

TRANSIENT FREE CONVECTIVE RADIATIVE FLOW BETWEEN VERTICAL PARALLEL PLATES HEATED/COOLED ASYMMETRICALLY WITH HEAT GENERATION AND SLIP CONDITION

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Investigation of an MHD convective flow of viscous, incompressible and electrically conducting fluid through a porous medium bounded by two infinite vertical parallel porous plates is carried out. Forchheimer-Brinkman extended Darcy model is assumed to simulate momentum transfer within the porous medium. A magnetic field of uniform strength is applied normal to the plates. The analytical results are evaluated numerically and the presented graphically to discuss in detail the effects of different parameter entering into the problem.

Key words: MHD, slip flow, heat generation, thermal radiation.

1. Introduction

Flows through fluid saturated media are important in many scientific and engineering problems such as geothermal energy utilization, heat exchangers, nuclear reactor repositories and chemical engineering for filtration processes. Comprehensive reviews of porous media thermal/species convection have been presented by (Kaviany [1]; Pop and Ingham [2]; Ingham and Pop [3]; Vadasz [4]; Vafai [5]; Neild and Bejan [6]). For any application of porous media it is important to account for non-Darcian effects which can be divided into the inertial (Forchheimer) and boundary (Brinkman) effects. A generalized model for the fluid flow through a porous medium of variable porosity was developed to account for inertial effects, and boundary effects. These effects are incorporated by using the general flow model known as the Brinkman-Forchheimer-extended Darcy model.

An analysis on the theoretical derivation of the Darcy and Forchheimer models was presented by Irmay [7]. Neale and Nader [8] showed that the Brinkman model considering continuity of the velocity and the shear stress at the interface gives the same results as obtained by using the Darcy model with Beavers-Joseph condition. Kavinay [9] and Nakayama *et al.* [10] obtained an analytical solution for a forced convection flow problem in a channel filled with a saturated Brinkman- Darcy porous medium. Flow through porous media, considering the Brinkman-Forchheimer extended Darcy model under different physical conditions has been studied by several authors [Cheng and Choudhary [11], Vafai and Kim [12], Nakayama *et al.* [13], Kladias and Prasad [14], Shenoy [15], Vafai and Kim [16], Whitakar [17], Neild *et al.* [18],

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Kuznetsov and Austria [19], Nakayama [20], Leong and Jin [21], Singh and Takhar [22], Singh *et al.* [23], Pal and Mondal [24]].

In recent years, considerable attention has been paid to the analysis of an MHD boundary layer flow and heat transfer of a Newtonian fluid from a vertical plate/channel immersed in a porous medium because of its wide spectrum of applications in engineering processes, especially in the enhanced recovery of petroleum resources, plasma studies, drying of porous solids, thermal insulation and MHD generators. An MHD natural convection flow bounded by parallel plates through porous media was investigated by Rapits *et al.* [25]. Attia and Kotb [26] analyzed a magnetohydrodynamic flow and heat transfer bounded by two parallel plates. Kim [27] investigated an unsteady MHD convective heat transfer from a semi-infinite vertical plate with inconstant suction through porous media. Attia [28] investigated the effects of variation in the physical variables on the MHD steady flow and heat transfer bounded by parallel plates through porous media. Ahmed [29] studied the effect of an MHD unsteady natural convective motion bounded by an infinite vertical porous media.

Radiation effects have important applications in the processes involving high temperatures and space technology. Recently, developments in hypersonic flights, space vehicles, gas turbines, nuclear power plants and gas cooled nuclear reactors have attracted researchers in. Radiative convective flows have important applications in environmental and industrial processes, e.g., space vehicle re-entry, astrophysical flows, evaporation from large open water reservoirs, fossil fuels and combustion. Radiative convective flows under different physical conditions have been studied by several authors [Das *et al.* [30], Bakier [31], Sanyal and Adhikari [32], Mebine [33], El-Hakim and Rashad [34], Muthucumarswamy and Kulandivel [35], Singh and Kumar [36], Singh and Garg [37]].

The study of MHD fluid flows and heat transfer in the slip flow regime has important applications in engineering, for example, electric transformers, heating elements, transmission lines, refrigeration coils and power generators. An MHD unsteady flow of a polar fluid with variable permeability past an infinite horizontal plate in a slip-flow regime through porous media was presented by Khandelwal *et al.* [38]. Transient natural convection viscous incompressible flows with inconstant suction from a vertical plate in a slip flow regime were presented by Sharma and Chaudhary [39]. The effects of periodic heat and mass transfer on the unsteady natural stream with a mean velocity over which a velocity exponentially varying with time is superimposed was investigated by Sharma [40]. Choudhary and Jha [41] studied an MHD micropolar fluid flow from a vertical plate with chemical reaction in a slip-flow regime. Singh and Pathak [42] investigated an MHD oscillatory convective flow past a rotating vertical channel with slip conditions, thermal radiation and Hall current through porous media.

2. Formulation of the problem

An unsteady convection flow of a viscous fluid bounded by two upright plates through porous media is considered. The coordinate axes x^* - and y^* are taken - along and perpendicular to one of the channel plate. Let d be the distance between the plates. Since the plates are of infinite extent, thus the flow variables depend only on y and t . Fluid characteristics, excluding density in the buoyancy force term, are assumed to be constant. Initially, temperatures of the plates and fluid are same as T_m^* . When $t^* > 0$ the temperatures of the plates at $y^* = 0$ and $y^* = d$ are instantaneously raised to T_h^* and T_c^* ($T_h^* > T_c^*$), such that $T = T_h^* + \varepsilon(T_h^* - T_c^*)e^{-nt}$ and which are thereafter maintained constant. A time dependent injection/suction velocity $v^* = -v_0(1 + \varepsilon e^{-nt^*})$ is applied at the plate $y^* = d$ and $y^* = 0$ respectively.

Therefore, under such assumptions, equations governing the flow relevant to the problem may be written as

$$\frac{\partial v^*}{\partial y^*} = 0, \tag{2.1}$$

$$\frac{\partial u^*}{\partial t^*} - \nu_0 \left(1 + \varepsilon e^{-n^* t^*} \right) \frac{\partial u^*}{\partial y^*} = \nu_{eff} \frac{\partial^2 u^*}{\partial y^{*2}} - \frac{\nu_f}{K} u^* - \frac{F}{\sqrt{K}} u^{*2} + g\beta_f (T^* - T_m^*) - \frac{\sigma}{\rho_f} B_0^2 u^*, \tag{2.2}$$

$$\frac{\partial T^*}{\partial t^*} - \nu_0 \left(1 + \varepsilon e^{-n^* t^*} \right) \frac{\partial T^*}{\partial y^*} = \frac{k}{\rho_f C_p} \frac{\partial^2 T^*}{\partial y^{*2}} - \frac{Q_0}{\rho_f C_p} (T^* - T_m^*) - \frac{\partial q_r^*}{\partial y^*} \frac{1}{\rho_f C_p}. \tag{2.3}$$

The relevant boundary conditions are

$$t^* \leq 0, \quad u^* = 0, \quad T^* = T_m^* \quad \text{at} \quad 0 \leq y^* \leq d,$$

$$u^* = L_l \frac{\partial u^*}{\partial y^*}, \quad T^* = T_h^* + \varepsilon (T_h^* - T_c^*) e^{-n^* t^*} \quad \text{at} \quad y^* = 0, \tag{2.4}$$

$$t^* > 0, \quad u^* = 0, \quad T^* = T_c^* \quad \text{at} \quad y^* = d$$

where
$$L_l^* = \left(\frac{2 - m_l}{m_l} \right) L.$$

Rosseland’s approximation is used for the radiative heat flux which is given below

$$q_r^* = - \left(\frac{4\sigma^*}{3k^*} \frac{\partial T^{*4}}{\partial y^*} \right). \tag{2.5}$$

