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Technical note

INFLUENCE OF THE MESH GEOMETRY EVOLUTION ON GEARBOX DYNAMICS DURING ITS MAINTENANCE

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Toothed gears constitute the necessary elements of power transmission systems. They are applied as stationary devices in drive systems of road vehicles, ships and crafts as well as airplanes and helicopters. One of the problems related to the toothed gears usage is the determination of their technical state or its evolutions. Assuming that the gear slippage velocity is attributed to vibrations and noises generated by cooperating toothed wheels, the application of a simple cooperation model of rolled wheels of skew teeth is proposed for the analysis of the mesh evolution influence on the gear dynamics. In addition, an example of utilising an ordinary coherence function for investigating evolutionary mesh changes related to the effects impossible to be described by means of the simple kinematic model is presented.

Key words: geometry of gears, slippage velocity, vibroacoustics.

1. Introduction

Toothed gears are widely used in drive systems of machines, devices and all kinds of vehicles moving on roads, water and in the air. Mesh geometries and their principle of operation have been known for years. A lot of various models were created, more or less complicated as well as abstract, based on their kinematics or dynamics. Some were used for determining the influence of the geometric and mechanic parameters on the generated vibroacoustic energy [1].

To present some examples of the influence of the mesh geometry on the gearbox dynamics the Authors decided to use a relatively simple kinematic model of involute mesh, on which basis the - variable in time - slippage velocity can be determined [2]. The adoption of such a model results from the idea that there is a relation between the mesh slippage velocity and the vibroacoustic energy generated by the mesh. The same essential signal components which are occurring in the spectrum of slippage velocity changes obtained from the model, should occur in the recorded signal of vibrations (or noises) accelerations. The most popular method of investigating phenomena related to intermeshing teeth is the analysis of vibrations accelerations of their housing [3]. The reducer compact structure causes that vibrations of the body, being in the direct vicinity of the object, are relatively easily measurable signals providing information on cooperating toothed wheels. However, it should be remembered that such measurement is taken at a certain distance from the vibrations source. Thus, for an accurate diagnosis the transfer function between the model and real object should be determined for each tested object.

The analysis of relations between basic operation parameters of the gear and its dynamic responses can be used in the task of diagnosing the technical state (e.g. related to wear or defects of wheels) or eventually at the initial time of operations to determine the quality of manufacturing and assembling.

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2. Geometry of the involute mesh

The involute mesh geometry is based on the principle of the constant transmission ratio in the function of the toothed wheels rotation [4]. For rolled toothed wheels of skew teeth it is reduced to a description in the frontal plane (perpendicular to the wheels axle of rotation). The basic rules for determining the rolled toothed gear geometry are presented in Fig.1.

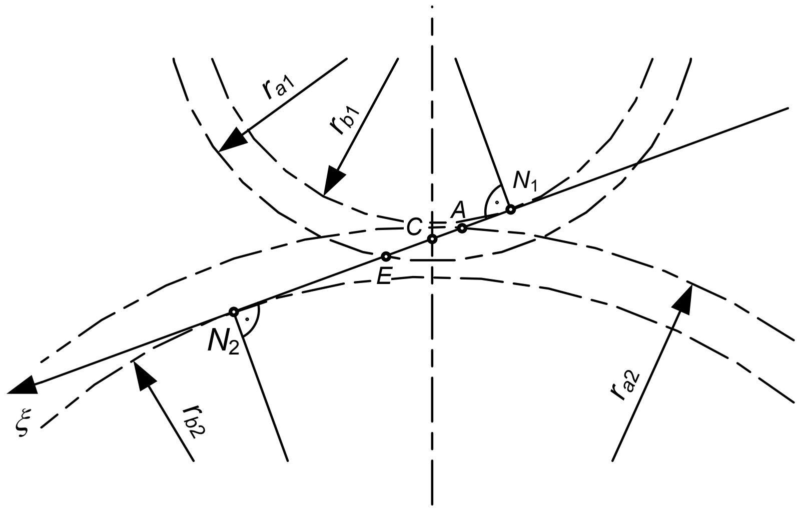

Fig.1. Line of action between a pair of toothed wheels with markers of characteristic points.

