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NANOFLUID MOTION PAST A SHRINKING SHEET IN POROUS MEDIA UNDER THE IMPACT OF RADIATION AND HEAT SOURCE/SINK

R.P. SHARMA*
Department of Basic & Applied Science, National Institute of Technology
Yupia, Papum Pare District, Arunachal Pradesh-791112, INDIA
E-mail: rpsharma@nitap.ac.in

A.K. JHA
Department of Mathematics, C M SCIENCE College Darbhanga (A constituent uniot of L N Mithila University Darbhanga)
Bihar-846004, INDIA

P.K. GAUR
Department of Mathematics, JECRC University
Jaipur-303905, INDIA

S.R. MISHRA
Siksha 'O' Anusandhan Deemed to be University
Khandagiri, Bhubaneswar-751030, Odisha, INDIA

An investigation has been carried out for the MHD 3-dimensional flow of nanofluid over a shrinking sheet saturating a porous media in the presence of thermal radiation and heat generation. Convective boundary conditions for the flow phenomena are used in the present analysis. The governing equations are reduced to ODEs employing suitable similarity transformations. The solutions of formulated differential equations have been attained mathematically by fourth order R-K technique along with the shooting method. The impact of the governing constraints on momentum, heat, and local Nusselt number, are explored. It is noticed that the momentum and heat decrease with raise in the porosity variable, temperature reduces with an enhance in the thermal radiation variable, and temperature enhances with an enhance in the heat source/sink parameter.

Key words: MHD, nanofluid, shrinking surface, thermal radiation, heat generation, porous medium, convective conditions.

1. Introduction

Nanofluid is illustrated as a liquid in which hard nanoparticles amid the measurement lengthwise sizes of 1-100 nm are suspended in a traditional heat transport basic fluid. Ethylene glycol, oil and water have low thermal conductivity and are known as conventional heat transfer fluids. By adding solid nanoparticles to conventional fluids the thermal conductivity increases. A very small quantity of nanoparticles, when scattered stably in the base fluid, provides significant improvements in the thermal properties of the base fluid. Nanoparticle fluid suspension (Nanofluid) is the name invented by Choi [1] to mark out this novel theory of a nanotechnology-based energy transport fluid that shows heat properties higher than those of the base fluid. Nanoparticles utilized in nanofluids are prepared of different materials, such as metals (Au, Ag, Cu), carbide ceramics (SiC, TiC), nitride ceramics (SiN, AlN), oxide ceramics

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* To whom correspondence should be addressed

(CuO, Al_2O_3) and carbon nanotubes. The aim of nanofluids is to get maximum feasible heat properties at the least feasible masses (in preference less than *1*% by volume) by constant scattering and firm deferment of nanoparticles in foundation liquids. To attain this be determined, it is essential to know how nanoparticles get better the heat transfer in fluids. In current years, a number of interests have been specified to convective transport of nanofluids. Extensive reviews on thermal conductivity of nanofluids have been published by some researchers (Eastman *et al.* [2]; Choi *et al.* [3]; Das *et al.*[4]; Wang and Majumdar [5, 6]; Kakac and Pramuanjaroenki [7]; Ho *et al.* [8]). An analysis of water based nanofluids in different physical conditions have presented by (Elif [9]; Salem *et al.*[10]; Sheikholeslami *et al.* [11, 12]).The motion and energy transport of a nanofluid past a extending sheet was reported by (Xu *et al.* [13]; Sheikholeslami *et al.* [14]; Ramzan and Yousaf [15]).

The investigation of fluid motion with magnetic field is significant in the polymer manufacturing, metallurgy, engineering, physics, and chemistry, etc. Also, the MHD flow plays a vital role in problems related to blood plasma, blood pump machines and physiological fluids. An electrically conducting liquid dependent on a magnetic field is practical in calculating the rate of cooling. The electro-conducting liquid has been growingly utilized in the manufacturing processes of semiconducting materials for instance silicon crystal gallium arsenide. The effect of the magnetic field could be very helpful in the modernization of technological processes. In consequence of their various significance these motions have been reported by numerous researchers, remarkable amongst them are (Turkyilmazoglu [16]; Hamad [17]; Sheikholesla [18]; Ibrahim and Makinde [19]).

Radiation is the power that arrives starting a resource and travels through some objects or throughout space. Sound, energy, and light are the kinds of radiation. The thermal radiation impact may cooperate a important task in calculating energy transfer procedure in the polymer processing engineering and in many manufacturing processes for instance solar power technology, astrophysical flows, fossil fuel combustion energy processes, gas turbines, the different impulsion devices for missiles, aircraft, satellites, missiles. Several authors reported the radiation impact on heat transport of a nanofluid in different physical conditions, notable amongst them are (Hady *et al.* [20]; Nadeem and Hag [21], [22]; Turkyilmazoglu and Pop [23]; Hsiao [24]; Ramzan and Bilal [25]; Hayat *et al.* [26]; Hag *et al.* [27]).

Rahman *et al.* [28] studied heat generation and slip influences on an MHD motion of H_2o based nanofluids in a wedge. The impact of energy generation and radiation on assorted convective flow of a nanofluid over a non-linear extending surface was reported by Lakshmi and Reddy [29]. Malvandi *et al.* [30] analyzed the heat generation impact on the stagnation-point motion of a nanofluid past a extending surface through porous media. Hayat *et al.* [31] recorded a 3- dimensional motion of a nanofluid over a extending sheet in the presence of a heat source/sink and thermal radiation.