The inertia coefficient term F appearing in the model can be evaluated by the following formula (Alazmi *et al.* [43]; Ergun [45])

$$F = \frac{1.75}{\sqrt{175 \varepsilon^3}}. \tag{2.6}$$

The dimensionless quantities are defined as

$$y = \frac{y^*}{d}, \quad t = \nu_f \frac{t^*}{d^2}, \quad n = \frac{d^2 n^*}{\nu_f}, \quad u = \frac{u^* \nu_f}{g\beta_f d^2 (T_d^* - T_c^*)}, \quad \theta = \frac{T^* - T_c^*}{T_d^* - T_c^*},$$

$$\nu_f = \frac{\mu_f}{\rho_f} \text{ (kinematic viscosity), } \lambda = \frac{\nu_{eff}}{\nu_f} \text{ (kinematic viscosity ratio), } S = \frac{d\nu_0}{\nu_f} \text{ (suction parameter),}$$

$$\begin{aligned}
M &= \frac{\sigma}{\nu_f \mu_f} d^2 B_0^2 \quad (\text{Hartman number}), \quad Q = \frac{Q_0 d^2}{\nu_f \rho_f C_p} \quad (\text{heat source/sink parameter}), \quad Da = \frac{K}{d^2} \quad (\text{Darcy} \\
&\text{number}), \quad R_4 = \frac{T_m^* - T_c^*}{T_d^* - T_c^*} \quad (\text{buoyancy force distribution parameter}), \quad Gr = g\beta d^3 \frac{(T_d' - T_c')}{\nu_f} \quad (\text{Grashof number}), \\
Pr &= \frac{\mu_f C_p}{k} \quad (\text{Prandtl number}), \quad N_R = \frac{kk^*}{4\sigma^* T_\infty^{*3}} \quad (\text{radiation parameter}), \quad h = \frac{L_1 \nu_0}{\nu} \quad (\text{slip flow parameter}), \\
k_2 &= \left(\frac{3N_R + 4}{3N_R} \right) \quad (\text{constant})
\end{aligned} \tag{2.7}$$

Now introducing the relation Eqs (2.5) and (2.7), into Eqs (2.2) and (2.3), we get

$$\frac{\partial u}{\partial t} - S(1 + \varepsilon e^{-nt}) \frac{\partial u}{\partial y} = \lambda \frac{\partial^2 u}{\partial y^2} - \frac{u}{Da} - \frac{FGr}{\sqrt{Da}} u^2 + \theta - R_4 - Mu, \tag{2.8}$$

$$\frac{\partial T}{\partial t} - S(1 + \varepsilon e^{-nt}) \frac{\partial T}{\partial y} = k_2 \frac{1}{Pr} \frac{\partial^2 T}{\partial y^2} - Q(\theta - R_4), \tag{2.9}$$

when $R_4 < 0$, $T_h > T_c > T_m$ while for $R_4 > 0$, $T_m > T_h > T_c$ and when $0 < R_4 < 1$, the wall temperature T_h and T_c straddle the fluid temperature T_m .

The dimensionless boundary conditions are

$$\begin{aligned}
u &= h \frac{\partial u}{\partial y}, \quad \theta = 1 + \varepsilon e^{-nt} \quad \text{at} \quad y = 0, \\
u &= 0, \quad \theta = 0 \quad \text{at} \quad y = 1.
\end{aligned} \tag{2.10}$$

3. Solution of the problem

To solve Eqs (2.8) and (2.9), we assumed $\varepsilon \ll 1$ (Gebhart and Pera [45]; Singh *et al.* [23]) and the solutions to the equations are as follows

$$u(y) = u_0(y) + \varepsilon u_1(y) e^{-nt}, \tag{3.1}$$

$$\theta(y) = \theta_0(y) + \varepsilon \theta_1(y) e^{-nt}. \tag{3.2}$$

Now using the above Eqs (3.1) and (3.2), in Eqs (2.8) to (2.9), we obtain the subsequent equations

$$\lambda \frac{d^2 u_0}{dy^2} + S \frac{du_0}{dy} - \frac{1}{Da} u_0 - Mu_0 = \frac{GrF}{\sqrt{Da}} u_0^2 - \theta_0 + R_4, \tag{3.3}$$

$$\lambda \frac{d^2 u_I}{dy^2} + S \frac{du_I}{dy} - \frac{I}{Da} u_I - Mu_I + nu_I = \frac{2GrF}{\sqrt{Da}} u_0 u_I - \theta_I, \tag{3.4}$$

$$k_2 \frac{d^2 \theta_0}{dy^2} + PrS \frac{d\theta_0}{dy} - PrQ\theta_0 = -PrR_4 Q, \tag{3.5}$$

$$k_2 \frac{d^2 \theta_I}{dy^2} + PrS \frac{d\theta_I}{dy} - Pr(Q-n)\theta_I = -PrS \frac{d\theta_0}{dy}. \tag{3.6}$$

The differential Eqs (3.3) and (3.4), are still coupled, so further we assume $F \ll 1$ (Chamkha [46]) and the solutions to the equations are as follows

$$u_0(y) = u_{00}(y) + Fu_{0I}(y), \quad u_I(y) = u_{I0}(y) + Fu_{II}(y). \tag{3.7}$$

Now using the above Eqs (3.7), in Eqs (3.3) to (3.4), we get the following equations

$$\lambda \frac{d^2 u_{00}}{dy^2} + S \frac{du_{00}}{dy} - E_1 u_{00} = -\theta_0 + R_4, \tag{3.8}$$

$$\lambda \frac{d^2 u_{0I}}{dy^2} + S \frac{du_{0I}}{dy} - E_1 u_{0I} = \frac{Gr}{\sqrt{Da}} u_{00}^2, \tag{3.9}$$

$$\lambda \frac{d^2 u_{I0}}{dy^2} + S \frac{du_{I0}}{dy} - E_2 u_{I0} = -\theta_I - Su_{00}, \tag{3.10}$$

$$\lambda \frac{d^2 u_{II}}{dy^2} + S \frac{du_{II}}{dy} - E_2 u_{II} = -Su_{0I}' + \frac{2Gr}{\sqrt{Da}} u_{00} u_{I0}. \tag{3.11}$$

The corresponding boundary conditions (2.10), reduce to the following form

$$u_{00} = h \frac{\partial u_{00}}{\partial y}, \quad u_{0I} = h \frac{\partial u_{0I}}{\partial y}, \quad u_{I0} = h \frac{\partial u_{I0}}{\partial y}, \quad u_{II} = h \frac{\partial u_{II}}{\partial y}, \quad \theta_0 = 1, \quad \theta_I = 1 \quad \text{at } y = 0, \tag{3.12}$$

$$u_{00} = 0, \quad u_{0I} = 0, \quad u_{I0} = 0, \quad u_{II} = 0, \quad \theta_0 = 0, \quad \theta_I = 0 \quad \text{at } y = l$$

The solutions to Eqs (3.5) and (3.6), satisfying boundary conditions (3.12) are

$$\theta_0 = F_1 e^{n_1 y} + F_2 e^{n_2 y} + R_4, \tag{3.13}$$

$$\theta_I = F_3 e^{n_4 y} + F_4 e^{n_5 y} - B_1 e^{n_2 y} - B_2 e^{n_3 y}. \tag{3.14}$$

Now using Eqs (3.13) and (3.14), in Eq.(3.2), we get $\theta(y)$.

The solutions to Eqs (3.8) to (3.11) satisfying the boundary conditions (3.12) are

$$u_{00} = F_5 e^{n_5 y} + F_6 e^{n_6 y} - B_9 e^{n_1 y} - B_{10} e^{n_2 y}, \quad (3.15)$$

$$u_{01} = F_7 e^{n_5 y} + F_8 e^{n_6 y} + B_{15} e^{2n_5 y} + B_{16} e^{2n_6 y} + B_{17} e^{2n_1 y} + B_{18} e^{2n_2 y} + B_{19} e^{n_9 y} - B_{20} e^{n_{10} y} - B_{21} e^{n_{11} y} - B_{22} e^{n_{12} y} - F_{23} e^{n_{13} y} + B_{24} e^{n_{14} y}, \quad (3.16)$$

$$u_{10} = F_9 e^{n_7 y} + F_{10} e^{n_8 y} + e^{n_1 y} (B_{28} + B_{32}) + e^{n_2 y} (B_{29} + B_{33}) - B_{30} e^{n_3 y} - B_{31} e^{n_4 y} - B_{34} e^{n_5 y} - B_{35} e^{n_6 y}, \quad (3.17)$$

$$u_{11} = F_{11} e^{m_7 y} + F_{12} e^{m_8 y} - e^{2n_5 y} B_{68} - e^{2n_6 y} B_{69} - e^{2n_2 y} B_{70} - e^{2n_1 y} B_{71} - B_{72} e^{n_9 y} + B_{73} e^{n_{10} y} + B_{74} e^{n_{11} y} + B_{75} e^{n_{12} y} + B_{76} e^{n_{13} y} - B_{77} e^{n_{14} y} + B_{78} e^{n_{19} y} + B_{79} e^{n_{20} y} - B_{80} e^{n_{21} y} - B_{81} e^{n_{22} y} + B_{82} e^{n_{23} y} + B_{83} e^{n_{24} y} - B_{84} e^{n_{25} y} - B_{85} e^{n_{26} y} - B_{86} e^{n_{27} y} - B_{87} e^{n_{28} y} + B_{88} e^{n_{29} y} + B_{89} e^{n_{30} y} - B_{90} e^{n_{31} y} - B_{91} e^{n_{32} y} + B_{92} e^{n_{33} y} + B_{93} e^{n_{34} y} - B_{94} e^{n_5 y} - B_{95} e^{n_6 y}. \quad (3.18)$$

Now using Eqs (3.15) to (3.18) in Eq.(3.7), we get $u_0(y)$ and $u_1(y)$ respectively, which finally yields $u(y)$ by Eq.(3.1).