Under the assumption of the constant transmission ratio the contact point of intermeshing teeth is moving along the line of action. The line of action is on the tangent to main wheels (of radiuses r_{b1} and r_{b2}) at points N_1 and N_2. On the basis of geometrical dependencies it is possible to determine the location of characteristic points, being on the line of action:

A – beginning of the line of action in the intersection of diameters of wheels vertex r_{a2};
C – mesh pole in the intersection of the line of action with the line joining wheels centres;
E – end of the line of action in the intersection of diameters of wheels vertex r_{a1};

The beginning of the local coordinate system was assumed at point N_1 ($\xi_{N1} = 0$). The value of the transverse pitch is suitable for describing the teeth cooperation along the line of action

$$p_{bt} = p_t \cos\alpha_t = \frac{\pi m_n}{\cos\beta}\cos\alpha_t \tag{2.1}$$

where

p_t – transverse pitch;
a_t – frontal pressure angle;
m_n – normal modulus;
β – pitch angle.

The frontal tooth contact ratio is one of the characteristic parameters determining the properties of toothed gears

$$\varepsilon_\alpha = \frac{\xi_E - \xi_A}{p_{bt}} \tag{2.2}$$

where ξ_A and ξ_E – coordinates of the beginning and end of the line of action.

For the steady motion, the coordinate ξ_i of the temporary position of the contact point of intermeshing teeth can be presented as a function of the mesh angle of rotation

$$\xi_i(t) = \xi_A - (i-1)p_{bt} + r_{b1}\omega_1 t \tag{2.3}$$

where:

i – number of the successive teeth pair entering into contact,

t – time counted from the moment the first wheel pair enters into contact.

Changes of the temporary contact point positions in time are presented in Fig.2 for successive cooperating teeth.

In-between coordinates ξ_A and ξ_B and in between ξ_D and ξ_E two-pair contact occurs, while in-between coordinates ξ_B and ξ_D one-pair contact.

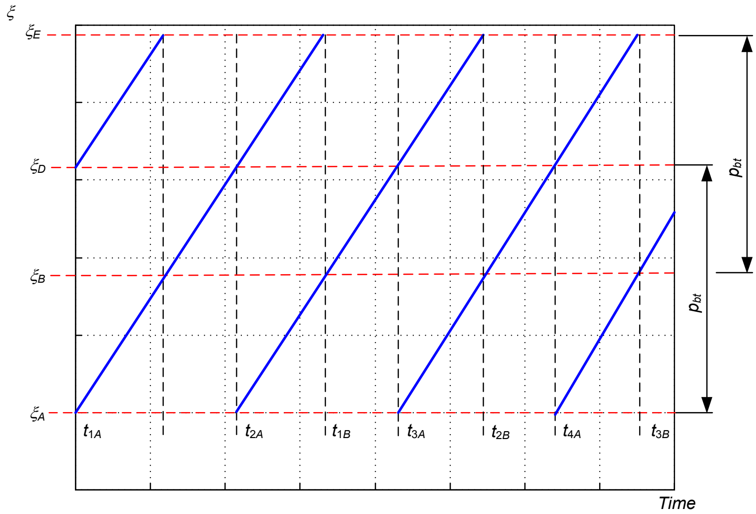

Fig.2. Changes of temporary contact point positions in time, for successive pairs of cooperating toothed wheels.

3. Total slippage velocity and its changes related to the mesh geometry

For deriving the equation describing the slippage velocity of cooperating teeth pairs of the involute profile, the identity of the linear velocity components, parallel to the line of action, is used. The slippage velocity is calculated as the difference of the linear velocity components perpendicular to the line of action of both cooperating teeth. However, such an approach requires accepting the assumption of maintaining point contacts between teeth along the whole line of action.

For a single teeth pair the temporary slippage velocity can be described by

$$\begin{gathered} v_{si}(t) = \omega_1 \xi_i(t) - \omega_2 \left[\xi_{N2} - \xi_i(t)\right] \\ \Lambda \\ \xi_i(t) \in <\xi_A, \xi_B> \end{gathered} \tag{3.1}$$

where

ω_1 – temporary angular velocity of the pinion;

ω_2 – wheel angular velocity;

ξ_{N2} – coordinate of point N_2.

Taking into account the transmission ratio

$$v_{si}(t) = \omega_1 \left(\xi_i(t) - \frac{z_1}{z_2} \left[\xi_{N2} - \xi_i(t) \right] \right), \qquad \underset{\xi_i(t) \in \langle \xi_A, \xi_B \rangle}{\Lambda} \tag{3.2}$$

The basis of the model operation is the total slippage velocity, being the algebraic sum of slippages of all cooperating – at a given moment - tooth pairs

$$v_s(t) = \sum_{i=1}^{z1} v_{si}(t) . \tag{3.3}$$

An exemplary spectrum of effective values of the total slippage velocity in the band around the 1-st harmonic of the mesh f_{m1} with components differing by the rotational frequency multiple f_{rot} (modulating components) is presented in Fig.3.