The flow of fluids through a porous medium is of great importance in energy elimination from nuclear energy garbage, alternative removal of radiative devastate material, cargo space of food stuff and oil exploration. A porous shrinking sheet, suction/inoculation of a liquid be able to significantly alter the motion field. inoculation of liquid via a permeable shrinking surface is of ordinary alarm in several purposes which engage boundary film be in command of purposes. These consist of outside layer of wires, silver screen cooling, polymer fiber covering. Alternatively, injection performs in the reverse way. Suction is useful to compound process to eradicate reactants. Injection is second-hand to insert reactants, prevent corrosion, reduce the drag and cool the surface and also suction or injection is significant in production activities for instance in the drawing of bearings, diffusers and oil recovery. Some authors have analyzed flow of a nanofluid through porous media in different physical conditions, notable amongst them are (Kahar *et al.* [32]; Chamkha and Ahmed [33]; Kuznetsov and Nield [34]; Sheikholeslami and Ganji [35]; Nandy and Pop [36]). Recently, Ramzan [37] and Hayat *et al.* [38] investigated a three dimensional flow of a nanofluid over a stretching sheet.

2. Formulation of the problem

Three-dimensional radiative nanofluid motion over a contracting sheet by a porous media is investigated in the present paper. The energy transport occurrence is enhanced by incorporating heat source/sink. The convective thermal boundary situation is also considered which affects the entire flow phenomena. An invariant magnetic pasture of magnitude B_0 is useful which is parallel to the z-axis. Due to low magnetic Reynolds number, the effects of induced magnetic field as well as electric field are neglected. Underneath such suppositions the governing equations for the current problem with their corresponding boundary conditions are

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0\,, \tag{2.1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}=\nu_{nf}\frac{\partial^2 u}{\partial z^2}-\frac{\sigma B_0^2}{\rho_{nf}}u-\frac{\nu_{nf}}{K}u\,, \tag{2.2}$$

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}=\nu_{nf}\frac{\partial^2 v}{\partial z^2}-\frac{\sigma B_0^2}{\rho_{nf}}v-\frac{\nu_{nf}}{K}v\,, \tag{2.3}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}+w\frac{\partial T}{\partial z}=\alpha_{nf}\frac{\partial^2 T}{\partial z^2}-\frac{1}{\left(\rho C_p\right)_{nf}}\frac{\partial q_r}{\partial z}+\frac{Q}{\left(\rho C_p\right)_{nf}}\left(T-T_\infty\right), \tag{2.4}$$

$$\begin{gathered}u=cx,\quad v=c\left(m_2-1\right)y,\quad w=-W,\quad -k_f\frac{\partial T}{\partial z}=h_1\left(T_f-T\right)\quad \text{at}\quad z=0,\\ u=0,\quad v=0,\quad T\to T_\infty\quad \text{at}\quad z\to\infty\,,\end{gathered} \tag{2.5}$$

here W, the suction, $c<0$, shrinking rate and h_1 convective heat transfer coefficient. It be interesting to recorded that for $m_2=0$, the surface contracts along x-direction only and for $m_2=2$ the surface contracts axis-symmetrically (Zheng *et al.* [39]).

$$\alpha_{nf}=\frac{k_{nf}}{\left(\rho C_p\right)_{nf}}, \tag{2.6}$$

$$\alpha_{nf}=\rho_f(1-\varphi)+\rho_s\varphi, \tag{2.7}$$

$$\mu_{nf}=\frac{\mu_f}{\left(1-\varphi\right)^{2.5}}, \tag{2.8}$$

$$(\rho C_p)_{nf}=(\rho C_p)_f(1-\varphi)+(\rho C_p)_s\varphi\,, \tag{2.9}$$

$$\frac{k_{nf}}{k_f}=\frac{k_s+2k_f-2\varphi(k_f-k_s)}{k_s+2k_f+2\varphi(k_f-k_s)} \tag{2.10}$$

here, α_{nf}, $(\rho C_p)_{nf}$, k_{nf}, ρ_{nf}, and μ_{nf} are the efficient thermophysical properties of nanofluid defined as thermal diffusivity, heat capacitance, thermal conductivity, density and dynamic viscosity respectively (Kakac [7]).

Employing Rosseland's approximation for radiation [40, 41], we get

$$q_r = -\left(\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial z}\right). \tag{2.11}$$

Here, σ^* be the Stefan–Boltzmann constant, α^* be the absorption coefficient of the nanofluid. More, we considered that the heat variation within the motion is such that T^4 might be expanded in a Taylor series. Hence, expanding T^4 about T_∞ and neglecting high order expressions, we obtain

$$T^4 \cong 4T_\infty{}^3 T - 3T_\infty{}^4 . \tag{2.12}$$

Using Eqs (2.11) and (2.12) in the energy Eq.(2.4) we obtain

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} + w\frac{\partial T}{\partial z} = \alpha_{nf}\left(\frac{3N_R + 4}{3N_R}\right)\frac{\partial^2 T}{\partial z^2} + \frac{Q}{(\rho C_p)_{nf}}\left(T - T_\infty\right) \tag{2.13}$$

where $N_R = \dfrac{k_{nf}k^*}{4\sigma^* T_\infty{}^3}$ is the radiation parameter.