4. Results and discussion

In order to study the nature of velocity, temperature, and mass transfer, numerical calculations are carried out for distinct values of R_4, S, Gr, N_R, t, M and Q which are listed in figures and the results are reported graphically.

Figures 1 to 3 show the effects of time and the buoyancy force parameter on the fluid velocity. When $R_4 < 0$ from Fig.1, it is noticed that near the heated plate ($y=0$) the velocity gets its maximum value and starts falling towards the cold plate ($y=1$) due to the negative value of the buoyancy force parameter, the temperature of both plates is greater than the fluid temperature. When $0 \leq R_4 \leq 1$ it is observed in Fig.2 that near the hot plate ($y=0$) the fluid velocity gets its maximum value and then drops all over the flow area. The reason is that the hot plate fluid is heated. When $R_4 > 1$ it is observed in Fig.3 that the temperature of the cooled plate is lower than the temperature of the fluid, thus near the cooled plate a reverse flow is occurring. The reason is that at the starting stage, the temperature of both the plates is greater than the fluid temperature. The figures it also show that the velocity enhances as the time increases and a steady state is obtained at $t=1$. Figure 4 presents the influence of time and the suction/injection parameter on the velocity. It is noticed that the velocity diminishes with the growing value of the suction/injection parameter. The reason is that the suction/injection parameter enhances the drag force nearby the channel plates. From the figures it also follows that the velocity enhances as the time increases and the steady state is obtained at $t=1$. Figure 5 illustrates the influence of time and the Grashof number on the velocity. It is found that the Grashof number has the leading effect on accelerating velocities. It is also observed that the velocity enhances as the time increases and the steady state is obtained at $t=1$. The influence of the radiation parameter is shown in Fig.6. It is found that the fluid velocity gets its maximum value nearby the heated plate and then diminishes gradually towards the cooled plate. It is also noticed that the velocity enhances as the time increases and the steady state is obtained at $t=1$. The influence of the Hartmann number on the

fluid velocity is presented in Fig.7. It is observed that the fluid velocity decelerates with the growing value of the Hartmann number the velocity enhances as the time increases and the steady state is obtained at $t = 1$.

Figures 8 and 9 represent the influence of heat source /sink Q on the temperature. It can be noticed that the temperature diminishes with the growing value of the heat sink parameter and a similar trend is seen in the case of the heat source parameter. Figure 10 depicts the effect of the radiation parameter on the temperature. It is observed that the temperature profile drops with the growing value of the radiation parameter. Figures 11 to 13 show the effect of the buoyancy force parameter on the temperature. When $R_4 < 0$, the temperature diminishes with rising values of R_4 . A similar behavior is noticed in the case of $0 \leq R_4 < 1$ and $R_4 > 0$. The reason is that the temperature of the cooled plate is lower than the temperature of the heated plate and the temperature of the fluid is lower than the temperature of both plates.

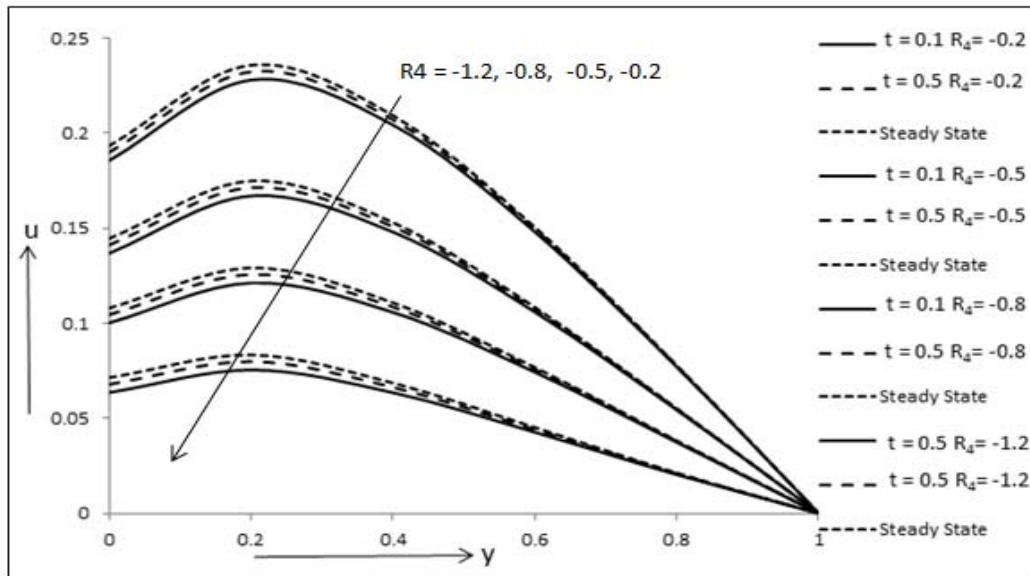


Fig.1. Velocity profiles for various values of the buoyancy force parameter ($R_4 < 0$).

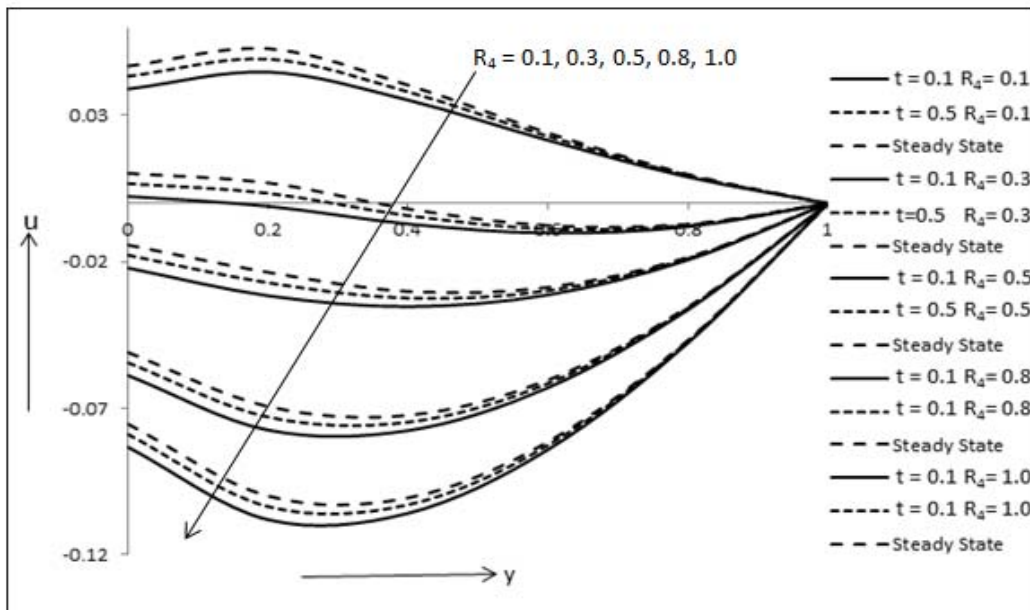


Fig.2. Velocity profiles for various values of the buoyancy force parameter ($R_4 > 0$).

