

## **ROLE OF BROWNIAN MOTION AND THERMOPHORESIS EFFECTS ON HYDROMAGNETIC FLOW OF NANOFUID OVER A NONLINEARLY STRETCHING SHEET WITH SLIP EFFECTS AND SOLAR RADIATION**

S.P. ANJALI DEVI and S. MEKALA\*

Department of Applied Mathematics, Bharathiar University  
Coimbatore-46, Tamilnadu, INDIA

E-mails: anjalidevi\_s\_p@yahoo.co.in; mekalam19@gmail.com

Hydromagnetic flow of water based nanofluids over a nonlinearly stretching sheet in the presence of velocity slip, temperature jump, magnetic field, nonlinear thermal radiation, thermophoresis and Brownian motion has been studied. The article focuses on Cu water nanofluid and Ag water nanofluid. The similarity transformation technique is adopted to reduce the governing nonlinear partial differential equations into nonlinear ordinary differential equations and then they are solved numerically utilizing the Nachistem – Swigert shooting method along with the fourth order Runge Kutta integration technique. The influence of physical parameters on the flow, temperature and nanoparticle volume fraction are presented through graphs. Also the values of the skin friction coefficient at the wall and nondimensional rate of heat transfer are given in a tabular form. A comparative study with previous published results is also made.

**Key words:** nanofluid, MHD, radiation, slip flow, stretching sheet.

### **1. Introduction**

Renewable energy generation is one of the most important challenges facing society today. The challenge lies in resourcefully collecting and converting this energy into something valuable. Solar energy offers a solution to this typical problem. It is a natural result of electromagnetic radiation released from the sun by the thermonuclear reaction occurring inside its core. It has produced energy for billions of years, so the utilization of solar energy has received significant attention. The major component of any solar energy system is the solar collector which is a special kind of heat exchanger that transforms solar radiation energy to internal energy of the transport medium. The novel concept of nanofluids (nanoparticles must be dispersed uniformly in the traditional fluid) has been utilized to improve the solar weighted absorption and increase the efficiency of solar collectors by replacing the working fluid. Water heating systems in homes, solar desalination, solar space heating, electricity production, solar drying devices and solar power plants are examples of universal applications of nanofluids in solar collectors.

In view of all these practical applications, studies on nanofluids with radiation heat transfer have attracted attention of researchers, scientists and engineers in recent times. The present work is concerned with a problem of such kind.

The study of hydromagnetic flow has attracted attention of researchers for the last few decades due to numerous applications in engineering and industries. Hydromagnetic flow over a stretching surface finds lots of practical applications. Hydromagnetic boundary layer flow of a viscous incompressible fluid which is caused by a sheet stretching according to a power law velocity distribution in the presence of a magnetic field was investigated by Chaim [1] following the lines of Sakiadis [2]. After Chaim, many investigations have been done on MHD flow field and its characteristics.

---

\* To whom correspondence should be addressed

Convective heat transfer in nanofluids is a topic of major contemporary interest in science and engineering. Choi [3] was the first to introduce the word nanofluid that represents the fluid in which nanoscale particles are suspended in the base fluid. Lee *et al.* [4], Wang *et al.* [5], Xuan [6] and Yu and Choi [7] investigated the properties of nanofluids containing metals and metal oxides nanoparticles. They studied the parameters which influence nanofluid properties.

Convective transport in nanofluids was studied by Buongiorno [8]. He established that among seven slip mechanisms, Brownian diffusion and thermophoresis effects are more important nanoparticle/base fluid slip mechanisms. Das *et al.* [9] studied the heat conduction effects in nanofluids. Effects of Brownian motion and thermophoresis diffusion on non equilibrium heat conduction of nanofluid were investigated by Yuwen *et al.* [10]. An analysis was carried out by Vajravelu *et al.* [11] to study the convective heat transfer in a nanofluid flow over a stretching surface. Hamad and Ferdows [12] studied the boundary-layer flow and heat transfer in a viscous fluid containing metallic nanoparticles over a nonlinear stretching sheet.

Further, convective flows with nonlinear thermal radiation have been widely studied in recent years due to their broad applications in many industrial processes. Khullar and Tyagi [13] concluded that the enhancement of the solar radiation absorption capacity leads to a higher heat transfer rate resulting in more efficient heat transfer.

A partial slip between the fluid and the moving surface may occur in some situations when the fluid is particulate such as emulsions, suspensions, foam and polymer solutions. In these cases, the proper boundary condition is replaced by Navier's condition. The flow of an electrically conducting fluid over a stretching sheet with slip effects and thermal radiation problem was studied by Mukhopadhyay [14].

Heat transfer characteristics of nanofluids under various physical situations were investigated by many authors such as Nadeem *et al.* [15], Khan *et al.* [16], Mohammad Mehdi Keshtkar and Babak Amiri [17], Malvandi *et al.* [18], Shateyi and Prakash [19], Krishnamurthy *et al.* [20], Falana *et al.* [21].

But so far, a laminar hydromagnetic flow of incompressible nanofluids over a nonlinearly stretching sheet with slip effects, nonlinear radiative heat transfer, Brownian motion and thermophoresis effects of nanofluids in two phase revised model has not been considered. This fact motivates the present investigation. By employing similarity transformations, governing nonlinear partial differential equations are converted into nonlinear ordinary differential equations and then are solved numerically. Numerical investigations of the effect of physical parameters on flow, temperature, nanoparticle volume fraction and skin friction coefficient and nondimensional rate of heat transfer are presented graphically and also provided in tabular form wherever necessity arises.

## 2. Mathematical formulation

A steady, laminar, two dimensional, hydromagnetic flow of nanofluids over a nonlinear stretching surface in the presence of velocity slip, temperature jump and nonlinear thermal radiation is considered. Here the fluid is considered to be viscous, incompressible, electrically conducting and radiating. Also the important slip mechanisms of nanoparticles such as the Brownian motion and thermophoresis effects are taken into consideration. Two types of nanofluids such as copper water nanofluid and silver water nanofluid are considered. The Cartesian coordinates  $x$  and  $y$  run along the flow direction of the nonlinear stretching sheet and normal to the sheet respectively. Considering Navier's slip condition, the velocity slip is assumed to be proportional to the local shear stress at the nonlinear stretching surface. The geometric model of the problem is presented in Fig.1.

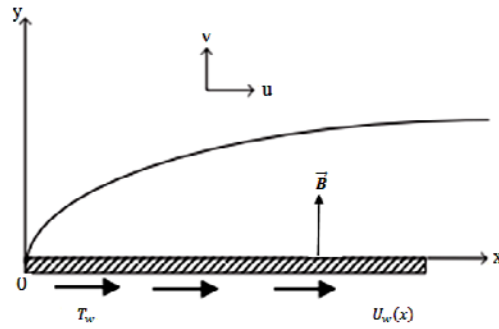


Fig.1. The geometry of the problem.

The sheet is considered to stretch with the surface velocity  $U_w(x)=cx^n$ ,  $c > 0$  is the stretching coefficient.

The radiating fluid is considered to be a grey, emitting, absorbing but non scattering medium. The thermal gradient applied in the  $x$  direction is neglected. The magnetic field is applied in the perpendicular direction. The magnetic Reynolds number  $R_m$  is assumed to be small ( $R_m \ll 1$ ) and hence the induced magnetic field is assumed to be neglected [Davidson & Ref. [2]]. Due to this and as  $B$  is independent of time,  $\text{curl } \mathbf{E} = 0$ . Also,  $\text{div } \mathbf{E} = 0$  in the absence of surface charge density. Hence  $\mathbf{E}=0$ .

Adopting the modified mass flux condition by Kuznetsov and Nield [22], mass flux at the sheet is considered to be zero.