At present establish the subsequent dimensionless parameters and similarity transformations [39]

$$\theta(\eta) = \frac{T - T_\infty}{T_f - T_\infty}, \quad w = -\sqrt{b\upsilon_f}\, m_2 f(\eta), \quad u = bx f'(\eta), \quad v = b(m_2 - 1)y f'(\eta), \quad \eta = \sqrt{\frac{b}{\upsilon_f}}\, z\,,$$

$$M_1 = \frac{1}{(1-\varphi)^{2.5}\left[1 - \varphi + \varphi\dfrac{\rho_s}{\rho_f}\right]} \text{ (Constant related to properties of the nanofluid),}$$

$$M_2 = \frac{1}{(1-\varphi)^{2.5}\left[1 - \varphi + \varphi\dfrac{(\rho C)_s}{(\rho C)_f}\right]} \text{ (Constant related to properties of the nanofluid),}$$

$$M_3 = \frac{k_{nf}}{k_f} = \frac{k_s + 2k_f - 2\varphi(k_f - k_s)}{k_s + 2k_f - \varphi(k_f - k_s)} \text{ (Constant related to properties of the nanofluid),}$$

$k_2 = \left(\dfrac{3N_R + 4}{3N_R}\right)$ (constant), $N_R = \dfrac{kk^*}{4\sigma^* T_\infty{}^3}$ (Radiation parameter), $\delta = c / b$ (shrinking parameter), $\kappa = \dfrac{h_1}{k}\sqrt{\dfrac{\upsilon}{b}}$ (Biot number), $Q = \dfrac{Q_0}{b(\rho C_p)_f}$ (heat source parameter),

$$S=\frac{W}{\sqrt{b\upsilon_f m_2}}\,\text{(suction/injection parameter)},\ \ \varsigma=\frac{\alpha_f}{bK}\,\text{(permeability parameter)},$$

$$\Pr=\frac{\upsilon_f\,(\rho C_p)_f}{k_f}\,\text{(Prandtl number) and }\ \mathrm{M}=\frac{\sigma B_0^{\,2}}{\rho_f\, b}\quad\text{(magnetic parameter)},\qquad(2.14)$$

where the prime denotes differentiation with respect to η, using Eq.(2.14), Eq.(2.1) is identically satisfied, and substituting into Eqs (2.2), (2.3) and (2.13) we obtain

$$M_1 f''' + m_2\, f\, f'' - f'^2 - MM_1(1-\varphi)^{2.5} f' - \Pr\varsigma M_1 f' = 0\,, \qquad(2.15)$$

$$\frac{k_2 M_2 M_3}{\Pr}\left(1-\varphi\right)^{2.5}\theta'' + m_2\, f\theta' + QM_2\left(1-\varphi\right)^{2.5}\theta = 0\,. \qquad(2.16)$$

The boundary conditions (2.5) reduce to

$$\begin{aligned}&f(0)=S,\qquad f'(0)=\delta,\qquad \theta'(0)=-\kappa\left[1-\theta(0)\right],\\&f'(\infty)\to 0,\qquad \theta(\infty)\to 0\,.\end{aligned}\qquad(2.17)$$

3. Method of Solution

Equations (2.15) and (2.16) are resolved with boundary conditions (2.17) by employing fourth order R-K technique along with the shooting method. We 1st reduce the Eqs (2.15) and (2.16) into 1st order DEs by pertaining

$$S_1 = f,\qquad S_2 = f',\qquad S_3 = f'',\qquad S_4 = \theta,\qquad S_5 = \theta'\,.$$

Thus we get

$$\begin{aligned}
&S_1' = S_2,\\
&S_2' = S_3,\\
&S_3' = -\frac{m_2}{M_1}S_1 S_3 + \frac{S_2^{\,2}}{M_1} + M\left(1-\varphi\right)^{2.5} S_2 + \Pr\varsigma S_2,\\
&S_4' = S_5,\\
&S_5' = \left[\left(\frac{k_2\Pr}{M_3}\right)\left\{-m_2\frac{S_1 S_5}{M_2\left(1-\varphi\right)^{2.5}} - QS_4\right\}\right].
\end{aligned}\qquad(3.1)$$

The corresponding initial conditions are

$$S_1 = S, \quad S_2 = \delta, \quad S_3 = u_1, \quad S_4 = u_2, \quad S_5 = -\kappa\left[1-u_2\right]. \tag{3.2}$$

Equations (3.1) and (3.2), explaind mathematically by employing 4th - order R-K technique along with the shooting method. In Eq.(3.2), u_1 and u_2 be unknown which are to be established in the mathematical result and the outcomes are demonstrated in figures and tables.

The local Nusselt number be describe like under

$$\mathrm{Nu} = \frac{x q_w}{k_f (T_f - T_\infty)}. \tag{3.3}$$

The sheet energy flux q_w be describe like under

$$q_w = k_{nf} \left.\frac{\partial \theta}{\partial Z}\right|_{Z=0}. \tag{3.4}$$

From Eqs (2.14), (3.3) and (3.4) we obtain.

$$\mathrm{Nu}\,\mathrm{Re}_x^{-\frac{1}{2}} = M_3 \theta'(0) \tag{3.5}$$

where $\mathrm{Re}_x = \dfrac{u_w x}{\upsilon_f}$ (local Reynolds number)

4. Results and discussion

In order to study the nature of the velocity distribution, heat transfer and Nu for Cu-$\mathrm{H_2o}$ based nanofluid, mathematical results be carried out for the different values of $\varsigma, \delta, \varphi, M$, S, N_R, Q and κ which be recorded in graphs and the outcomes be examined graphically. In this chapter, we use the thermo physical properties of water and nanoparticles as given in Tab.1.