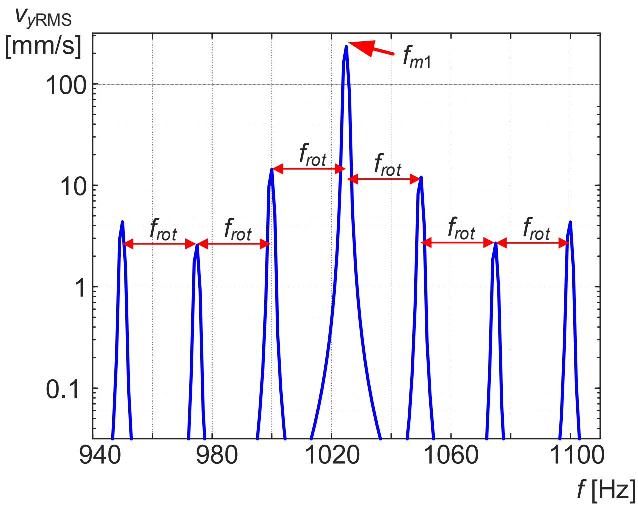

Fig.3. Example of the RMS spectrum of the total slippage velocity.

Changes in the total slippage velocity will be reflected in changes of the ratio of modulating component amplitudes to the component amplitude of a frequency f_{m1}. Similar dependencies will occur in bands around other harmonic frequencies of the mesh according to

$$f_{mi} = i \cdot f_{rot} \cdot z_1 \tag{3.4}$$

where i is the number of the successive harmonic.

The signals presented in examples were generated for the following gear parameters:

- normal modulus, $m_n = 1.5\ mm$;
- pinion teeth number, $z_1 = 41$;
- wheel teeth number, $z_2 = 51$;
- normal pressure angle, $\alpha_n = 20°$;
- axle distance, $a_w = 71\ mm$;

– teeth pitch angle, $\beta = 9.0°$ and $13.6°$ (in dependence on the total mesh correction coefficients);
– pinion rotational velocity: $n = 1500$ *rot/min* (rotational frequency $f_{rot} = 25$ Hz).

The total value of the mesh correction coefficients influences the transverse contact ratio ε_α. In accordance with Eq.(2.2), the contact ratio change is related to the line of action length (and to the proportion of the single-pair to the two-pair line of action). In accordance with these changes in the mesh geometry, for various values of the total correction coefficients the slippage velocity should be changing (Fig.4).

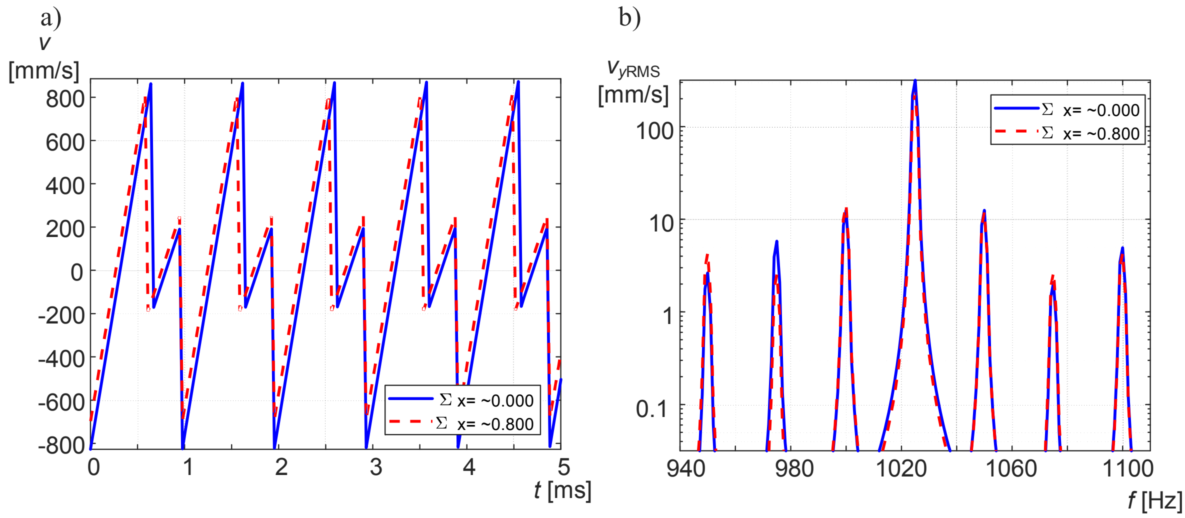

Fig.4. Influence of the sum of the correction coefficients on: a) the change in time of the gear slippage, b) the spectrum of these changes.