Considering the above assumptions, the governing boundary layer equations of the problem (Buongiorno [8]) are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{2.1}$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu_{nf} \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B^2}{\rho_{nf}} u, \tag{2.2}$$

$$(\rho c_p)_{nf} \left( u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = k_{nf} \frac{\partial^2 T}{\partial y^2} + (\rho c_p)_s \left\{ D_B \left( \frac{\partial T}{\partial y} \frac{\partial \phi}{\partial y} \right) + \frac{D_T}{T_\infty} \left( \frac{\partial T}{\partial y} \right)^2 \right\} - \frac{\partial q_r}{\partial y}, \tag{2.3}$$

$$u \frac{\partial \phi}{\partial x} + v \frac{\partial \phi}{\partial y} = D_B \frac{\partial^2 \phi}{\partial y^2} + \frac{D_T}{T_\infty} \frac{\partial^2 T}{\partial y^2}. \tag{4}$$

The boundary conditions are

$$u = U_w(x) + u_s, \quad v = 0, \quad T = T_w + T_s, \quad D_B \frac{\partial \phi}{\partial y} + \frac{D_T}{T_\infty} \frac{\partial T}{\partial y} = 0 \quad \text{at } y=0, \tag{2.5}$$

$$u = 0, \quad T \rightarrow T_\infty, \quad \phi \rightarrow \phi_\infty \quad \text{as } y \rightarrow \infty.$$

Here  $u$  and  $v$  are the velocity components in the  $x$  and  $y$  directions respectively.  $T$  is the temperature of the nanofluid,  $T_w$  is the constant surface temperature,  $T_\infty$  is the ambient temperature,  $\phi$  is the nanoparticle

volumetric fraction,  $\rho_{nf}$  is the density of the nanofluid,  $k_{nf}$  is the thermal conductivity of the nanofluid,  $(c_p)_{nf}$  is the specific heat capacity of the nanofluid,  $(c_p)_s$  is the specific heat capacity of the nanoparticles,  $D_B$  is the Brownian diffusion coefficient,  $D_T$  is the thermophoresis diffusion coefficient which represent the active control of nanoparticle volume fraction at the boundary (Nield and Kuznetsov [22]).  $u_s$  is the slip velocity

which is given by  $u_s = \lambda^* \left( \frac{\partial u}{\partial y} \right)_{y=0}$  where  $\lambda^* = \lambda_1 x^{\frac{1-n}{2}}$  is the initial value of the velocity slip factor which

changes with  $x$ ,  $\lambda_1 = \left( \frac{2-f_1}{f_1} \right) \xi_1$  is the initial value of velocity slip. Here  $\xi_1 = \left( \frac{\pi}{2p\rho} \right)^{\frac{1}{2}}$  is the mean free path,  $f_1$  is the Maxwell's reflection coefficient,  $p$  is the fluid pressure.

Further,  $T_s$  is the thermal jump factor and is given by  $T_s = D^* \left( \frac{\partial T}{\partial y} \right)$ , where  $D^* = D_1 x^{\frac{1-n}{2}}$  is the thermal slip which changes with  $x$ ,  $D_1 = \left( \frac{2-b}{b} \right) \xi_2$ , is the initial value of the thermal slip factor and  $\xi_2 = \left( \frac{2\gamma_1}{\gamma_1 + 1} \right) \frac{\xi_1}{Pr}$ ,  $\gamma_1$  is the ratio of specific heats and  $b$  is the thermal accommodation coefficient.

The Rosseland [23] approximation for optically thick media is applied which is given by the expression

$$q_r = -\frac{16\sigma^*}{3k^*} \left( T^3 \frac{\partial T}{\partial y} \right). \quad (2.6)$$

Hence 
$$\frac{\partial}{\partial y} (q_r) = -\frac{16\sigma^*}{3k^*} \left( 3T^2 \left( \frac{\partial T}{\partial y} \right)^2 + T^3 \frac{\partial^2 T}{\partial y^2} \right). \quad (2.7)$$

Substituting Eq.(2.7) into Eq.(2.3) becomes

$$\begin{aligned} (\rho c_p)_{nf} \left( u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) &= k_{nf} \frac{\partial^2 T}{\partial y^2} + \\ &+ (\rho c_p)_s \left\{ D_B \left( \frac{\partial T}{\partial y} \frac{\partial \phi}{\partial y} \right) + \frac{D_T}{T_\infty} \left( \frac{\partial T}{\partial y} \right)^2 \right\} + \frac{16\sigma^*}{3k^*} \left( 3T^2 \left( \frac{\partial T}{\partial y} \right)^2 + T^3 \frac{\partial^2 T}{\partial y^2} \right). \end{aligned} \quad (2.8)$$

### 3. Method of solution

Equations (2.2), (2.8) and (2.4) along with the boundary conditions can be transformed to the corresponding nonlinear ordinary differential equations by establishing the following similarity transformations (Anjali Devi and Mekala [24])

$$\psi = \sqrt{\frac{2cv_f}{(n+1)}} x^{\frac{n+1}{2}} f(\eta), \quad \eta = y \sqrt{\frac{c(n+1)}{2\nu_f}} x^{\frac{n-1}{2}}, \quad \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \quad \Phi(\eta) = \frac{\phi - \phi_\infty}{\phi_\infty}. \quad (3.1)$$

Defining the stream function  $\psi$  in the common way such as

$$u = \frac{\partial \psi}{\partial y} \quad \text{and} \quad v = -\frac{\partial \psi}{\partial x}, \tag{3.2}$$

which identically satisfies Eq.(2.1) and substituting Eqs (3.1) and (3.2) into Eqs (2.2), (2.4) and (2.8), the following nonlinear ordinary differential equations are obtained. In order to have a similarity solution, it is assumed that the imposed magnetic field  $B$  is proportional to  $x^{n-1/2}$ . Hence  $B = B_0 x^{n-1/2}$ ,  $B_0$  is a positive constant [Ref.2].

$$f''' + L_1 \left( L_2 \left( ff'' - \frac{2n}{n+1} (f')^2 \right) - M^2 f' \right) = 0, \tag{3.3}$$

$$\begin{aligned} \theta'' \left( I + \frac{4}{3} \frac{k_f}{N_R k_{nf}} \{ I + (\theta_w - I) \theta \}^3 \right) + \frac{4}{N_R k_{nf}} (I + (\theta_w - I) \theta)^2 (\theta_w - I) \theta'^2 = \\ = -\frac{k_f}{k_{nf}} \left( \text{Pr} \cdot L_3 (f \theta') + Nb \Phi' \theta' + Nt (\theta')^2 \right), \end{aligned} \tag{3.4}$$

$$\Phi'' + \text{Pr} \text{Le} f \Phi' + \frac{Nt}{Nb} \theta'' = 0. \tag{3.5}$$

Here  $L_1, L_2$  and  $L_3$  are constants whose values are provided in the Appendix.

The boundary conditions (3.5) are transformed to

$$\begin{aligned} f(0) = 0, \quad f'(0) = I + \lambda f''(0), \quad \theta(0) = I + D \theta'(0), \\ Nb \Phi'(0) + Nt \theta'(0) = 0 \quad \text{at} \quad \eta = 0, \end{aligned} \tag{3.6}$$

$$f'(\infty) = 0, \quad \theta(\infty) \rightarrow 0, \quad \Phi(\infty) \rightarrow 0 \quad \text{as} \quad \eta \rightarrow \infty$$

where

$$M^2 = \frac{2\sigma B_0^2}{c(n+1)\rho_f}, \text{ the magnetic interaction parameter,}$$

$$\lambda = \lambda_1 \sqrt{\frac{c(n+1)}{2\nu_f}}, \text{ the slip parameter,}$$

$$D = D_1 \sqrt{\frac{c(n+1)}{2\nu_f}}, \text{ the temperature jump parameter,}$$

$$N_R = \frac{k_f k^*}{4\sigma^* T_\infty^3}, \text{ the radiation parameter,}$$

$$\text{Pr} = \frac{v_f (\rho c_p)_f}{k_f}, \text{ the Prandtl number,}$$

$$\theta_w = \frac{T_w}{T_\infty}, \text{ the temperature ratio parameter,}$$

$$\text{Nb} = \frac{(\rho c_p)_s D_B (\phi_\infty)}{(\rho c_p)_f \alpha_f}, \text{ the Brownian motion parameter,}$$

$$\text{Nt} = \frac{(\rho c_p)_s D_T (T_w - T_\infty)}{(\rho c_p)_f T_\infty \alpha_f}, \text{ the thermophoresis parameter,}$$

$$\text{Le} = \frac{\alpha_f}{D_B}, \text{ the Lewis number.}$$

### 3.1. Skin friction coefficient, Nusselt number, Sherwood number

From the engineering point of view, the important characteristics of the flow are the skin friction coefficient, Nusselt number and Sherwood number which are defined as

$$C_f = \frac{\tau_w}{\rho_f u_w^2} \quad \text{where} \quad \tau_w = \mu_{nf} \left( \frac{\partial u}{\partial y} \right)_{y=0}, \quad (3.7)$$

$$C_f (\text{Re}_x)_f^{1/2} = \frac{\sqrt{n+1}}{(1-\phi)^{2.5}} f''(0), \quad \text{Nu}_x = \frac{q_w x}{k_f (T_w - T_\infty)}$$

where surface heat flux is

$$q_w = - \left[ k_{nf} + \frac{16\sigma^*}{3k^*} T^3 \right] \left( \frac{\partial T}{\partial y} \right)_{y=0}$$

$$\frac{\text{Nu}_x}{\sqrt{\text{Re}_x}} = - \frac{k_{nf}}{k_f} \sqrt{n+1} \left( 1 + \frac{k_f}{k_{nf}} \frac{4}{3N_R} \theta_w^3 \right) \theta'(0). \quad (3.8)$$

Here  $(\text{Re}_x)_f$  is the local Reynolds number and is given by

$$(\text{Re}_x)_f = \frac{2c x^{n+1}}{v_f}.$$

The Sherwood number tends to zero since the mass flux boundary condition is assumed to be zero [Ref.22].