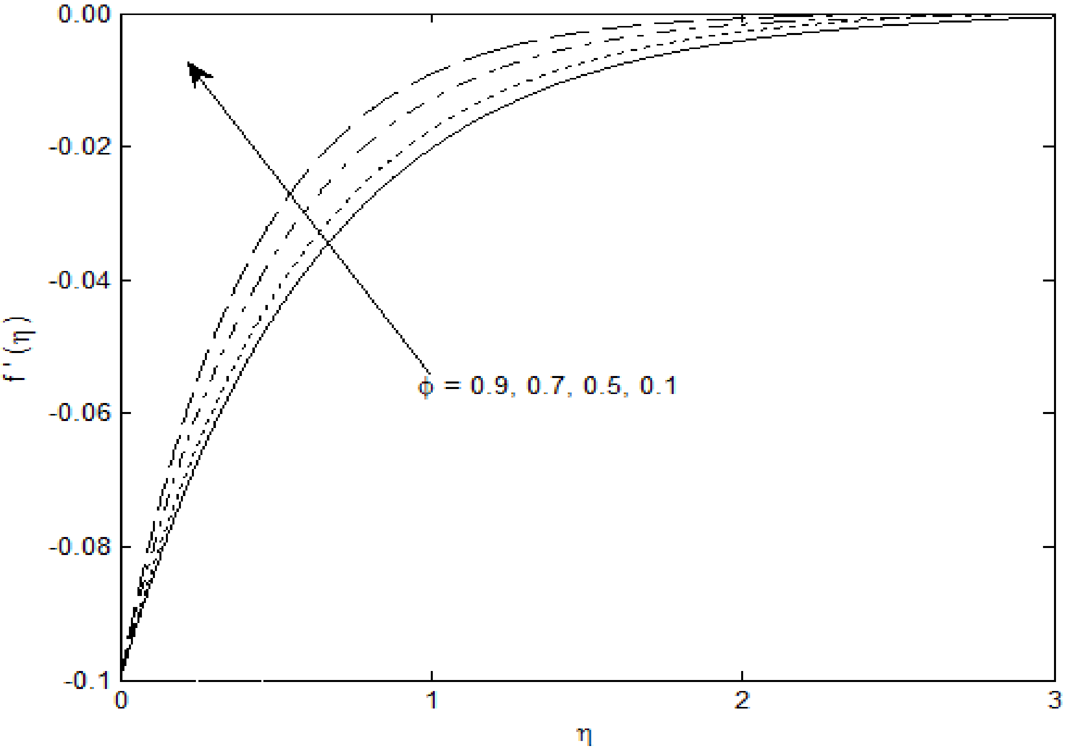
Table 1. Thermo physical characteristics of H_2O and nano particles.

	$\rho\left(Kg/m^3\right)$	$Cp(j/kgk)$	$k(W/m.k)$	$\beta \times 10^5\left(K^{-1}\right)$
Water $(\mathrm{H_2O})$	997.1	4179	0.613	21
Silver (Ag)	10500	235	429	1.89
Copper (Cu)	8933	385	40	1.67
Titanium oxide ($\mathrm{TiO_2}$)	4250	686.2	8.9538	0.9
Alumina ($\mathrm{Al_2O_3}$)	3970	765	40	0.85

Figure 1 depicts the influence of nanoparticle volume fraction on the momentum. It be obvious that, as the ϕ rises, the velocity diminishes because when the volume of nanoparticles increases, the k_{nf} grows, and the thickness of the velocity boundary layer decreases. Figures 2-5 depict the impact of the shrinking parameter, porosity parameter, M and S on the velocity, respectively. It is apparent that, as the magnitude of such parameters increases, the velocity diminishes. The reason is that the growing value of these parameters decreases the velocity boundary layer thickness.

Figure 6 depicts the impact of ϕ on the energy profile. It is surveyed that, as the ϕ grows, the temperature upsurges because while the volume of nanoparticles rises, the k_{nf} upsurges and the width of the thermal boundary layer lessens. Figures 7 and 8 illustrate the influence of the porosity variable and shrinking parameter on the heat description, respectively. It is clear that, for the growing value of the porosity parameter and shrinking parameter the heat description diminishes. Figure 9 illustrates the effect of heat source/sink variable on the energy description. It is observed that for the growing value of source/sink variable the energy profile upsurges and it be obvious that the energy in the case of heat source is superior than in the case of sink. The influence of the N_R on the energy description is showed in Fig.10. It is noted that the thermal radiation yields a diminution in the temperature profile because the thermal radiation parameter diminishes the thickness of the thermal boundary layer. The effect of the κ and S on the heat profile are exposed in Figs 11-12. It be noticed that the heat grows as κ enhances while it diminishes as S increases. Figure 13 depicts the effect of the magnetic variable on the heat profile. It be observed that the heat profile upsurges as M increases. The reason is that the Lorentz force resists the motion of fluid, thus heat is produced and therefore the width of the thermal boundary layer upsurges.

From Tab.2, we observed that the value of $\mathrm{Nu}\left(\mathrm{Re}_x\right)^{-\frac{1}{2}}$ upsurges for the growing value of the κ, ϕ and M magnetic parameter.



S

Fig.1. Velocity description for different values of the nanoparticles volume fraction.

