K_1^* – coefficient of thermal conductivity

m_{yz} – tangential couple stress
T_1 – thermodynamic temperature
$\vec{u}^I$ – displacement vector
α_{t_1} – coefficient of linear thermal expansion
ρ_1 – density
σ_{ij}^I – stress tensor
υ_1 – constitutive coefficient

Medium-II

K_2^* – coefficient of thermal conductivity
$k_1, k_2, k_3, k_4,$
$k_5, k_6, a_1, \upsilon_2$ – constitutive coefficients
q_{ij}^{II} – first heat flux moment tensor
T_2 – thermodynamic temperature
$\boldsymbol{u}^{II}$ – displacement vector
$\boldsymbol{w}^{II}$ – microtemperature vector
α_{t_2} – coefficent of linear thermal expansion
$\lambda_2,\ \mu_2$ – Lame’s constants
ρ_2 – density
σ_{ij}^{II} – stress tensor

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