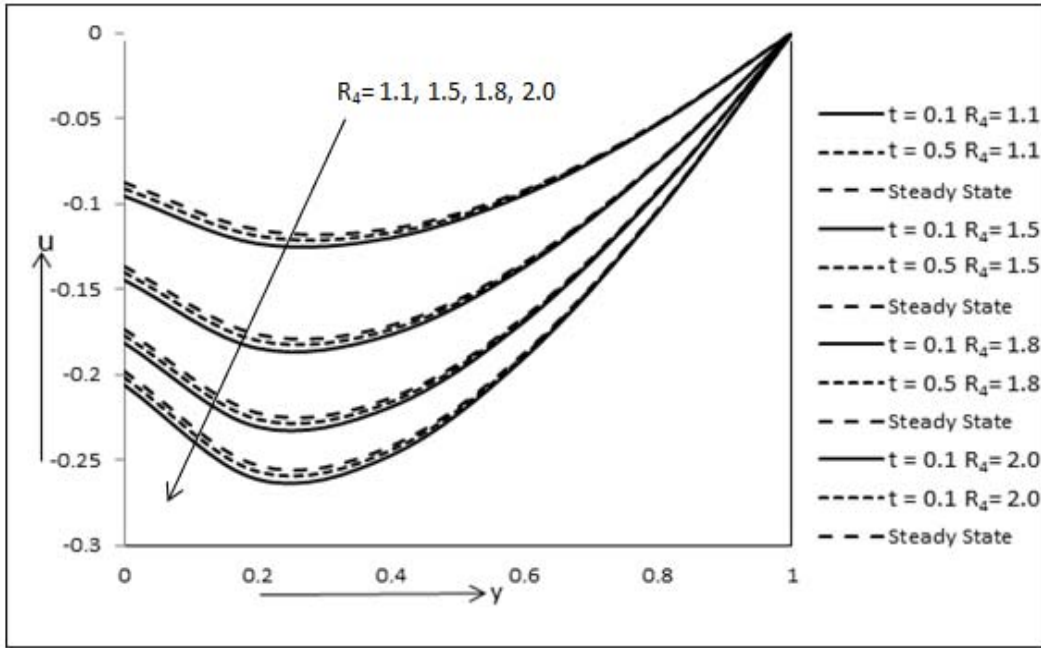


Fig.3. Velocity profiles for various values of the buoyancy force parameter ($R_4 > 1$).

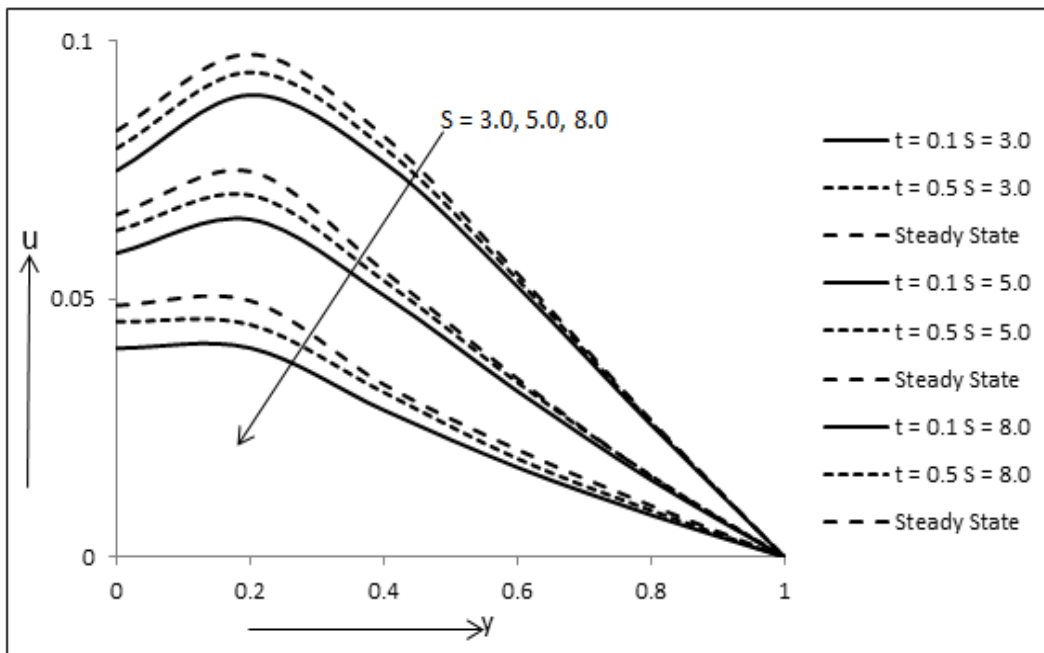


Fig.4. Velocity profiles for various values of the suction/injection parameter.

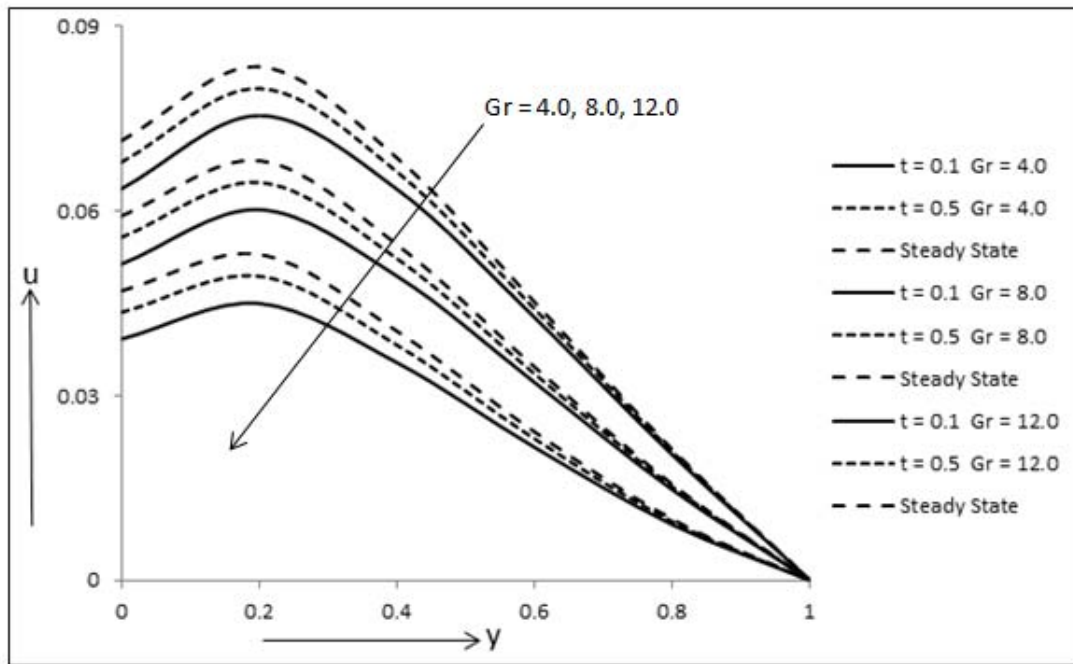


Fig.5. Velocity profiles for various values of the Grashof number.

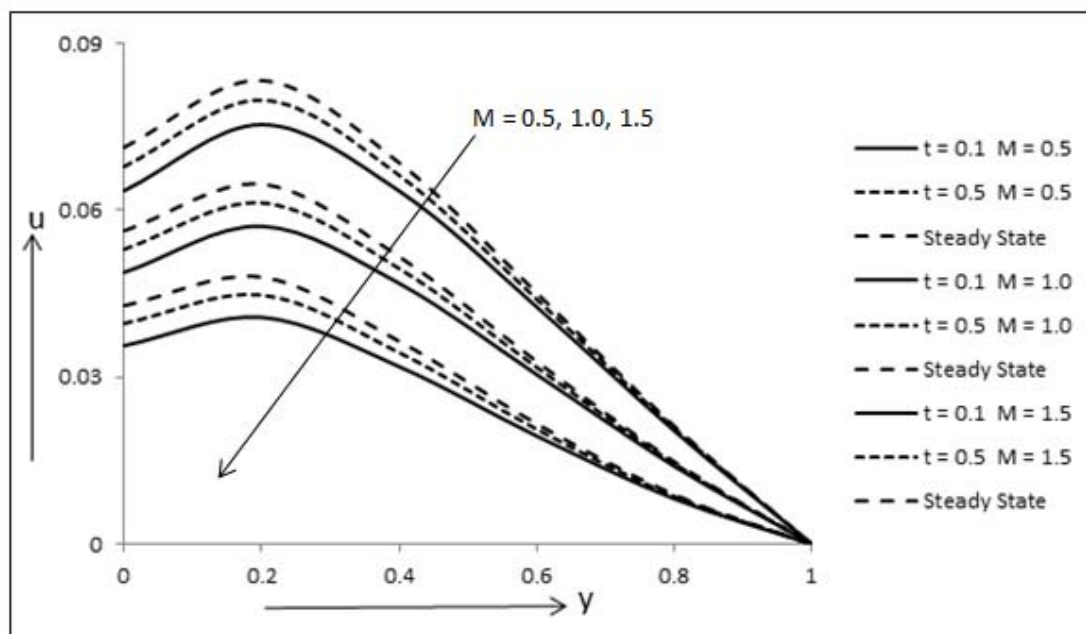


Fig.6. Velocity profiles for various values of the Hartmann number.