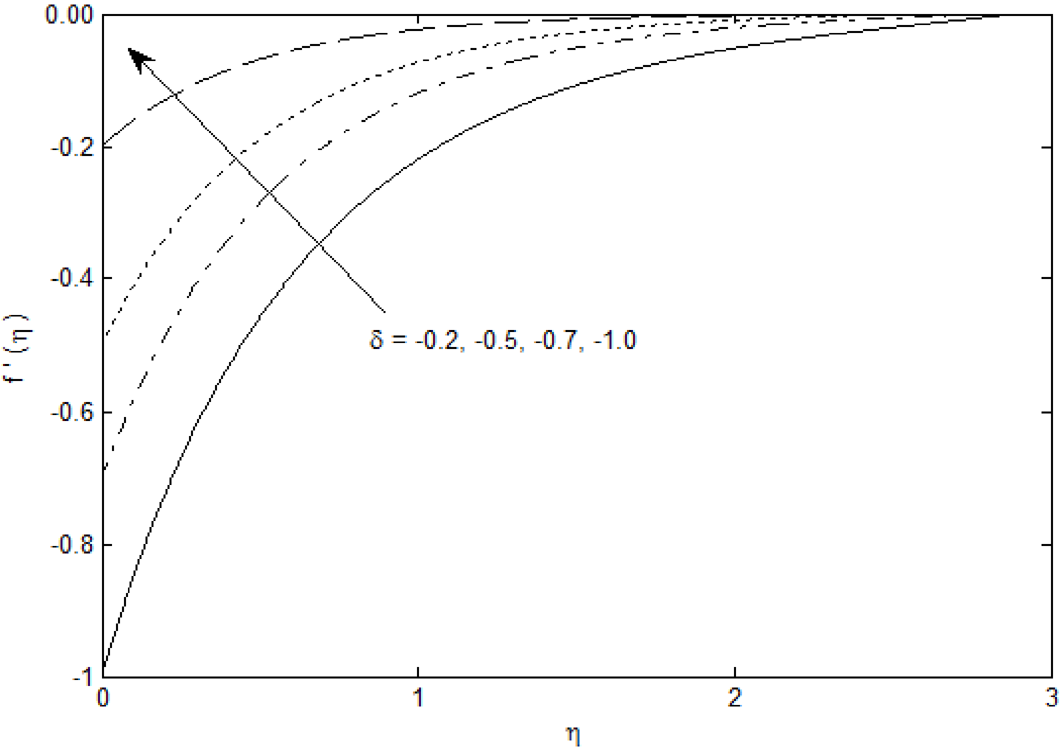


Fig.2. Velocity description for dissimilar values of the δ .