Differences in the slippage velocity for various correction coefficients are clearly seen on the time waveform. These changes are also present in the spectrum but they are not easily identified. When the correction coefficients are divided according to the ISO standard the waveform of the total slippage velocity is relatively uniformly distributed versus *0 m/s* value. The influence of the arbitrary division of correction coefficients on the intermeshed slippage change and on the spectrum of these changes, is seen in Fig.5.

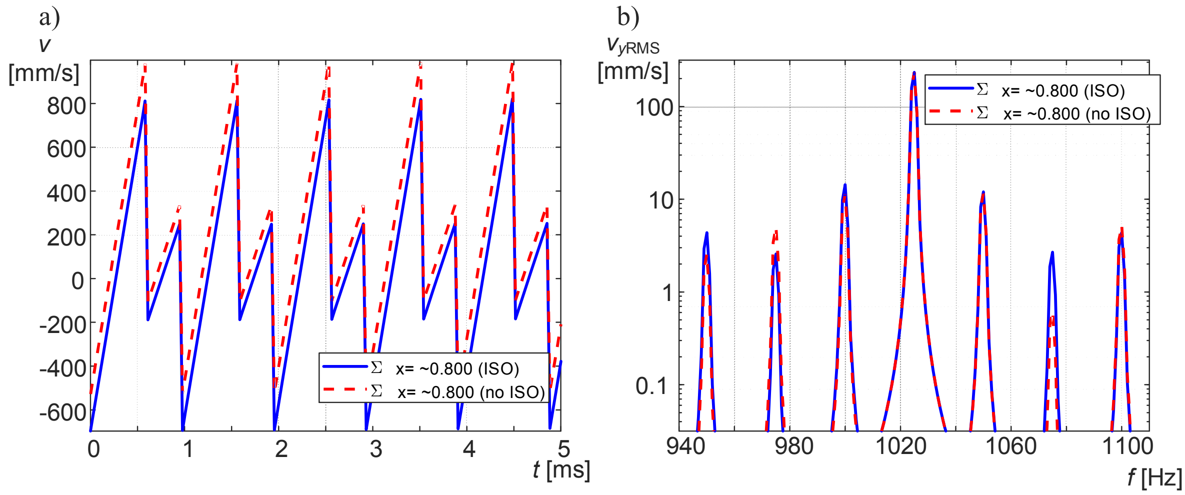

Fig.5. Influence of dividing the sum of correction coefficients - in agreement and not in agreement with ISO: a) on the change in time of the intermeshed slippage, b) on the spectrum of these changes.

The arbitrary division of the sum of correction coefficients causes shifts versus *0 m/s* value, which is also reflected in the signal spectrum.

The way in which errors of some parameters (inherently related to the accuracy of manufacturing) influence the slippage velocity, requires consideration. Two examples of such errors are selected:

– pitch errors;
– lateral whip errors.

Allowable errors of the circular pitch depend on the accuracy class of the produced toothed wheels and are derivatives of errors of the normal base pitch. The division into two-pair and single-pair tooth contacts - according to Eq.(2.2) - depends on the normal base pitch. Pitch deviation values versus the theoretical value were randomly generated for each teeth pair. Differences between the 'ideal' mesh geometry and the geometry with pitch errors not higher than $5\ \mu m$ are shown in Fig.6, while in Fig.7 the geometry with errors up to $5\ \mu m$ is compared to the geometry with errors twice as big (up to $10\ \mu m$).