### 4. Numerical scheme

The set of Eqs (3.3) – (3.5) along with boundary conditions (3.6) is highly nonlinear which cannot be solved analytically. The numerical solutions of the nonlinear equations are obtained using the most efficient shooting method such as the Nachtsheim – Swigert shooting iteration scheme to satisfy asymptotic boundary conditions, along with the fourth order R – K integration technique. The difficulty lies in guessing the values for  $f'(0)$ ,  $\theta'(0)$  and  $\Phi'(0)$ . The level of accuracy for convergence is chosen as  $10^{-5}$ .

### 5. Validation of numerical scheme

A comparative study of the results for the skin friction coefficient and nondimensional rate of heat transfer with those of Cortell [25] is presented in Tabs 1 and 2. It is seen from these tables that the authors' results are in very good agreement with previously published results of Cortell [25] in the absence of nanoparticles, magnetic field, slip effects and nonlinear thermal radiation.

Table 1. Comparison of results for  $-f''(0)$  when  $\phi = 0$  and  $M^2 = 0.0$ .

$n$	Cortell [25]	Present work
0.0	0.627547	0.62753
0.2	0.766758	0.76683
0.5	0.889477	0.88954
0.75	0.953786	0.95396
1.0	1.0	1.00000
1.5	1.061587	1.06166
3.0	1.148588	1.14859
7.0	1.216847	1.22097
10.0	1.234875	1.23561

Table 2. Comparison of results for  $-\theta'(0)$  when  $\phi = 0$  and  $N_R \rightarrow \infty$ .

$N$	Pr = 1.0		Pr = 5.0	
	Cortell [25]	Present work	Cortell [25]	Present work
0.2	0.610262	0.61021	1.607175	1.60779
0.5	0.595277	0.59520	1.586744	1.58678
1.5	0.574537	0.57503	1.557463	1.55724
3.0	0.564472	0.56452	1.542337	1.54205
10.0	0.554960	0.55499	1.528573	1.52836

### 6. Results and discussion

Numerical solutions for the velocity, temperature and nanoparticle volume fraction distribution, skin friction coefficient and nondimensional rate of heat transfer are presented graphically and are tabulated for various physical parameters which are involved in this work for some fixed value of the Prandtl number  $Pr = 6.2$  when  $\phi = 0.05$ .

The following figures highlight the effects of various physical parameters on the nanofluid velocity, temperature and nanoparticle volume fraction distribution.

Figure 2 displays the effect of the magnetic interaction parameter on the flow field. It is seen that the magnetic interaction parameter decelerates the velocity. This is due to the presence of the transverse magnetic field which sets the Lorentz force effect, which results in the retarding effect on the velocity field. As the values of the magnetic interaction parameter  $M^2$  increase, the retarding force increases and consequently the velocity gets decelerated. It also reveals the fact that the boundary layer thickness reduces as the magnetic interaction parameter increases.

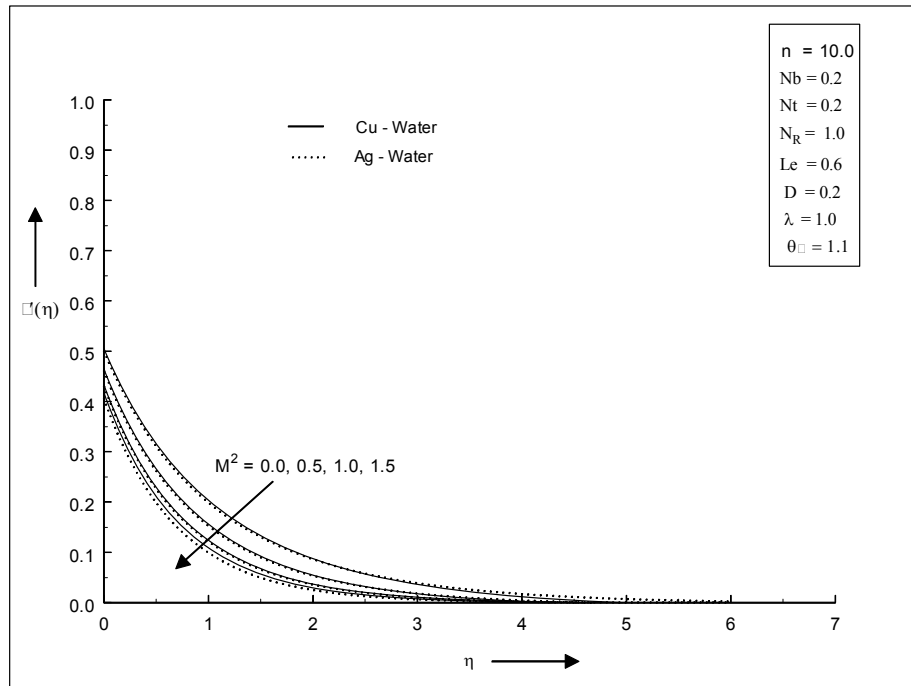


Fig.2. Dimensionless velocity profiles for various values of  $M^2$ .

Figure 3 depicts the effect of the magnetic field ( $M^2$ ) on the dimensionless temperature. As expected, it is observed that the thermal boundary layer thickness enhances with the increasing values of  $M^2$ , leading to a raise in the temperature. Figure 4 shows the influence of the magnetic interaction parameter on the nanoparticle volume fraction distribution. A rise in the values of the magnetic interaction parameter leads to the diminishing of the nanoparticle volume fraction distribution.

The dimensionless velocity distribution within the boundary layer region for different values of the slip velocity parameter is illustrated in Fig.5. When slip occurs, the flow velocity near the sheet is no longer equal to the stretching velocity of the sheet. When slip velocity increases, nanofluid velocity decelerates under the slip condition leading to thinning of momentum boundary layer thickness. In Fig.6, the behavior of temperature distribution for increasing values of the slip parameter is shown. It is noted that nanofluid temperature boosts up with the influence of the velocity slip parameter resulting in the thickening of thermal boundary layer thickness.



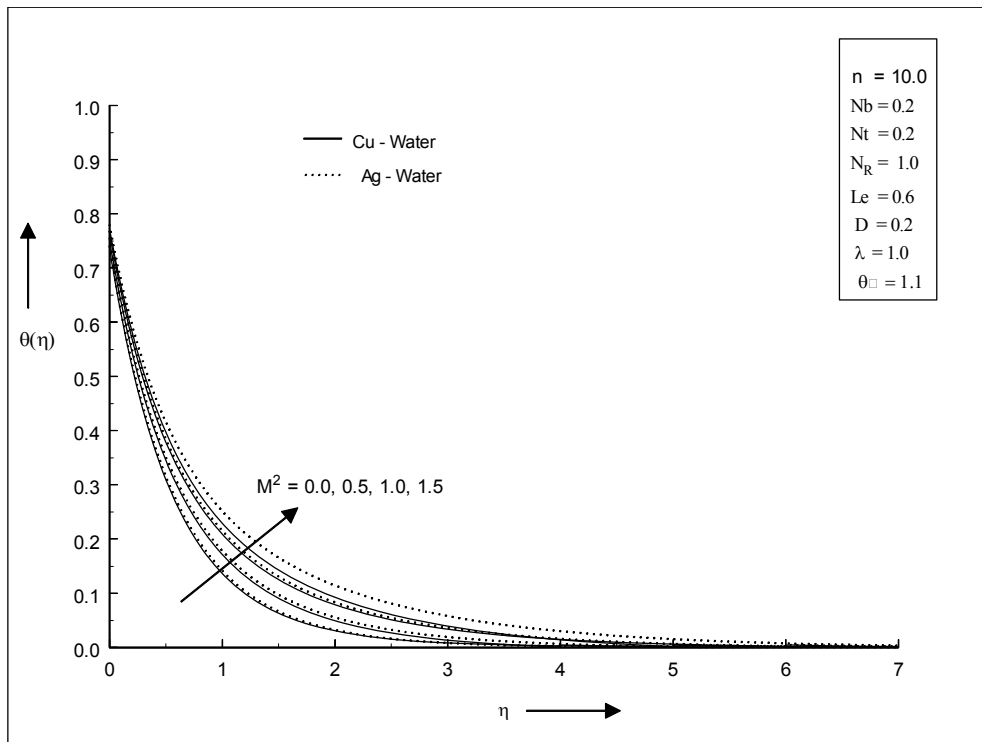


Fig.3. Temperature distribution for various values of  $M^2$ .