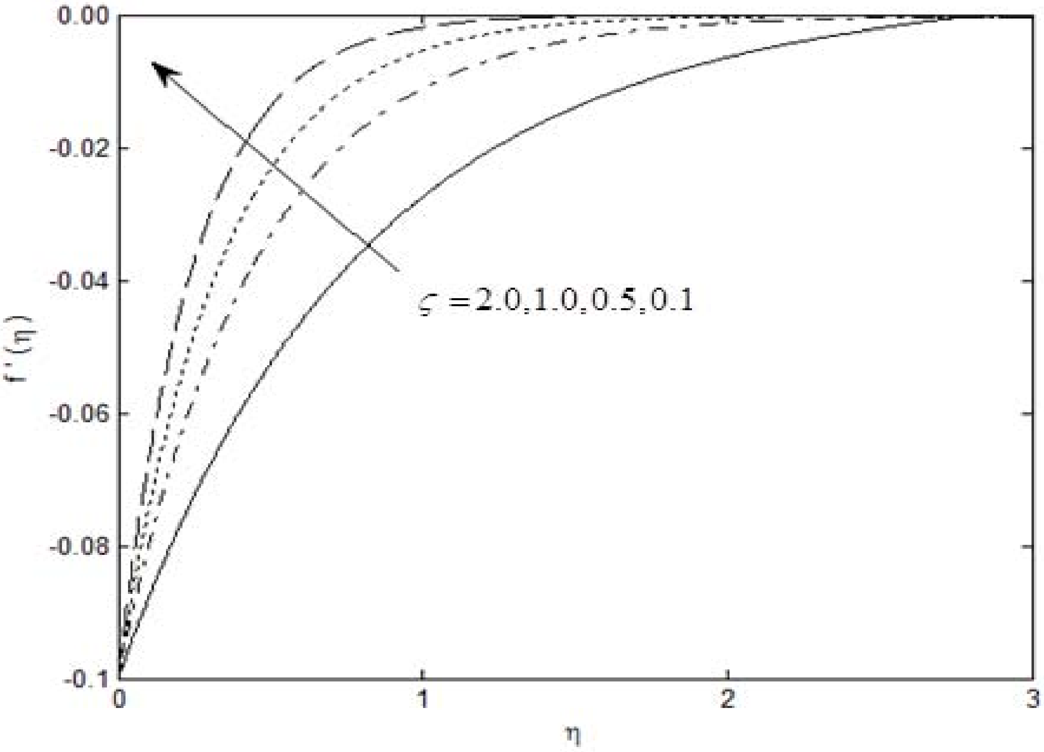


Fig.3. Velocity description for dissimilar values of the porosity parameter

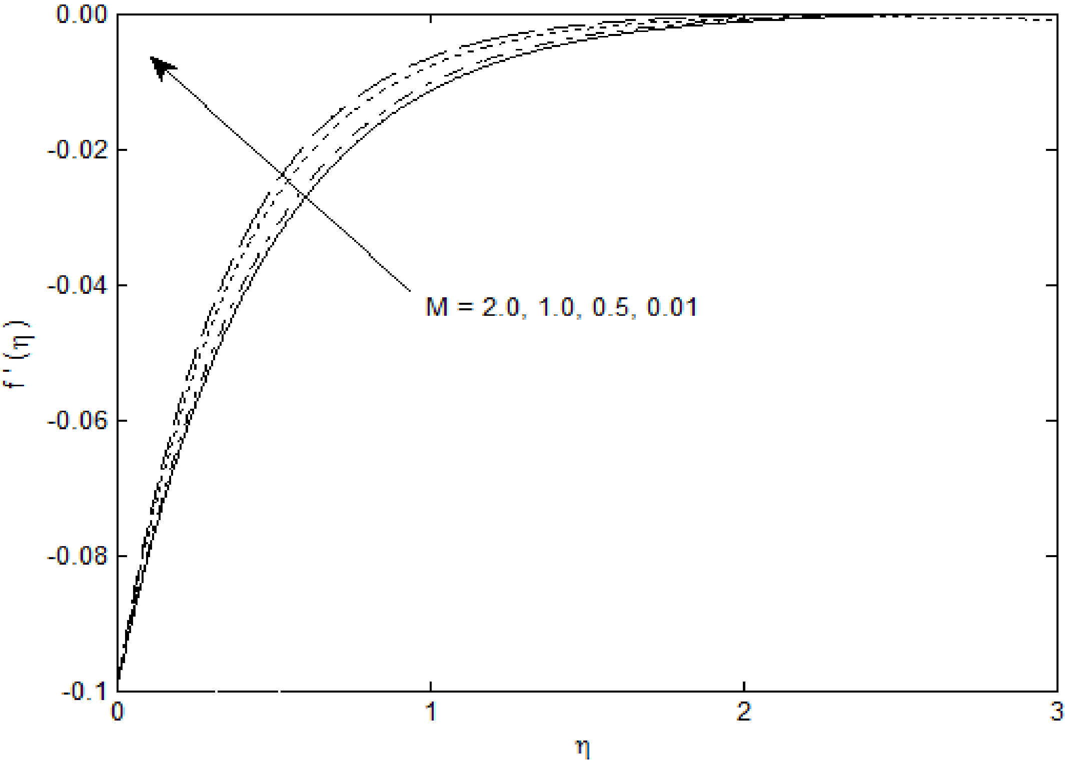


Fig.4. Velocity descriptions for dissimilar values of the magnetic parameter.

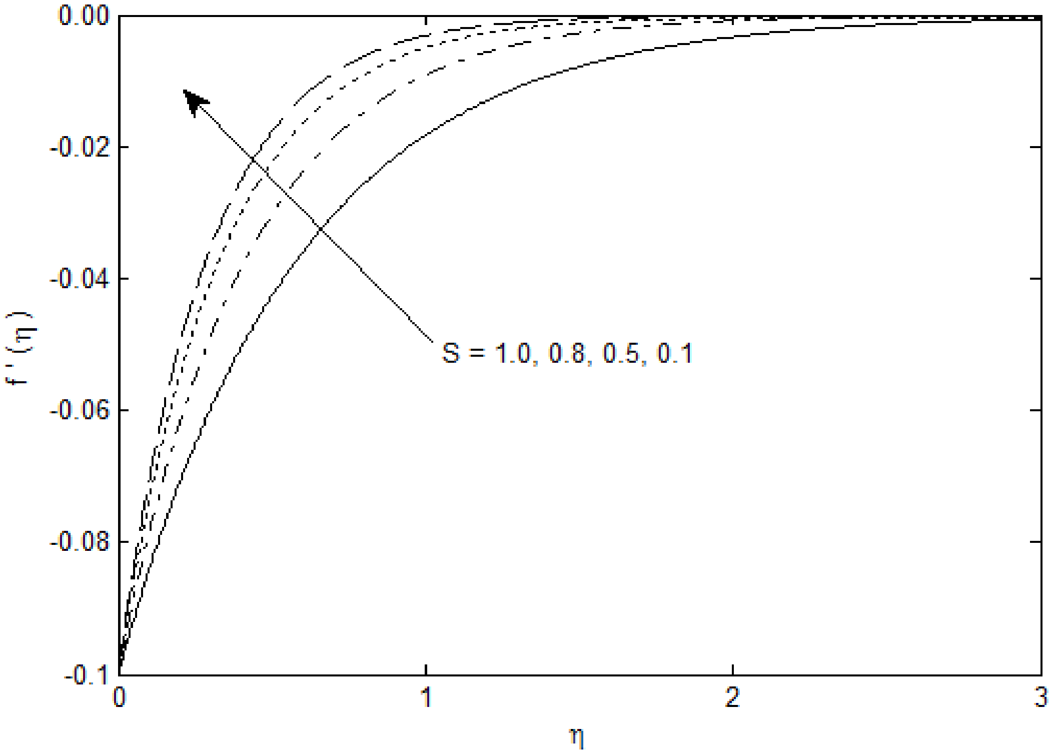


Fig.5. Velocity descriptions for dissimilar values of the S .

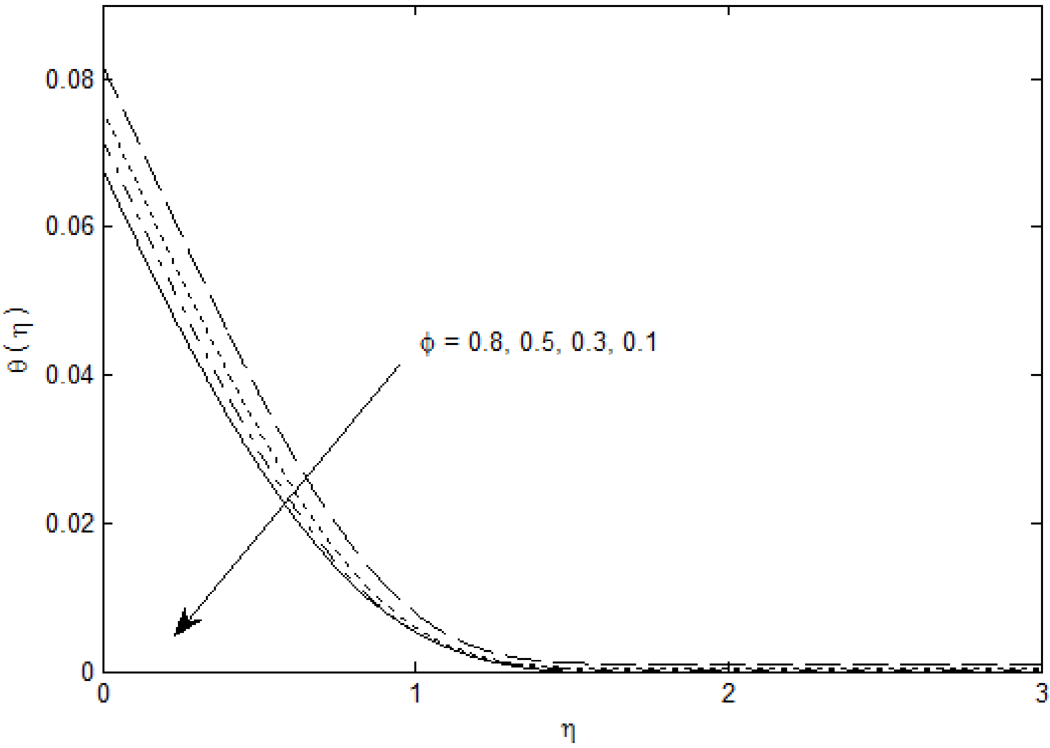


Fig.6. Temperature descriptions for diverse values of ϕ.