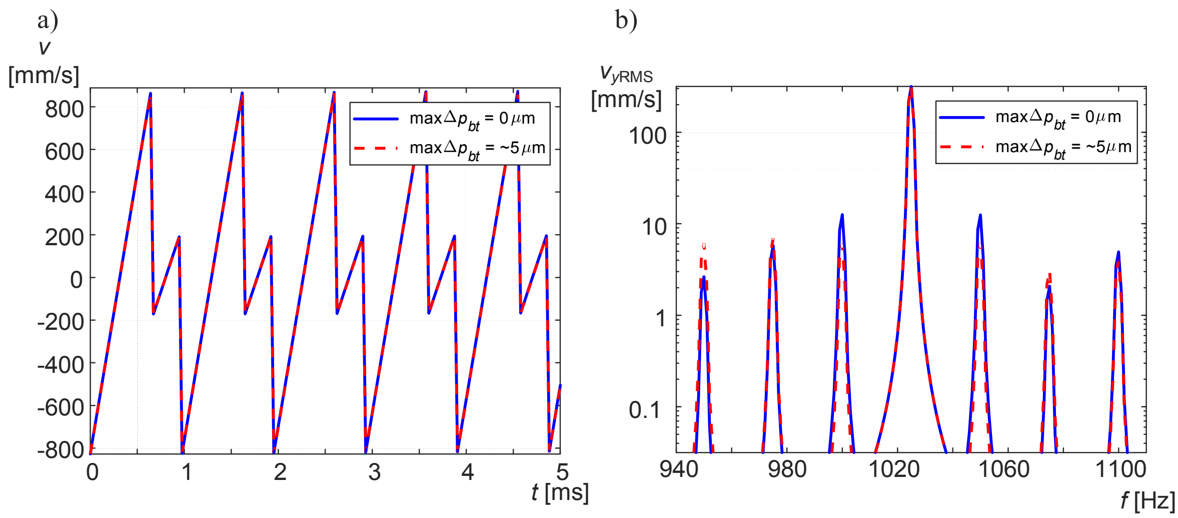

Fig.6. Comparison of the influence of the mesh pitch errors with the 'ideal' geometry: a) on change in time of the intermeshed slippage, b) on these changes spectrum.

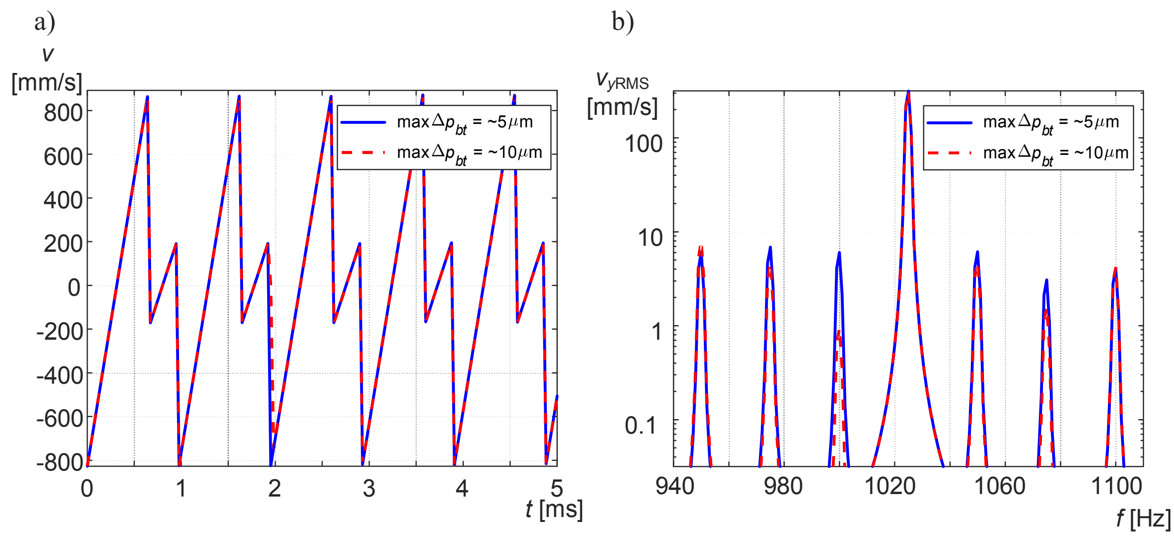

Fig.7. Influence of the twice increased errors of the mesh pitch: a) on the changed in time intermeshed slippage, b) on these changes spectrum.

Differences are noticeable in time waveforms of slippage velocities, however differences in signals spectra are more essential.

The analysis of the influence on errors of the lateral whips was performed in a similar fashion. Results are presented in Fig.8.

Similarly as in the previous case, random errors of the lateral whip can be assessed in the signal spectrum of slippage velocity changes.