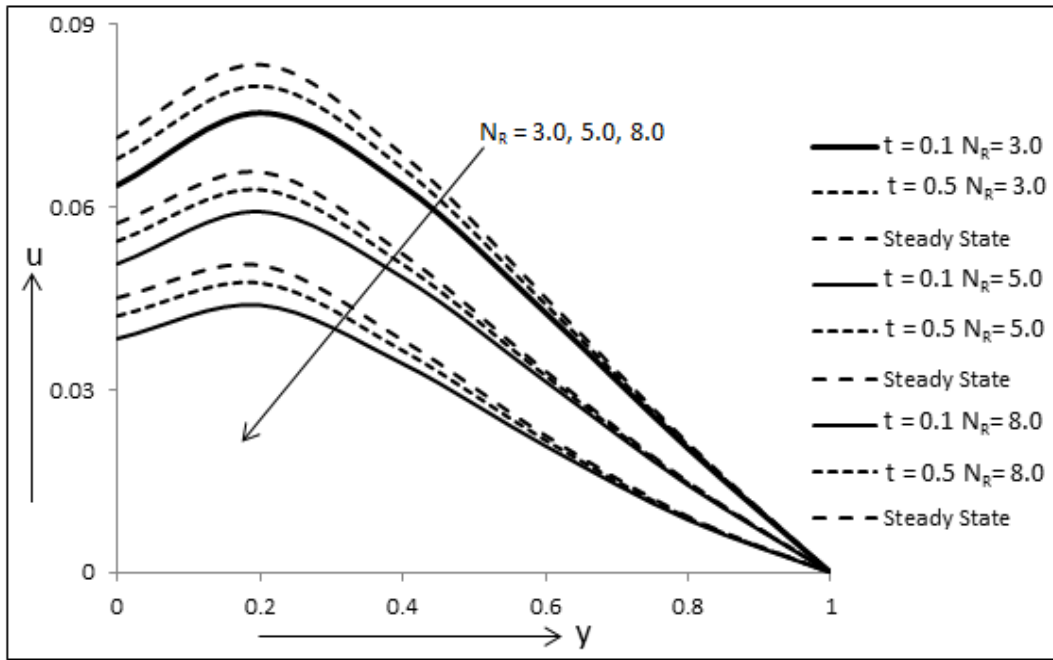


Fig.7. Velocity profiles for various values of the radiation parameter.

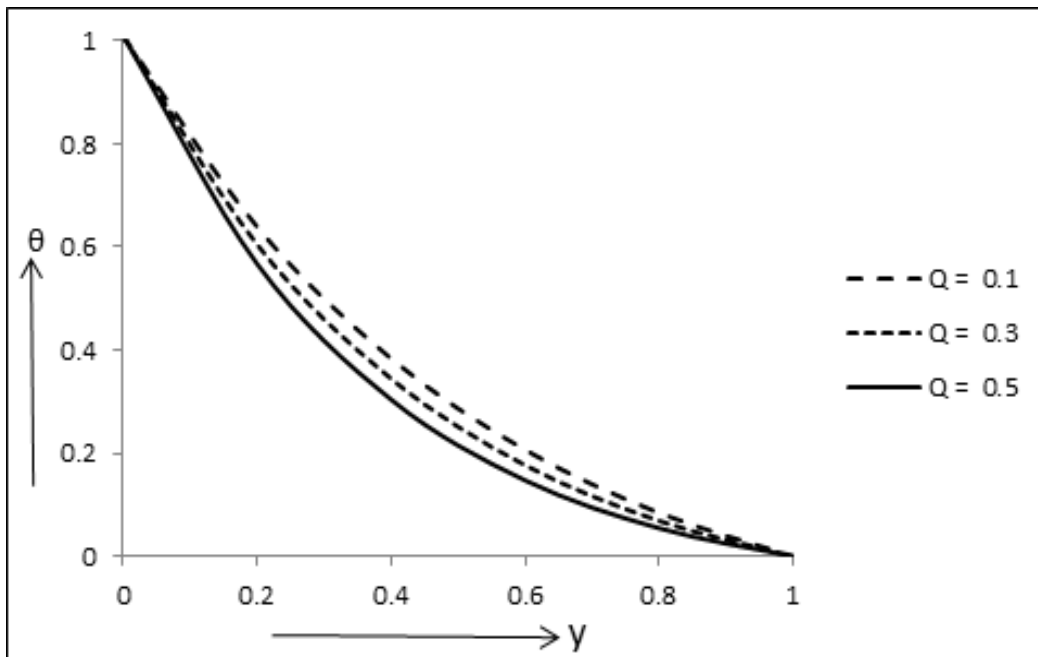


Fig.8. Temperature profiles for various values of the heat source parameter N .

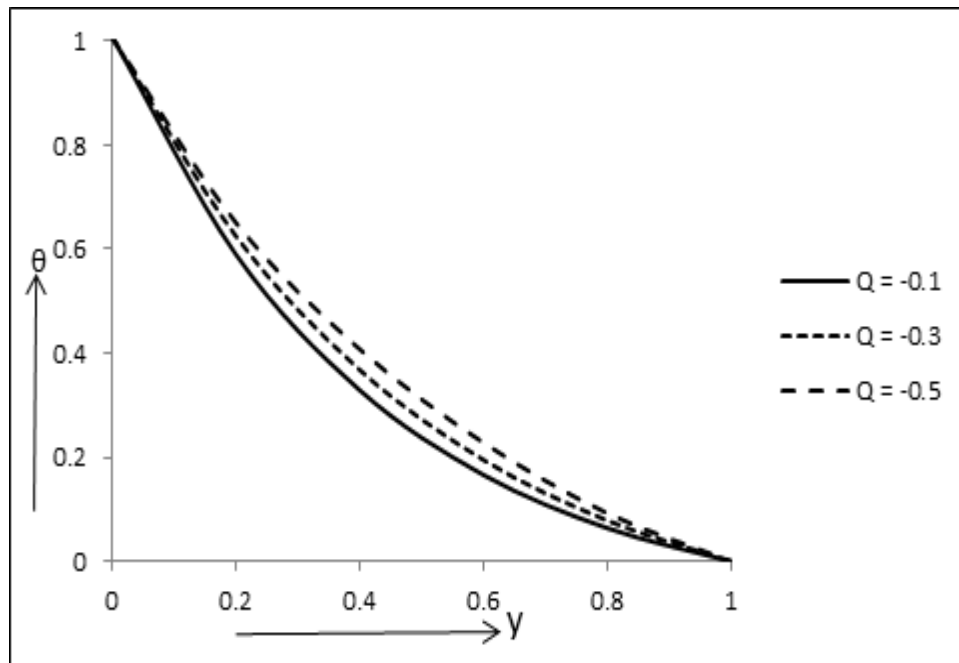


Fig.9. Temperature profiles for various values of the heat sink parameter.

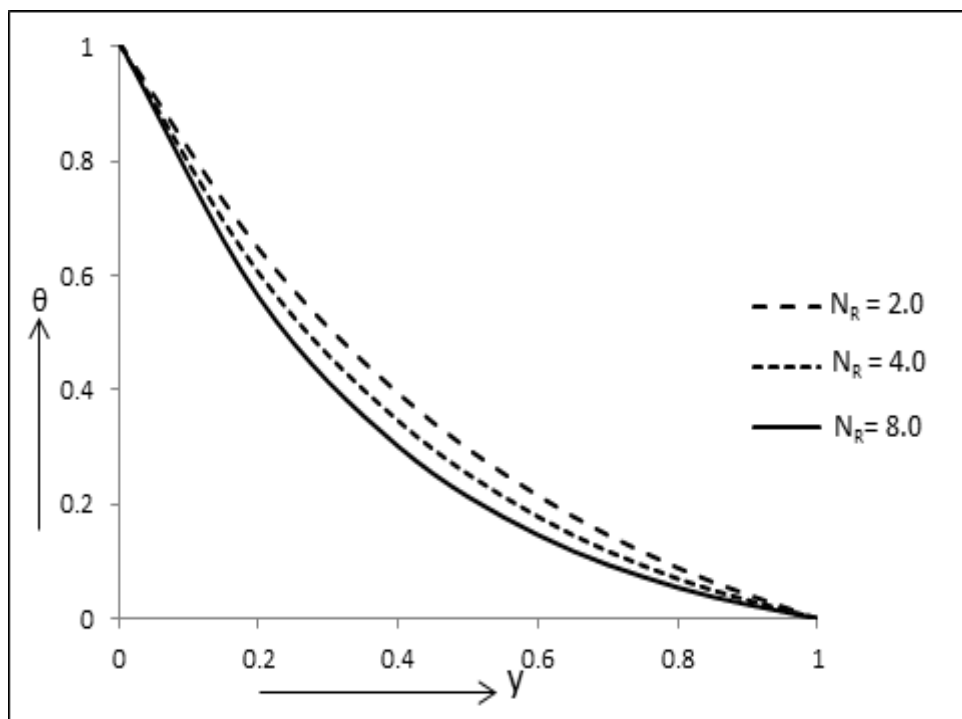


Fig.10. Temperature profiles for various values of the radiation parameter.