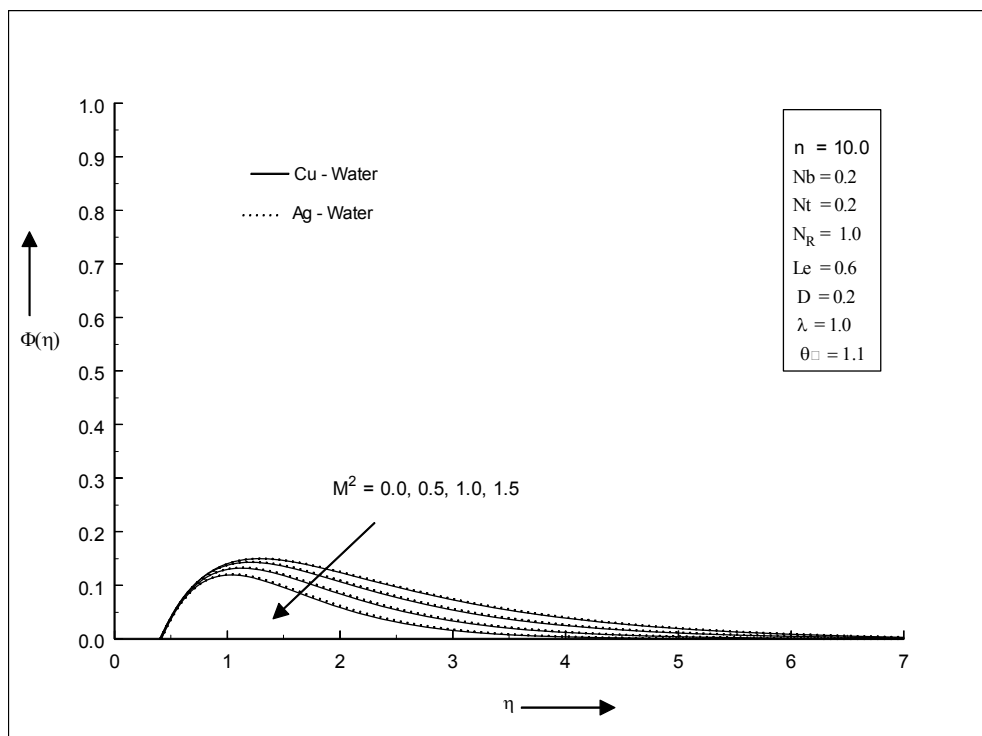


Fig.4. Dimensionless nanoparticle concentration for various values of  $M^2$ .

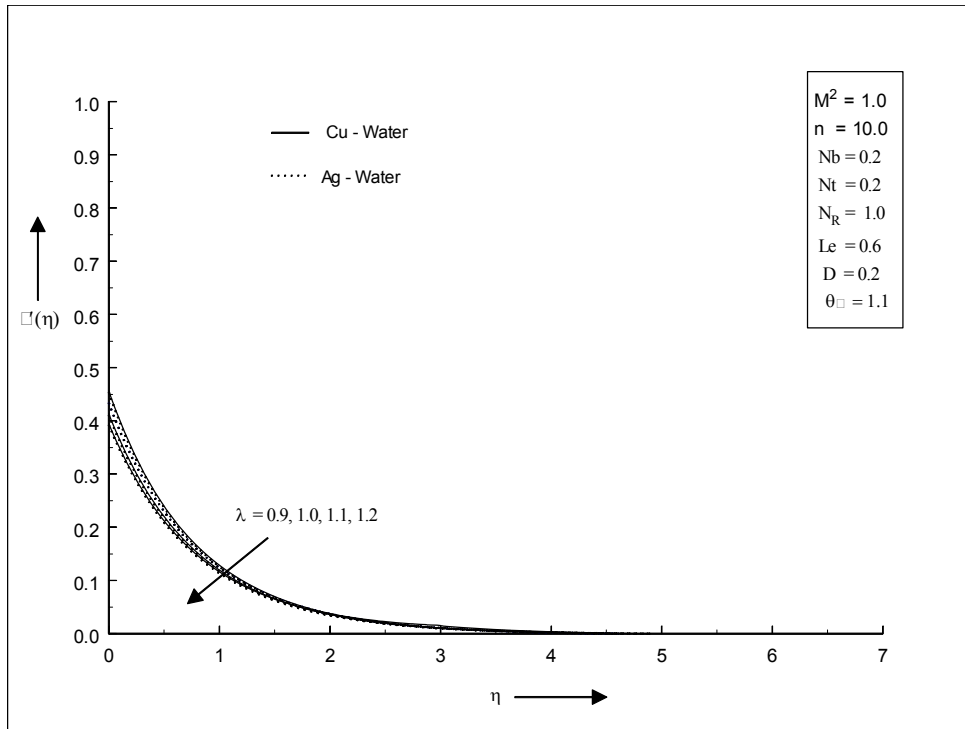


Fig.5. Dimensionless velocity profiles for different  $\lambda$ .

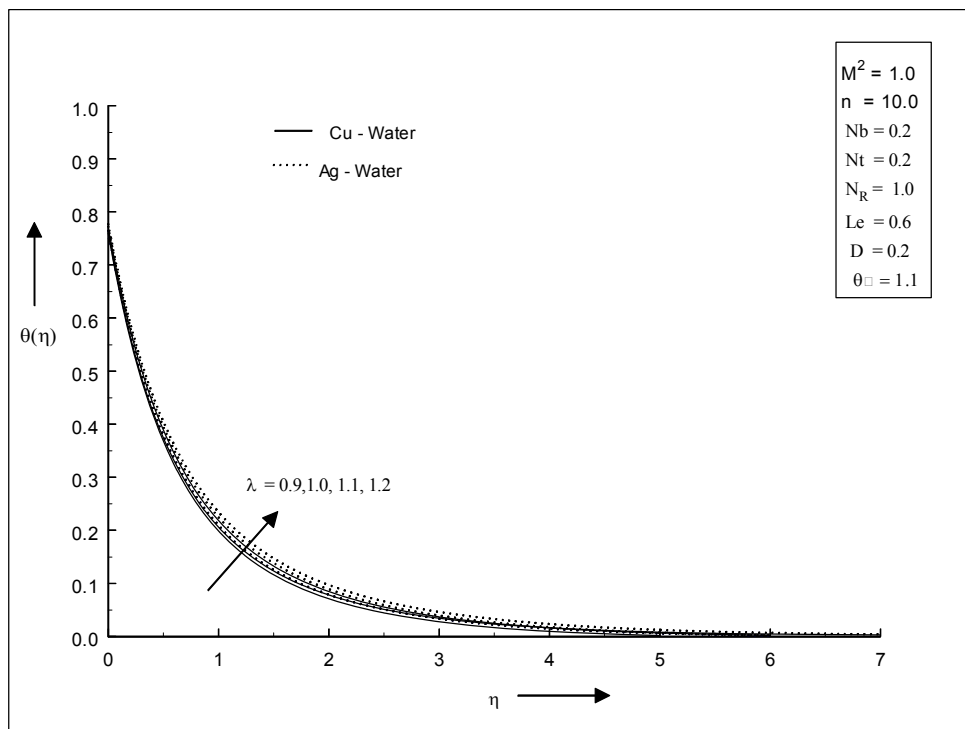


Fig.6. Temperature distribution for different values of slip parameter.

Variation in temperature distribution due to temperature jump parameter are presented in Fig.7. The results show that the temperature jump at the nonlinear stretching surface can deeply affect the temperature distribution so as to reduce it as well as the thermal boundary layer thickness.

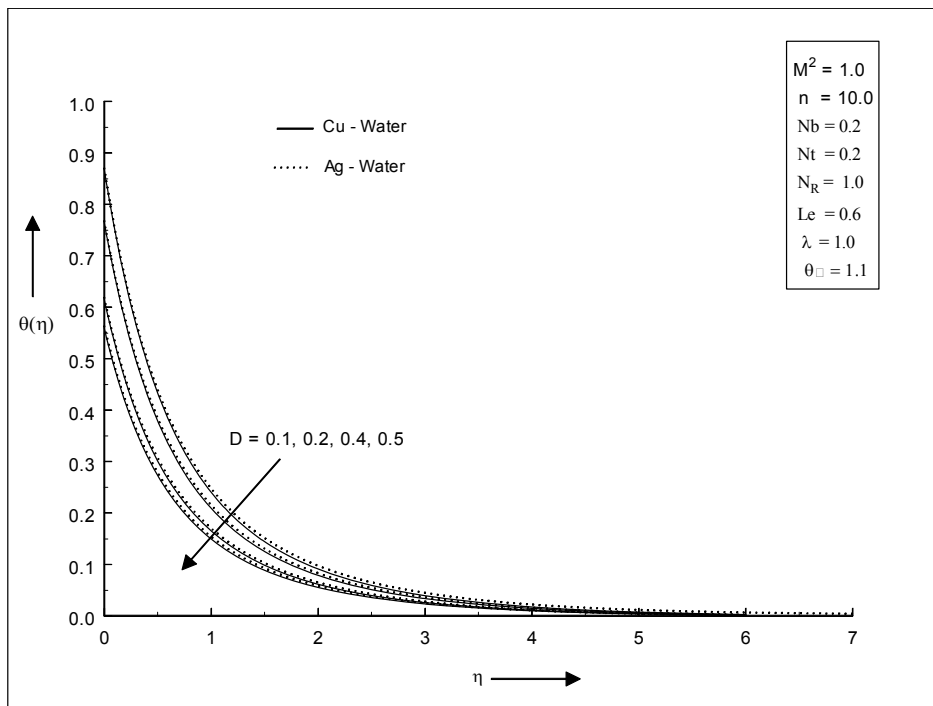


Fig.7. Temperature distribution for temperature jump parameter.