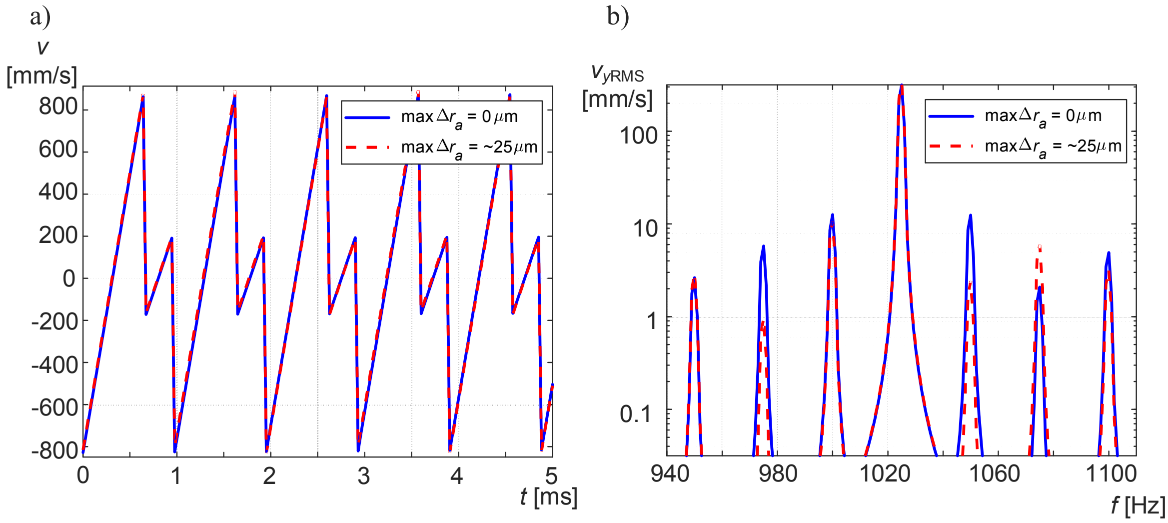

Fig.8. Influence of the lateral whip: a) on the changed in time intermeshed slippage velocity, b) on these changes spectrum.

4. Proposal of utilising the ordinary coherence function

The model applied in the examples given above is the linear model of phenomena occurring in intermeshing teeth. In real mechanical systems, including toothed gears, defects evolutions cause non-linear effects disturbing the model spectra waveforms [5, 6]. Due to this reason, such an approach to the influence of the mesh geometry evolution - during operations - on the gear dynamics is proposed, which allows finding non-linear disturbances of the theoretical 'ideal' spectra [7, 8]. Thus, it is proposed to utilise the properties of the ordinary coherence function related to the analysis of non-linear effects. The calculation algorithm was presented in more detail in [9]. In selected narrow spectrum bands, we are looking for the maximum coherence function of the reference signal (generated during the analysis) and the investigated signal. The obtained maximum of the coherence function indicates that the standard frequency was matched with the signal frequency. By applying the generated signal it is possible to determine the analysis accuracy. The application of the algorithm makes it possible to determine 'small' changes of the mesh frequency f_{mi} not available by other methods.

Let us consider what can be causing these changes. During the initial operational period, for new wheels, this will be caused by production errors. For wheels with skew teeth there is a pseudo involute outline in the frontal plane, which is a projection of the involute profile from the normal plane to the tooth line. The example of the analysis result is presented in Fig.9a. Next, a 'running in' of wheels mesh occurs and mesh gear operation is in steady state (Fig.9b). The evolution of wearing and defects causes disturbances of the involute profile and irregular gear operations (Figs 9c and 9d).