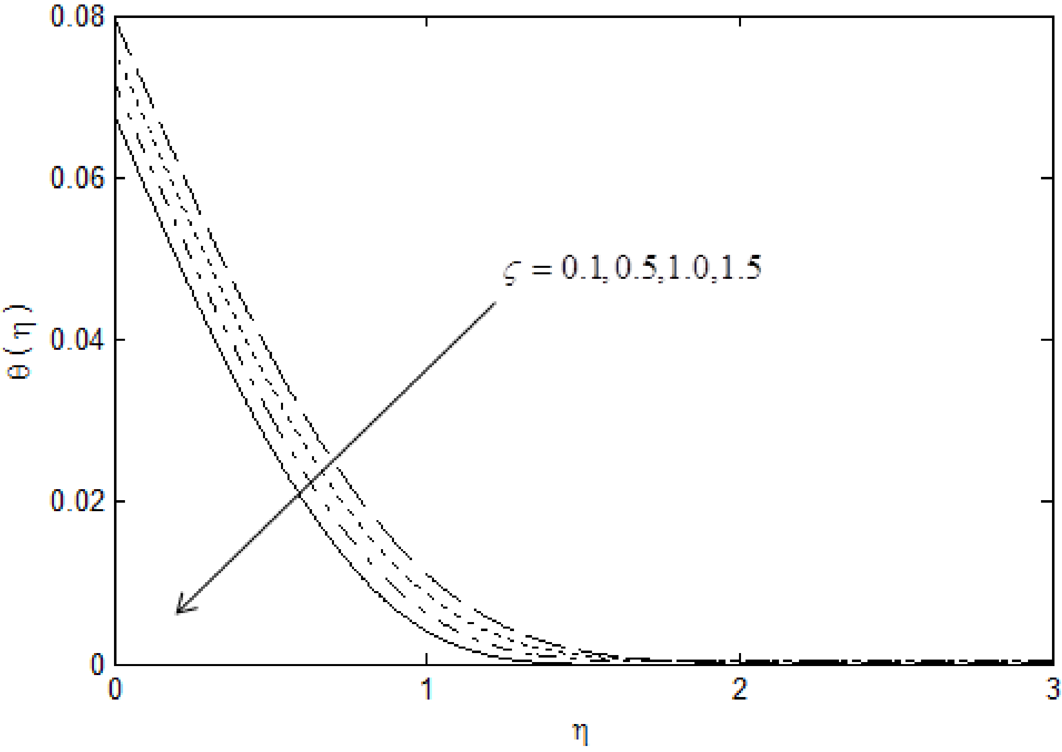


Fig.7. Energy descriptions for assorted values of the porosity parameter.

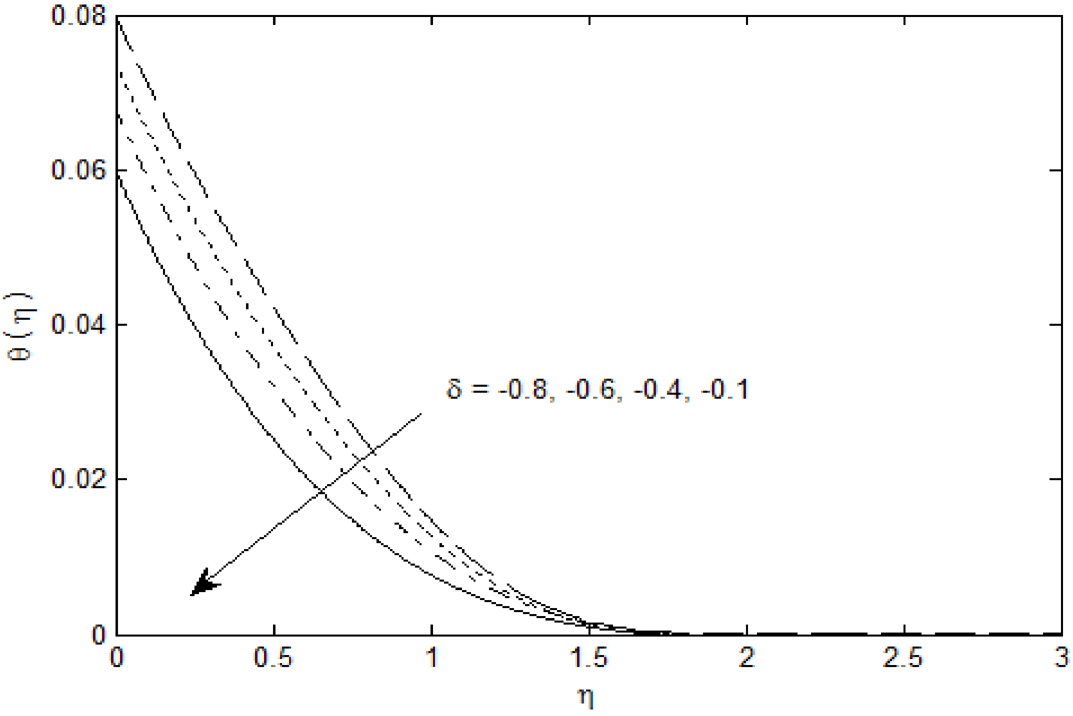


Fig.8. Energy descriptions for assorted values of the shrinking parameter.