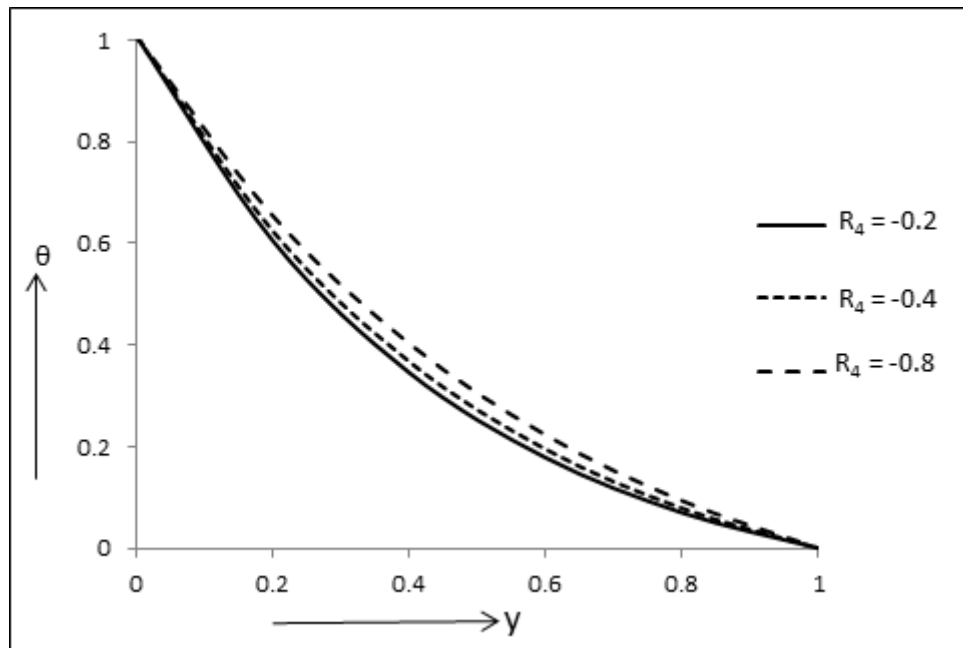


Fig.11. Temperature profiles for various values of the buoyancy force parameter ($R_4 < 0$).

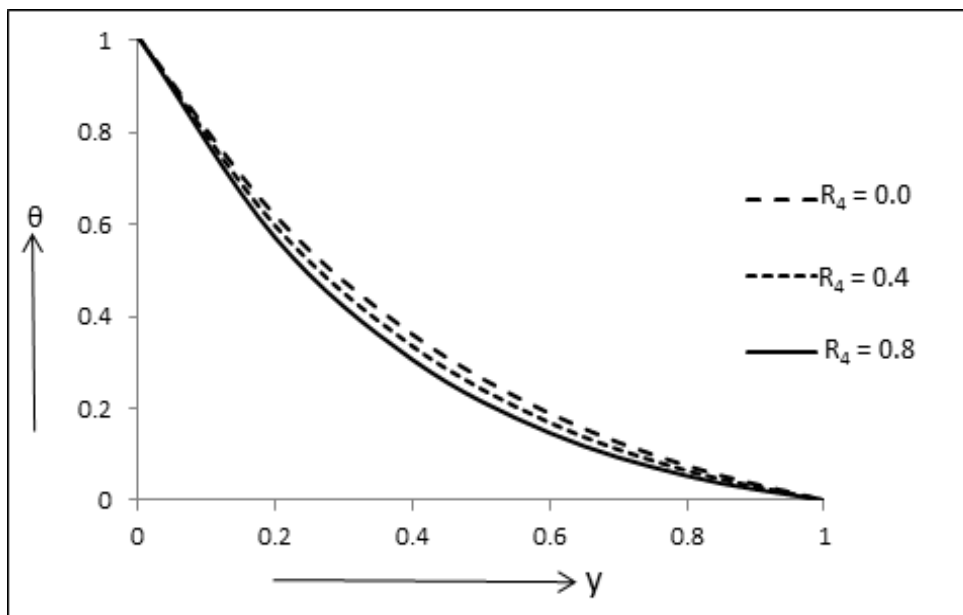


Fig.12. Temperature profiles for various values of the buoyancy force parameter ($R_4 > 0$).

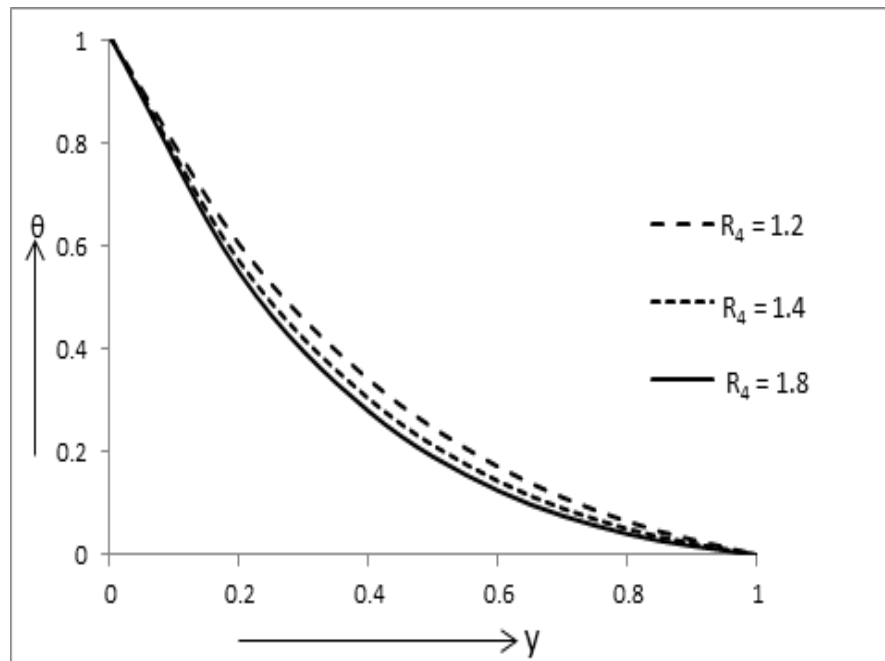


Fig.13. Temperature profiles for various values of the buoyancy force parameter ($R_4 > 1$).

Nomenclature

- B_0 – uniform magnetic field
- C_p – specific heat at constant pressure
- Da – Darcy number
- F – Forchheimer constant
- Gr – Grashof number
- g – acceleration due to gravity
- d – distance between vertical walls
- K – permeability of the porous medium
- k – thermal conductivity
- M – Hartmann number
- n – non-dimensional positive constant
- n^* – small positive constant
- Pr – Prandtl number
- Q – non-dimensional constant heat source
- Q_0 – dimensional constant heat source
- R_4 – buoyancy force distribution parameter
- T^* – temperature of the fluid
- T_m^* – initial temperature of the fluid
- T_h^* – temperature of the heated wall
- T_c^* – temperature of the cooled wall
- t – time in non-dimensional form
- t^* – time
- u^* – velocity of the fluid
- u – fluid velocity in non-dimensional form

- v_0 – dimensional constant suction
 S – suction parameter
 y – non-dimensional co-ordinate perpendicular to the walls
 y^* – co-ordinate perpendicular to the walls
 β – coefficient of thermal expansion
 λ – kinematic viscosity ratio
 ε – porosity-perturbation parameter
 μ_f – dynamic viscosity of the fluid
 ν_{eff} – effective kinematic viscosity of the porous region
 ν_f – kinematic viscosity of fluid
 ρ_f – density of the fluid
 σ – electrical conductivity of the fluid