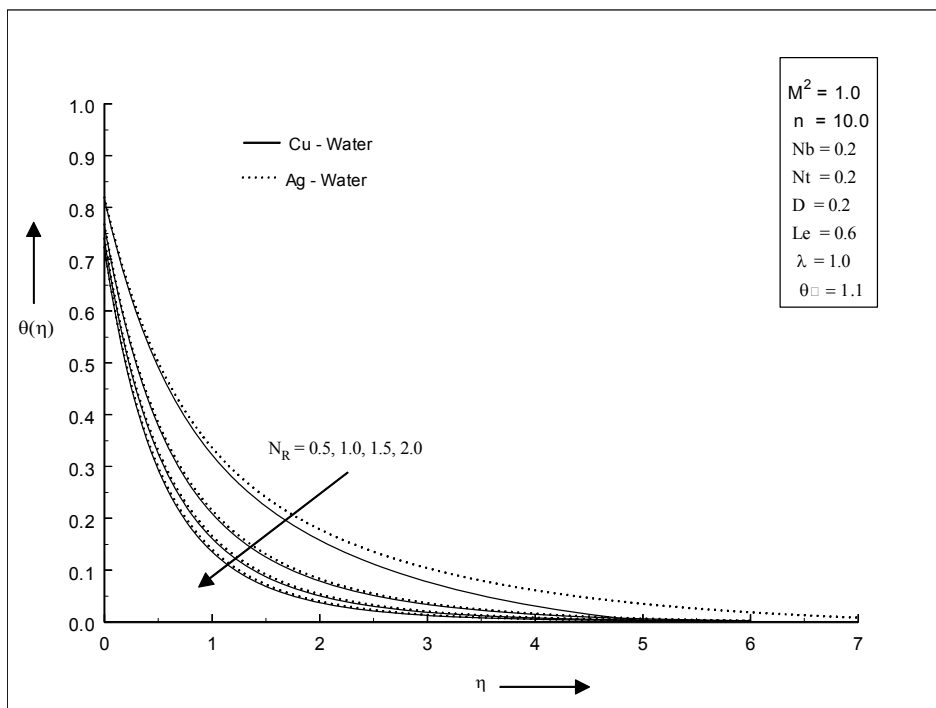


Fig.8. Temperature distribution for various values of  $N_R$ .

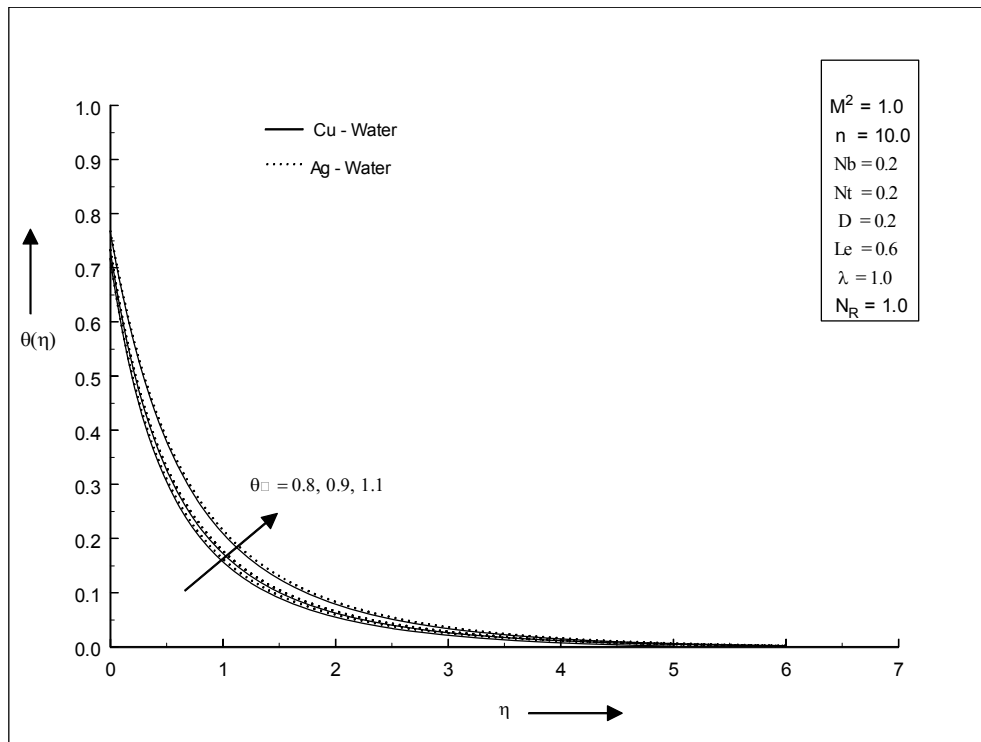


Fig.9. Effect of the temperature ratio parameter on the non-dimensional temperature distribution.

The effect of the nonlinear radiation parameter on the dimensionless temperature distribution is displayed in Fig.8. It is seen that the temperature reduces due to an increase in  $N_R$ , thus leading to a higher heat transfer rate between the nanofluids and the surface. Figure 9 has been plotted to examine the variations of the dimensionless temperature distribution for various values of the temperature ratio parameter  $\theta_w$ . It is interesting to note that increasing values of  $\theta_w$  lead to a rise in the dimensionless temperature and in the thermal boundary layer thickness as well as.

The random motion of nanoparticles suspended in a base fluid (water) resulting from their collision with the quick atoms or molecules in the fluid in micro convection is called a Brownian motion. Effects of a Brownian motion parameter on the dimensionless temperature and nanoparticle volume fraction distribution are plotted in Fig.10 and Fig.11 respectively. It is observed that while the effect of the Brownian motion is dominant over the nanoparticle volume fraction distribution, its effect is less significant over the nondimensional temperature distribution. A further increase in the Brownian motion parameter decreases the nanofluid volume fraction distribution.

Figures 12 and 13 illustrate the effect of thermophoresis parameter on the dimensionless temperature and nanofluid volume fraction distribution. As the values of the thermophoresis parameter increase, both the temperature and nanoparticle volume fraction distribution enhance. In consequence, the thermophoresis parameter enriches the thickness of the thermal boundary layer. Figure 14 demonstrates the dimensionless nanofluid volume fraction distribution for different values of the Lewis number. It reveals the fact that an increase in the Lewis number results in a decrease of the nanofluid volume fraction distribution.

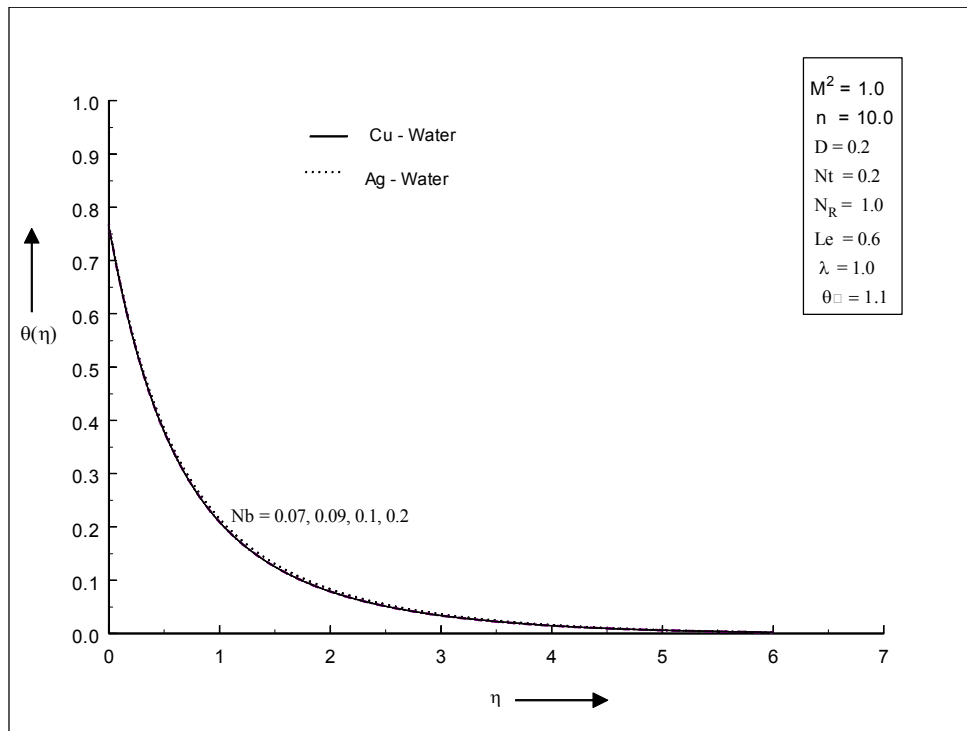


Fig.10. Dimensionless temperature distribution for  $Nb$ .