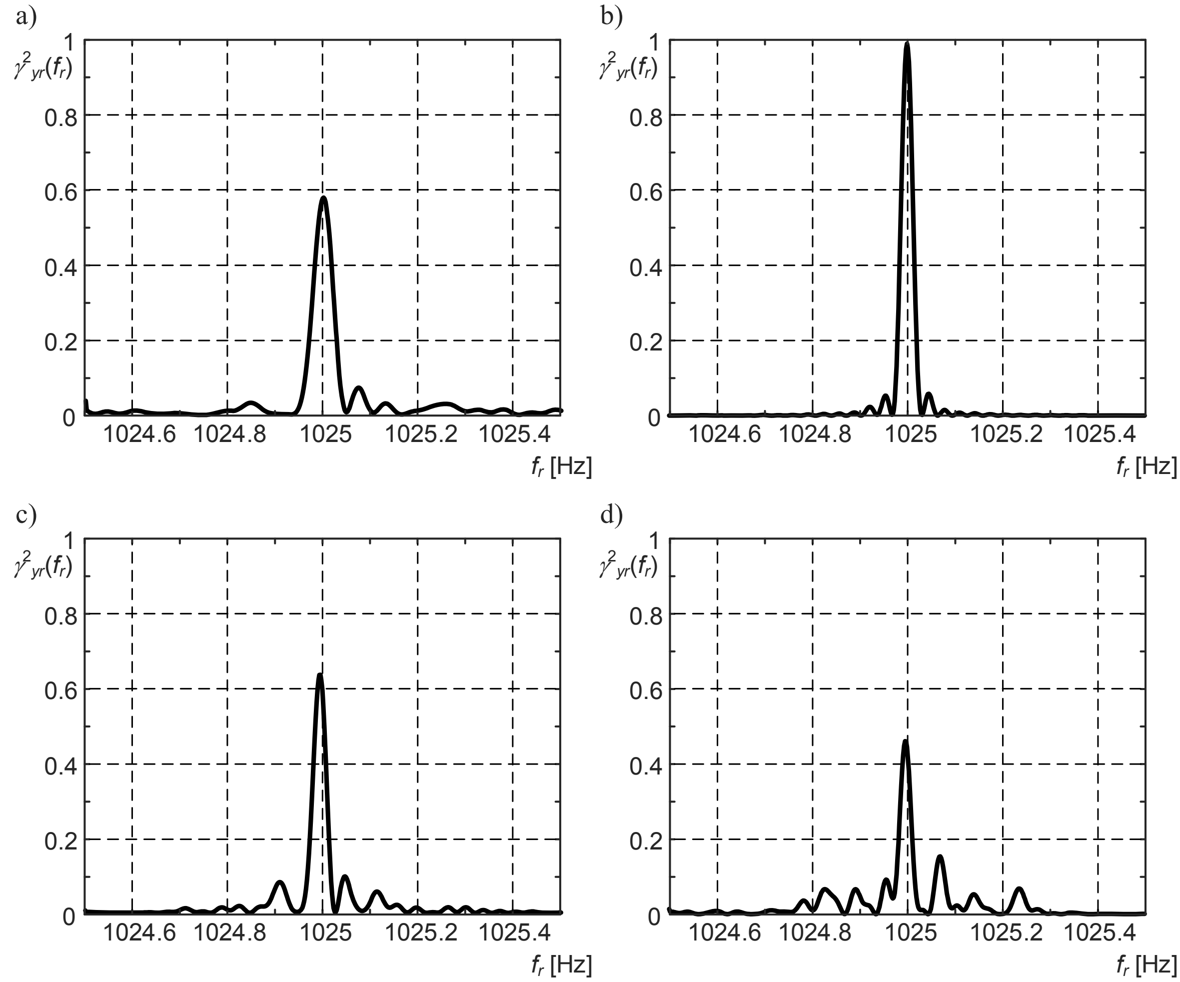

Fig.9. Examples of the analytical results utilising the ordinary coherence function: a) New toothed wheel, b) After wheels 'running in', c) Initial 'defect' phase, d) Developing 'defect' phase.

Attention should be directed to another phenomenon, strongly related to the toothed gear operations. The important cause of the non-linear response is the mesh variable stiffness along the line of action and teeth deflections related to it. Disturbance of the involute profile must occur even for the ideal mesh geometry.

The studies performed according to the presented algorithm, in the whole lifespan gear history, make possible the determination of nonlinear disturbances values and the degree of gear damage evolution.

5. Conclusions

The model of generating the total slippage velocity can be used for the diagnostics of the toothed gears state only in a limited way. However, it is a good method for defining accurately the 'standard' signal in the proposed coherence approach. By analysing the presented examples a few measures can be developed for determining the wheels production accuracy on the basis of changes in amplitudes of the selected harmonic components. An isolated research stand, with a small load ensuring constant mesh contact without its excessive deflection, will enable matching the vibroacoustic response to the geometric model.

The application of the algorithm with the ordinary coherence function allows, at periodic measuring vibrations of investigated objects, the development of the evolution history of general changes in the mesh geometry related to wear and defects [10].

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