Appendix

$$E_1 = \frac{l}{Da} + M, \quad E_2 = \frac{l}{Da} + M - n,$$

$$n_1 = \frac{-PrS + \sqrt{Pr^2 S^2 + 4k_2 Pr Q}}{2k_2}, \quad n_2 = \frac{-PrS - \sqrt{Pr^2 S^2 + 4k_2 Pr Q}}{2k_2},$$

$$n_4 = \frac{-PrS - \sqrt{Pr^2 S^2 + 4k_2 Pr(Q-n)}}{2}, \quad B_1 = \frac{Pr S n_1 F_1}{k_2 n_1^2 + Pr S k_2 - Pr(Q-n)},$$

$$B_2 = \frac{Pr S n_2 F_2}{k_2 n_2^2 + Pr S n_2 - Pr(Q-n)}, \quad B_3 = e^{n_3} (1 + B_1 + B_2),$$

$$B_4 = B_1 e^{n_1} + B_2 e^{n_2}, \quad B_5 = e^{n_3} - e^{n_4}, \quad B_6 = e^{n_4} (1 + B_1 + B_2),$$

$$B_7 = B_1 e^{n_1} + B_2 e^{n_2}, \quad B_8 = e^{n_4} - e^{n_3}$$

$$F_1 = \frac{e^{n_2} (R_4 - l) - R_4}{e^{n_1} - e^{n_2}}, \quad F_2 = \frac{R_4 - e^{n_1} (R_4 - l)}{e^{n_1} - e^{n_2}},$$

$$F_3 = \frac{B_6 - B_7}{B_8}, \quad F_4 = \frac{B_3 - B_4}{B_5},$$

$$n_5 = \frac{-S + \sqrt{S^2 + 4E_1 \lambda}}{2\lambda}, \quad n_6 = \frac{-S - \sqrt{S^2 + 4E_1 \lambda}}{2\lambda},$$

$$\begin{aligned}
 B_9 &= \frac{F_1}{\lambda n_1^2 + S n_1 - E_1}, & B_{10} &= \frac{F_2}{\lambda n_2^2 + S n_2 - E_1}, \\
 B_{11} &= B_9 \left[e^{n_6} n_{35} - e^{n_1} n_{15} \right], & B_{12} &= B_{10} \left[e^{n_6} n_{36} - e^{n_2} n_{15} \right], \\
 B_{13} &= \left[e^{n_6} n_{16} - e^{n_5} n_{15} \right], & B_{14} &= \frac{\text{Gr}}{\sqrt{\text{Da}}}, \\
 B_{15} &= \frac{B_{14} F_5^2}{4 n_5^2 \lambda + 2 S n_5 - E_1}, & B_{16} &= \frac{B_{14} F_6^2}{4 n_6^2 \lambda + 2 S n_6 - E_1}, \\
 F_5 &= \frac{B_{11} + B_{12}}{B_{13}}, & F_6 &= \frac{B_{13} + B_{14}}{B_{15}}, \\
 B_{17} &= \frac{B_{14} B_9^2}{4 n_1^2 \lambda + 2 S n_1 - E_1}, & B_{18} &= \frac{B_{14} B_{10}^2}{4 n_2^2 \lambda + 2 S n_2 - E_1}, \\
 B_{19} &= \frac{2 F_5 F_6 B_{14}}{n_9^2 \lambda + S n_9 - E_1}, & B_{20} &= \frac{2 F_5 B_9 B_{14}}{n_{10}^2 \lambda + S n_{10} - E_1}, \\
 B_{21} &= \frac{2 F_5 B_{10} B_{14}}{n_{11}^2 \lambda + S n_{11} - E_1}, & B_{22} &= \frac{2 F_6 B_9 B_{14}}{n_{12}^2 \lambda + S n_{12} - E_1}, \\
 B_{23} &= \frac{2 F_6 B_{10} B_{14}}{n_{13}^2 \lambda + S n_{13} - E_1}, & B_{24} &= \frac{2 B_9 B_{10} B_{14}}{n_{14}^2 \lambda + S n_{14} - E_1}, \\
 B_{25} &= B_{15} + B_{16} + B_{17} + B_{18} + B_{19} - B_{20} - B_{21} - B_{22} - B_{23} + B_{24}, \\
 B_{26} &= B_{15} 2 n_5 + B_{16} 2 n_6 + B_{17} 2 n_1 + B_{18} 2 n_2 + B_{19} n_9 + \\
 &\quad - B_{20} n_{10} - B_{21} n_{11} - B_{22} n_{12} - B_{23} n_{13} + B_{24} n_{14}, \\
 B_{27} &= B_{15} e^{2 n_5} + B_{16} e^{2 n_6} + B_{17} e^{2 n_1} + B_{18} e^{2 n_2} + B_{19} e^{n_9} + \\
 &\quad - B_{20} e^{n_{10}} - B_{21} e^{n_{11}} - B_{22} e^{n_{12}} - B_{23} e^{n_{13}} + B_{24} e^{n_{14}}, \\
 F_7 &= \frac{e^{n_6} h B_{26} - e^{n_6} B_{25} + B_{27} n_{15}}{e^{n_6} n_{16} - e^{n_5} n_{15}}, & F_8 &= \frac{e^{n_5} h B_{26} - e^{n_5} B_{25} + B_{27} n_{16}}{e^{n_5} n_{15} - e^{n_6} n_{16}}, \\
 n_7 &= \frac{-S + \sqrt{S^2 + 4 E_2 \lambda}}{2 \lambda}, & n_8 &= \frac{-S - \sqrt{S^2 + 4 E_2 \lambda}}{2 \lambda},
 \end{aligned}$$

$$\begin{aligned}
n_9 &= n_5 + n_6, & n_{10} &= n_1 + n_5, & n_{11} &= n_2 + n_5, & n_{12} &= n_1 + n_6, \\
n_{13} &= n_2 + n_6, & n_{14} &= n_1 + n_2, & n_{15} &= l - hn_6, & n_{16} &= l - hn_5, \\
n_{17} &= l - hn_8, & n_{18} &= l - hn_7, & n_{19} &= n_5 + n_7, & n_{20} &= n_5 + n_8, \\
n_{21} &= n_3 + n_5, & n_{22} &= n_4 + n_5, & n_{23} &= n_6 + n_7, & n_{24} &= n_6 + n_8, \\
n_{25} &= n_3 + n_6, & n_{26} &= n_4 + n_6, & n_{27} &= n_1 + n_7, & n_{28} &= n_1 + n_8, \\
n_{29} &= n_1 + n_3, & n_{30} &= n_1 + n_4, & n_{31} &= n_2 + n_7, & n_{32} &= n_2 + n_8, \\
n_{33} &= n_2 + n_3, & n_{34} &= n_2 + n_4, & n_{35} &= l - hn_1, & n_{36} &= l - hn_2,
\end{aligned}$$

$$B_{28} = \frac{B_1}{n_1^2 \lambda + Sn_1 - E_2}, \quad B_{29} = \frac{B_2}{n_2^2 \lambda + Sn_2 - E_2}, \quad B_{30} = \frac{F_3}{n_3^2 \lambda + Sn_3 - E_2},$$

$$B_{31} = \frac{F_4}{n_4^2 \lambda + Sn_4 - E_2}, \quad B_{32} = \frac{SB_9}{n_1^2 \lambda + Sn_1 - E_2}, \quad B_{33} = \frac{SB_{10}}{n_2^2 \lambda + Sn_2 - E_2},$$

$$B_{34} = \frac{SF_5}{n_5^2 \lambda + Sn_5 - E_2}, \quad B_{35} = \frac{SF_6}{n_6^2 \lambda + Sn_6 - E_2},$$

$$B_{36} = B_{28} + B_{29} - B_{30} - B_{31} + B_{32} + B_{33} - B_{34} - B_{35},$$

$$B_{37} = n_1(B_{28} + B_{32}) + n_2(B_{29} + B_{33}) - n_3 B_{30} - n_4 B_{31} - n_5 B_{33} - n_6 B_{35},$$

$$B_{38} = e^{n_1}(B_{28} + B_{32}) + e^{n_2}(B_{29} + B_{33}) - e^{n_3} B_{30} - e^{n_4} B_{31} - e^{n_5} B_{33} - e^{n_6} B_{35},$$

$$F_9 = \frac{e^{n_8} h B_{37} - e^{n_8} B_{36} + B_{38} n_{17}}{e^{n_8} n_{18} - e^{n_7} n_{17}}, \quad F_{10} = \frac{e^{n_7} h B_{37} - e^{n_7} B_{36} + B_{38} n_{18}}{e^{n_7} n_{17} - e^{n_8} n_{18}},$$