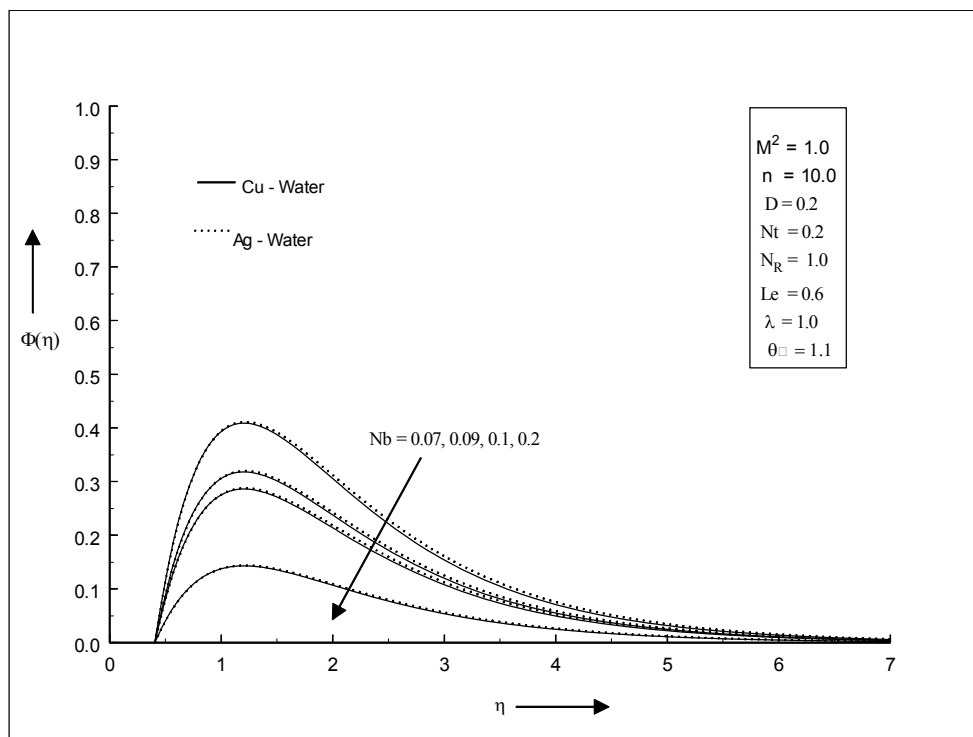


Fig.11. Dimensionless nanoparticle volume fraction distribution for the Brownian motion parameter.

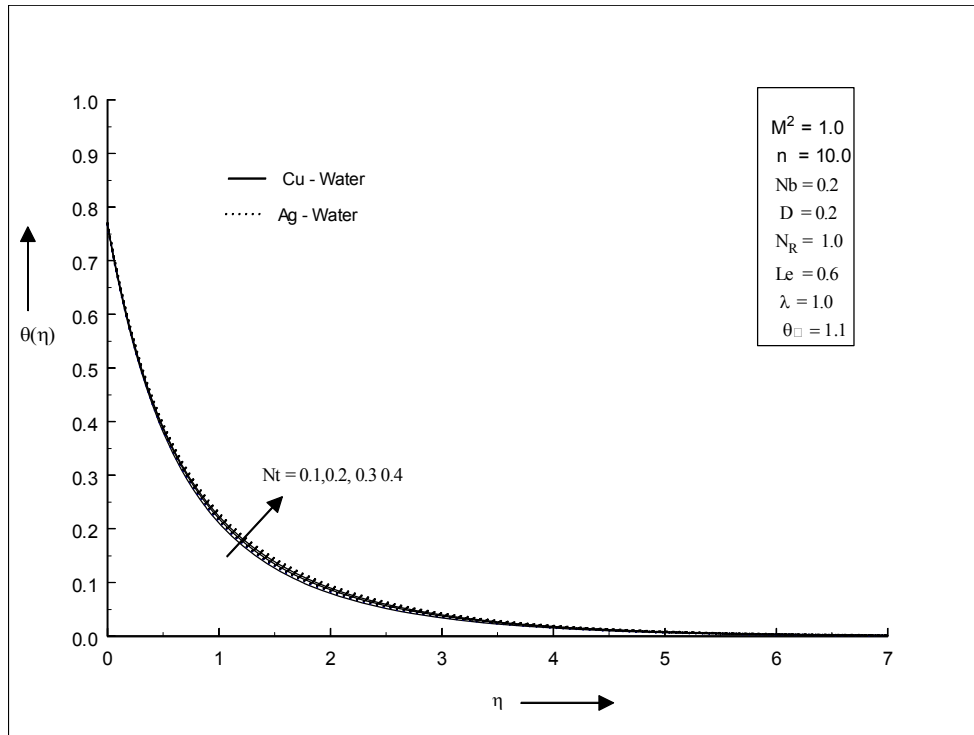


Fig.12. Dimensionless temperature distribution for  $Nt$ .

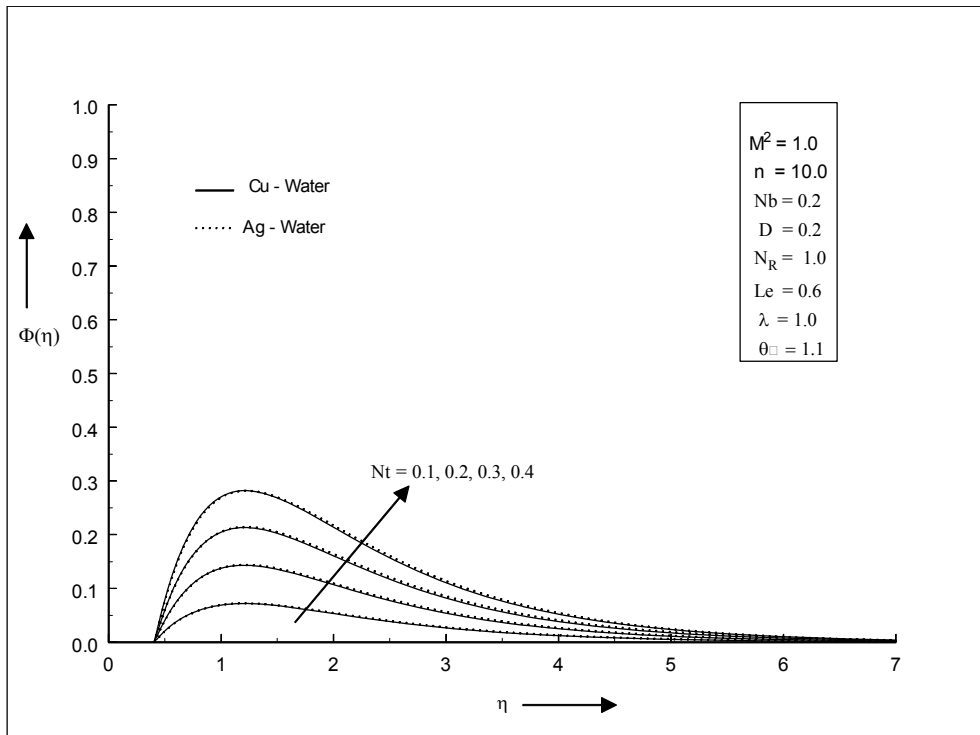


Fig.13. Dimensionless nanoparticles volume fraction distribution for  $Nt$ .