$$B_{39} = \frac{2Gr}{\sqrt{Da}}, \quad B_{40} = F_5 B_{34} B_{39} - S 2n_5 B_{15}, \quad B_{41} = F_6 B_{35} B_{39} + S 2n_6 B_{16},$$

$$B_{42} = [B_{39} B_{10} (B_{33} + B_{29}) + 2n_2 B_{18} S], \quad B_{43} = [B_{39} B_9 (B_{28} + B_{32}) + 2n_1 B_{17} S],$$

$$B_{44} = [B_{39} (F_5 B_{35} + F_6 F_{34}) + B_{19} S (n_5 + n_6)], \quad B_{45} = [B_{39} \{F_5 (B_{28} + B_{32}) + B_9 B_{34}\} + B_{20} S n_{10}],$$

$$B_{46} = [B_{39} \{F_5 (B_{29} + B_{33}) + B_{10} B_{34}\} + B_{21} S n_{11}], \quad B_{47} = [B_{39} \{F_6 (B_{28} + B_{32}) + B_9 B_{35}\} + B_{22} S n_{12}],$$

$$B_{48} = [B_{39} \{F_6 (B_{29} + B_{33}) + B_9 B_{35}\} + B_{23} S n_{12}],$$

$$\begin{aligned}
 B_{49} &= [B_{39}\{B_{10}(B_{28} + B_{32}) + B_9(B_{33} + B_{29})\} + B_{24} Sn_{13}], & B_{50} &= B_{39}F_5F_9, \\
 B_{51} &= B_{39}F_5F_{10}, & B_{52} &= B_{39}F_5B_{33}, & B_{53} &= B_{39}F_5B_{31}, & B_{54} &= B_{39}F_6F_9, \\
 B_{55} &= B_{39}F_6F_{10}, & B_{56} &= B_{39}F_6B_{30}, & B_{57} &= B_{39}F_6B_{31}, & B_{58} &= B_{39}F_9B_9, \\
 B_{59} &= B_{39}F_{10}B_9, & B_{60} &= B_{39}B_9B_{30}, & B_{61} &= B_{39}B_9B_{31}, & B_{62} &= B_{39}F_9B_{10}, \\
 B_{63} &= B_{39}F_{10}B_{10}, & B_{64} &= B_{39}B_{10}B_{30}, & B_{65} &= B_{39}B_{10}B_{31}, & B_{66} &= Sn_5F_7, \\
 B_{67} &= Sn_6F_8, & B_{68} &= \frac{B_{40}}{\lambda 4n_5^2 + 2n_5S - E_2}, & B_{69} &= \frac{B_{41}}{\lambda 4n_6^2 + 2n_6S - E_2}, \\
 B_{70} &= \frac{B_{42}}{\lambda 4n_2^2 + 2n_2S - E_2}, & B_{71} &= \frac{B_{43}}{\lambda 4n_1^2 + 2n_1S - E_2}, & B_{72} &= \frac{B_{44}}{\lambda n_9^2 + Sn_9 - E_2}, \\
 B_{73} &= \frac{B_{45}}{\lambda n_{10}^2 + Sn_{10} - E_2}, & B_{74} &= \frac{B_{46}}{\lambda n_{11}^2 + Sn_{11} - E_2}, & B_{75} &= \frac{B_{47}}{\lambda n_{12}^2 + Sn_{12} - E_2}, \\
 B_{76} &= \frac{B_{48}}{\lambda n_{13}^2 + Sn_{13} - E_2}, & B_{77} &= \frac{B_{49}}{\lambda n_{14}^2 + Sn_{14} - E_2}, & B_{78} &= \frac{B_{50}}{\lambda n_{19}^2 + Sn_{19} - E_2}, \\
 B_{79} &= \frac{B_{51}}{\lambda n_{20}^2 + Sn_{20} - E_2}, & B_{80} &= \frac{B_{52}}{\lambda n_{21}^2 + Sn_{21} - E_2}, & B_{81} &= \frac{B_{53}}{\lambda n_{22}^2 + Sn_{22} - E_2}, \\
 B_{82} &= \frac{B_{54}}{\lambda n_{23}^2 + Sn_{23} - E_2}, & B_{83} &= \frac{B_{55}}{\lambda n_{24}^2 + Sn_{24} - E_2}, & B_{84} &= \frac{B_{56}}{\lambda n_{25}^2 + Sn_{25} - E_2}, \\
 B_{85} &= \frac{B_{57}}{\lambda n_{26}^2 + Sn_{26} - E_2}, & B_{86} &= \frac{B_{58}}{\lambda n_{27}^2 + Sn_{27} - E_2}, & B_{87} &= \frac{B_{59}}{\lambda n_{28}^2 + Sn_{28} - E_2}, \\
 B_{88} &= \frac{B_{60}}{\lambda n_{29}^2 + Sn_{29} - E_2}, & B_{89} &= \frac{B_{61}}{\lambda n_{30}^2 + Sn_{30} - E_2}, & B_{90} &= \frac{B_{62}}{\lambda n_{31}^2 + Sn_{31} - E_2}, \\
 B_{91} &= \frac{B_{63}}{\lambda n_{32}^2 + Sn_{32} - E_2}, & B_{92} &= \frac{B_{64}}{\lambda n_{33}^2 + Sn_{33} - E_2}, & B_{93} &= \frac{B_{65}}{\lambda n_{34}^2 + Sn_{34} - E_2}, \\
 B_{94} &= \frac{B_{66}}{\lambda n_5^2 + Sn_5 - E_2}, & B_{95} &= \frac{B_{67}}{\lambda n_6^2 + Sn_6 - E_2},
 \end{aligned}$$

$$B_{96} = -B_{68} - B_{69} - B_{70} - B_{71} - B_{72} + B_{73} + B_{74} + B_{75} + B_{76} - B_{77} + B_{78} + B_{79} + \\ -B_{80} - B_{81} + B_{82} + B_{83} - B_{84} - B_{85} - B_{86} - B_{87} + B_{88} + B_{89} - B_{90} - B_{91} + \\ +B_{92} + B_{93} - B_{94} - B_{95},$$

$$F_{11} = \frac{e^{n_8} B_{97} - e^{n_8} B_{96} + n_{17} B_{98}}{[e^{n_8} n_{18} - e^{n_7} n_{17}]}, \quad F_{12} = \frac{e^{n_7} B_{97} - e^{n_7} B_{96} + n_{18} B_{98}}{[e^{n_7} h_{17} - e^{n_8} n_{18}]},$$

$$B_{97} = [-2n_5 B_{68} - 2n_6 B_{69} - 2n_2 B_{70} - 2n_1 B_{71} - B_{72} n_9 + \\ +B_{73} n_{10} + B_{74} n_{11} + B_{75} n_{12} + B_{76} n_{13} - B_{77} n_{14} + B_{78} n_{19} + \\ +B_{79} n_{20} - B_{80} n_{21} - B_{81} n_{22} + B_{82} n_{23} + B_{83} n_{24} - B_{84} n_{25} - B_{85} n_{26} - B_{86} n_{27} + \\ -B_{87} n_{28} + B_{88} n_{29} + B_{89} n_{30} - B_{90} n_{31} - B_{91} n_{32} + B_{92} n_{33} + B_{93} n_{34} - n_5 B_{94} - n_6 B_{95}]h,$$

$$B_{98} = -e^{2n_5} B_{68} - e^{2n_6} B_{69} - e^{2n_2} B_{70} - e^{2n_1} B_{71} - B_{72} e^{n_9} + B_{73} e^{n_{10}} + B_{74} e^{n_{11}} + B_{75} e^{n_{12}} + B_{76} e^{n_{13}} + \\ -B_{77} e^{n_{14}} + B_{78} e^{n_{19}} + B_{79} e^{n_{20}} - B_{80} e^{n_{21}} - B_{81} e^{n_{22}} + B_{82} e^{n_{23}} + B_{83} e^{n_{24}} - B_{84} e^{n_{25}} - B_{85} e^{n_{26}} - B_{86} e^{n_{27}} + \\ -B_{87} e^{n_{28}} + B_{88} e^{n_{29}} + B_{89} e^{n_{30}} - B_{90} e^{n_{31}} - B_{91} e^{n_{32}} + B_{92} e^{n_{33}} + B_{93} e^{n_{34}} - B_{94} e^{n_5} - B_{95} e^{n_6}.$$

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