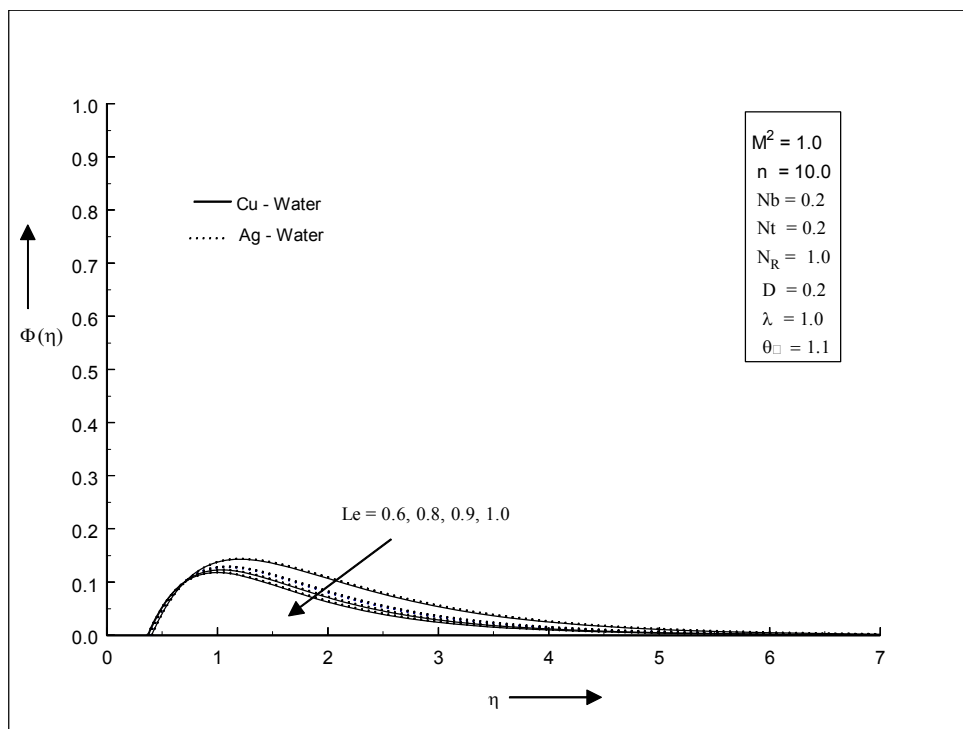


Fig.14. Dimensionless nanoparticle volume fraction distribution for the Lewis number.

Tables 3 and 4 illustrate the skin friction coefficient and nondimensional rate of heat transfer for various values of physical parameters. When the magnetic interaction parameter and slip velocity parameter enhance, the skin friction coefficient reduces whereas the nonlinear stretching parameter is to rise the skin friction coefficient for both Cu – water nanofluid and Ag – water nanofluid as shown in Tab.3.

Table 3. Skin friction coefficient for different values of  $M^2$ ,  $\lambda$  and  $n$ .

$M^2$	$n$	$\lambda$	Cu – H <sub>2</sub> O	Ag – H <sub>2</sub> O
			$-\frac{\sqrt{n+1}}{(1-\phi)^{2.5}} f''(0)$	$-\frac{\sqrt{n+1}}{(1-\phi)^{2.5}} f''(0)$
0.0	5.0	1.0	1.42733	1.41051
0.5			1.30913	1.29852
1.0			1.21813	1.21031
1.5			1.14540	1.13938
1.0	1.0	1.0	0.73078	0.72961
	2.0		0.87734	0.87216
	3.0		1.00363	0.99711
	5.0		1.21813	1.21031
1.0	5.0	0.9	1.28229	1.27418
		1.0	1.21813	1.21031
		1.1	1.16074	1.15325
		1.2	1.09653	1.10204

Table 4. Nondimensional heat transfer rate for different values of  $M^2$ ,  $\lambda$ ,  $n$ ,  $N_R$ ,  $Nb$ ,  $Nt$  and  $\theta_w$  when  $\phi = 0.05$  and  $Pr = 6.2$ .

$M^2$	$n$	$\lambda$	$D$	$N_R$	$\theta_w$	$Nb$	$Nt$	Cu – H <sub>2</sub> O	Ag – H <sub>2</sub> O
								$-\frac{k_{nf}}{k_f}\sqrt{n+1}\left(1+\frac{k_f}{k_{nf}}\frac{4}{3N_R}\theta_w^3\right)\theta(0)$	$-\frac{k_{nf}}{k_f}\sqrt{n+1}\left(1+\frac{k_f}{k_{nf}}\frac{4}{3N_R}\theta_w^3\right)\theta(0)$
0.0	5.0	1.0	0.2	2.0	1.1	0.5	0.5	2.65180	2.66507
0.5								2.70929	2.72022
1.0								2.75967	2.76966
1.5								2.80468	2.81399
2.0	1.0	1.0	0.2	2.0	1.1	0.5	0.5	1.68314	1.68812
	2.0							2.00138	2.00778
	3.0							2.28052	2.30524
	5.0							2.75967	2.76966
2.0	5.0	0.9	0.2	2.0	1.1	0.5	0.5	2.73668	2.74775
		1.0						2.75967	2.76966
		1.1						2.77866	2.78970
		1.2						2.77170	2.80635
1.0	5.0	1.0	0.1	2.0	1.1	0.5	0.5	3.19645	3.20309
			0.2					2.75967	2.76966
			0.4					2.15802	2.17039
			0.5					1.94368	1.95627
2.0	5.0	1.0	0.2	0.5	1.1	0.5	0.5	7.51027	7.53057
				1.0				4.29836	4.31208
				1.5				3.26599	3.27661
				2.0				2.75967	2.76966
2.0	5.0	1.0	0.2	2.0	0.8	0.5	0.5	1.86507	1.87215
					0.9			2.09876	2.10663
					1.1			2.75967	2.76966
					1.2			3.20457	3.21580
2.0	5.0	1.0	0.2	2.0	1.1	0.2	0.5	2.76410	2.77692
						0.3		2.76058	2.77257
						0.4		2.75990	2.77105
						0.5		2.75967	2.76966
2.0	5.0	1.0	0.2	2.0	1.1	0.5	0.2	2.75714	2.76678
							0.3	2.75793	2.76757
							0.5	2.75967	2.76966
							0.6	2.76020	2.77018

From Tab.4, it is noted that the nondimensional rate of heat transfer gets enhanced due to the influence of the magnetic interaction parameter, slip velocity parameter, temperature ratio parameter, nonlinear stretching parameter and thermophoresis parameter for their increasing values, while the effect of the temperature jump parameter, thermal radiation parameter and Brownian motion parameter are to reduce the nondimensional rate of heat transfer. Further, it is interesting to observe that the nondimensional rate of heat transfer for Ag – water nanofluid is greater than that of Cu – water nanofluid.



### 7. Conclusion

The slip effects on a steady hydromagnetic flow of nanofluids over a nonlinearly stretching sheet along with nonlinear thermal radiation, Brownian motion and thermophoresis effects are analysed numerically. The effects of different physical parameters on the flow field, temperature, nanofluid volume fraction distribution, skin friction coefficient and nondimensional rate of heat transfer are obtained and discussed.

The following conclusions can be drawn from the results and discussion.

- The numerical values of the skin friction coefficient and nondimensional rate of heat transfer are identical with those of previously published results.
- For increasing values of  $M^2$  and  $\lambda$  the momentum boundary layer thickness is reduced whereas an opposite trend is noticed in the case of thermal boundary layer thickness. Variation in the volume fraction distribution is less significant due to the influence of the magnetic field.
- The temperature of the nanofluids reduces due to the influence of the thermal slip parameter and thermal radiation parameter. Consequently, an opposite trend occur in the temperature for the increased values of the temperature ratio parameter and thermophoresis parameter. Further, it is noted that the temperature is not affected by the Brownian motion parameter.
- An increment in the thermophoresis parameter enriches the nanofluid volume fraction distribution whereas an opposite trend is observed for rising values of the Brownian motion parameter and Lewis number.
- The skin friction coefficient decreases for increasing values of the magnetic interaction parameter and slip parameter whereas it increases due to the effect of the nonlinear stretching parameter.
- For increasing values of the magnetic interaction parameter, velocity slip parameter, temperature ratio parameter, nonlinear stretching parameter and thermophoresis parameter, the nondimensional rate of heat transfer is found to decrease. An opposite result is observed for the nondimensional rate of heat transfer when the values of the temperature jump parameter, thermal radiation parameter and Brownian motion parameter increase.
- Due to the new type of mass flux boundary condition, the Sherwood number vanishes.

This specific work may find applications in many industrial processes especially in Solar Energy Harvesting using nanofluids.

### Appendix

The expressions for the thermophysical quantities of the nanofluid are given as Ahmad *et al.* [26]

$$\rho_{nf} = (1 - \phi)\rho_f + \phi\rho_s, \quad \mu_{nf} = \frac{\mu_f}{(1 - \phi)^{2.5}}, \quad k_{nf} = k_f \left[ \frac{k_s + 2k_f - 2\phi(k_f - k_s)}{k_s + 2k_f + \phi(k_f - k_s)} \right],$$

$$(\rho c_p)_{nf} = (1 - \phi)(\rho c_p)_f + \phi(\rho c_p)_s.$$

Here  $\phi$  is the solid volume fraction,  $c_p$  is the specific heat at constant pressure,  $\rho_f$  and  $\rho_s$  are the densities of the base fluid and the nanoparticles,  $(\rho c_p)_f$  and  $(\rho c_p)_s$  are the specific heat capacities of the base fluid and the nanoparticles respectively and  $k_f$  and  $k_s$  are the thermal conductivities of the base fluid and the nanoparticles.

The values of the constants are as follows,

$$L_1 = (1 - \phi)^{2.5}, \quad L_2 = \left( (1 - \phi) + \phi \left( \frac{\rho_s}{\rho_f} \right) \right), \quad L_3 = \left( (1 - \phi) + \phi \left( \frac{(\rho c_p)_s}{(\rho c_p)_f} \right) \right).$$

## Nomenclature

- $B_0$  – magnetic induction  
 $c$  – stretching coefficient  
 $c_p$  – specific heat due to constant pressure  
 $(c_p)_{nf}$  – heat capacity of the nanofluid  
 $D$  – temperature jump parameter  
 $D_B$  – Brownian diffusion coefficient  
 $D_T$  – thermophoretic diffusion coefficient  
 $Ec$  – Eckert number  
 $f$  – dimensionless stream function  
 $k^*$  – Rosseland mean absorption coefficient  
 $Le$  – Lewis number  
 $M^2$  – magnetic field parameter  
 $n$  – nonlinear stretching parameter  
 $Nb$  – Brownian motion parameter  
 $Nt$  – thermophoresis parameter  
 $Nu_x$  – local Nusselt number  
 $Pr$  – Prandtl number  
 $q_r$  – radiative heat flux  
 $Re_x$  – local Reynolds number  
 $T$  – temperature of the nanofluid within the boundary layer  
 $T_0$  – temperature of the fluid below the surface  
 $T_w$  – temperature at the surface of the sheet  
 $T_\infty$  – temperature of the ambient fluid  
 $u$  – velocity along the surface of the sheet  
 $u_s$  – velocity slip  
 $T_s$  – thermal jump  
 $v$  – velocity normal to the surface of the sheet  
 $(x, y)$  – Cartesian coordinates  
 $\alpha_{nf}$  – thermal diffusivity of the nanofluid  
 $\eta$  – similarity variable  
 $\theta$  – dimensionless temperature  
 $\theta_w$  – surface wall temperature  
 $\kappa_{nf}$  – thermal conductivity of the nanofluid  
 $\lambda$  – velocity slip parameter  
 $\mu_{nf}$  – viscosity of the nanofluid  
 $\rho_{nf}$  – density of the nanofluid  
 $\sigma$  – electrical conductivity  
 $\sigma^*$  – Stefan-Boltzmann constant  
 $\tau$  – nanoparticle heat capacity ratio  
 $\nu_{nf}$  – kinematic viscosity of the nanofluid  
 $\varphi$  – dimensionless rescaled nanoparticle volume fraction  
 $\psi$  – stream function

## Subscripts

- $w$  – surface conditions  
 $\infty$  – conditions far away from the surface

## Superscripts

- $'$  – differentiation with respect to  $\eta$

## References

- [1] Chaim C.(1995): *Hydromagnetic flow over a surface stretching with a power law velocity.* – International Journal of Engineering Science, vol.33, No.3, pp.429-435.
- [2] Sakiadis B.C. (1961): *Boundary layer behaviour on continuous solid surfaces: II boundary layer on a continuous flat surface.* – AIChE J., vol.7, pp.221–225.
- [3] Choi S. (1995): *Enhancing thermal conductivity of fluids with nanoparticles.* – In: Sidiner D.A., Wang H.P., (eds) *Developments and applications of non-Newtonian flows.* – ASMEFED, 231/MD, pp.99-105.
- [4] Lee S., Choi S., Li S. and Eastman J.A. (1999): *Measuring thermal conductivity of fluids containing oxide nanoparticles.* – Journal of Heat Transfer, vol.121, No.2, pp.280-289.
- [5] Wang X., Xu .X. and Choi S. (1999): *Thermal conductivity of nanoparticle-fluid mixture.* – Journal of Thermophysics and Heat Transfer, vol.13, pp.474–480.
- [6] Xuan Y. and Li Q. (2000): *Heat transfer enhancement of nanofluids.* – International Journal of Heat and Fluid Flow, vol.21, No.1, pp.58–64.
- [7] Yu W. and Choi S.U.S. (2003): *The role of interfacial layers in the enhanced thermal conductivity of nanofluids: a renovated Maxwell model.* – J. Nanopart. Res., vol.5, pp.167-171.
- [8] Buongiorno J. (2006): *Convective transport in nanofluids.* – ASME Journal of Heat Transfer, vol.128, No.3, pp.240-250.
- [9] Das S.K., Choi S., Yu W. and Pradeep T. (2007): *Conduction Heat Transfer in Nanofluids.* – Science and Technology (Hoboken, NJ: Wiley).
- [10] Yuwen Zhang, Ling Li, Ma H.B. and Mo Yang (2009): *Effect of Brownian and thermophoretic diffusions of nanoparticles on nonequilibrium heat conduction in a nanofluid layer with periodic heat flux.* – Numerical Heat Transfer Part A 56, pp.325-341.
- [11] Vajravelu K., Prasad K.V., Jinho Lee, Changhoon Lee, Pop I., Robert A. and Van Gorder (2011): *Convective heat transfer in the flow of viscous Ag–water and Cu– water nanofluids over a stretching surface.* – Int. J. Therm. Sci., vol.50, pp.843–851.
- [12] Hamad M. and Ferdows M. (2012): *Similarity solutions to viscous flow and heat transfer of nanofluid over nonlinearly stretching sheet.* – Applied Mathematics & Mechanics, vol.33, No.7, pp.923.
- [13] Vikrant Khullar and Himanshu Tyagi (2012): *Solar energy harvesting using nanofluids-based concentrating solar collector.* – ASME Journal of Nanotechnology in Engineering and Medicine, vol.3, No.3.
- [14] Mukhopadhyay S. (2013): *Slip effects on MHD boundary layer flow over an exponentially stretching sheet with suction/blowing and thermal radiation.* – Ain Shams Engineering Journal, vol.4, pp.485-491.
- [15] Sohail Nadeem, Rizwan Ul Haq and Zafar Hayat Khan (2014): *Heat transfer analysis of water-based nanofluid over an exponentially stretching sheet.* – Alexandria Engineering Journal, vol.53, pp.219-224.
- [16] Md. Shakhaoath Khan, Md. Mahmud Alam and Ferdows M. (2011): *MHD radiative boundary layer nanofluid flow past a stretching sheet.* – In: Proceedings of the International Conference on Mechanical Engineering and Renewable Energy,(ICMERE2011), Chittagong, Bangladesh, pp.22-24.
- [17] Mohammad Mehdi Keshtkar and Babak Amiri (2013): *MHD Flow and heat transfer nanofluid over a permeable stretching sheet.* – International Journal of Engineering and Innovative Technology (IJEIT), vol.3, No.3.
- [18] Malvandi, Hedayati F. and Ganji D.D. (2014): *Slip effects on unsteady stagnation point flow of a nanofluid over a stretching sheet.* – Powder Technology, vol.253, pp.377-384.
- [19] Shateyi S. and Prakash J. (2014): *A new numerical approach for MHD laminar boundary layer flow and heat transfer of nanofluids over a moving surface in the presence of thermal radiation.* – Boundary Value Problems, vol.2.

- [20] Krishnamurthy M.R., Prasannakumara B.C., Gireesha B.J. and Gorla R.S.R. (2015): *Effect of viscous dissipation on hydromagnetic fluid flow and heat transfer of nanofluid over an exponentially stretching sheet with fluid-particle suspension*. – Applied and Interdisciplinary Mathematics, vol.2, pp.1-18.
- [21] Falana A., Ojewale O.A., Adeboje T.B. (2016): *Effect of Brownian motion and thermophoresis on a nonlinearly stretching permeable sheet in a nanofluid*. – Advances in Nanoparticles, vol.5, pp.123-134.
- [22] Nield D.A. and Kuznetsov A.V. (2014): *Thermal instability in a porous medium layer Saturated by a nanofluid: A revised model*. – Int. J. Heat Mass Transfer, vol.68, pp.211-214.
- [23] Rosseland S. (1936): *Theoretical Astrophysics*. – Oxford: Clarendon Press.
- [24] Anjali Devi S.P. and Mekala S. (2015): *Thermal radiation effects on hydromagnetic flow of nanofluid over a nonlinearly stretching sheet in the existence of variable heat generation and viscous dissipation*. – Int. J. of Science and Research, vol.4, No.3, pp.935-943.
- [25] Cortell R. (2007): *Viscous flow and heat transfer over a nonlinearly stretching sheet*. – Applied Maths and Computation, vol.184, pp.864-873.
- [26] Ahmad S., Rohni A.M. and Pop I. (2011): *Blasious and Sakiadis problems in nanofluids*. – Acta Mechanica, vol.218, pp.195-204.

Received: July 4, 2018

Revised: November 1, 2018