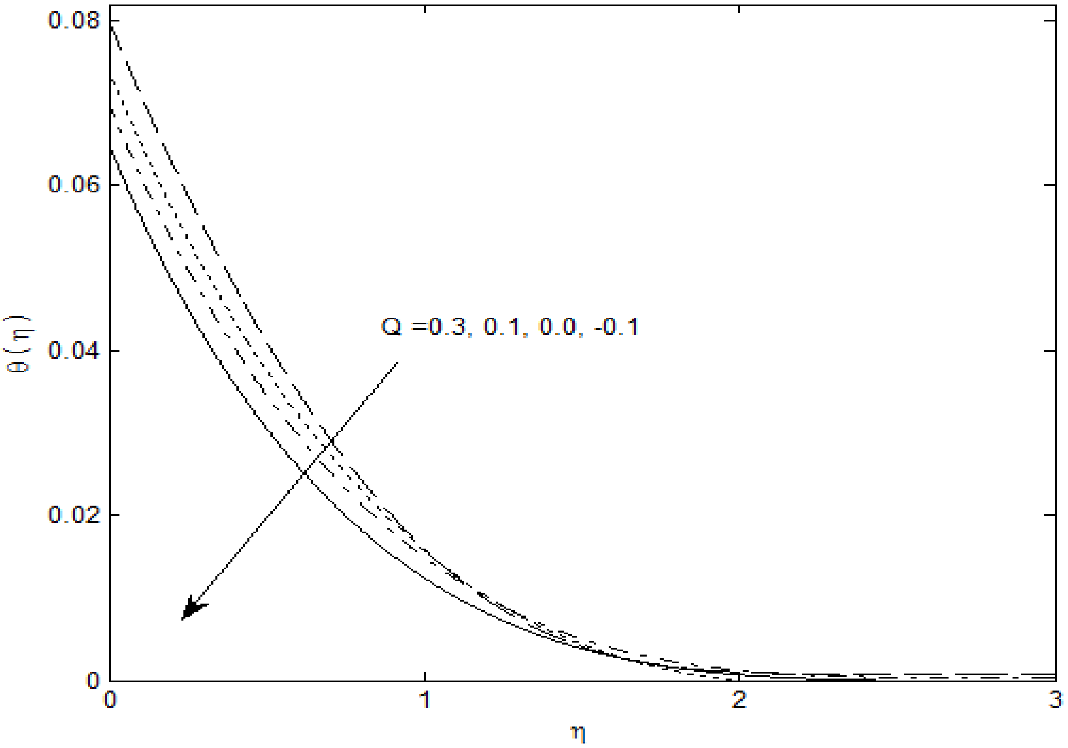


Fig.9. Energy descriptions for diverse values of Q .

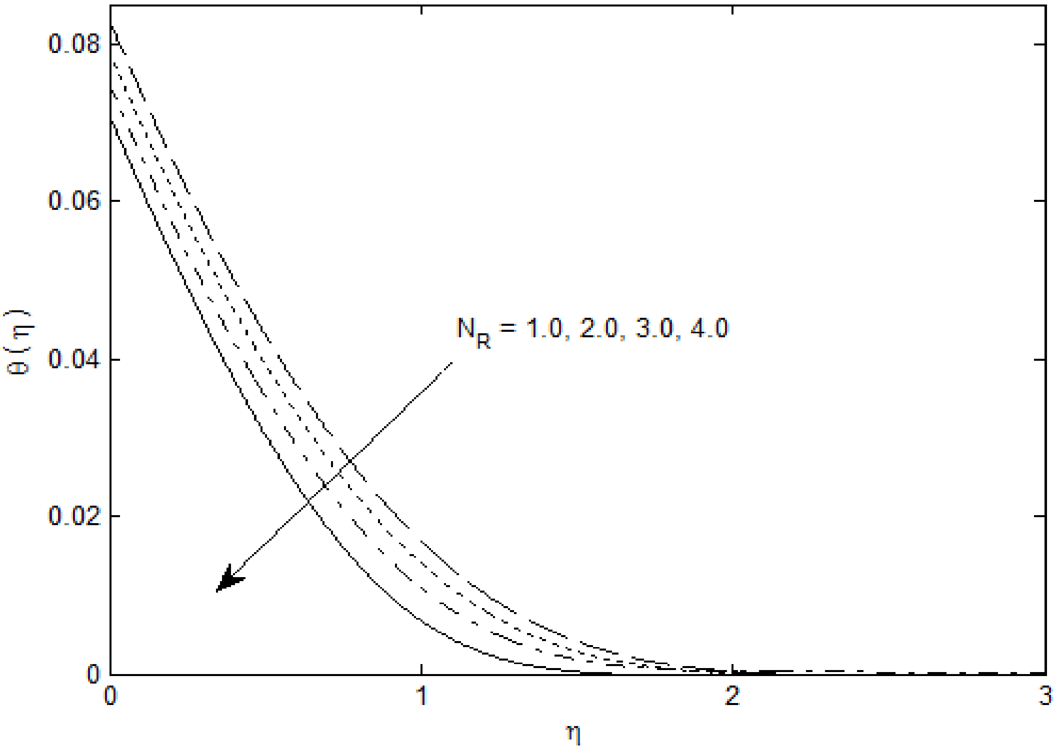


Fig.10. Energy descriptions for diverse values of the radiation parameter.

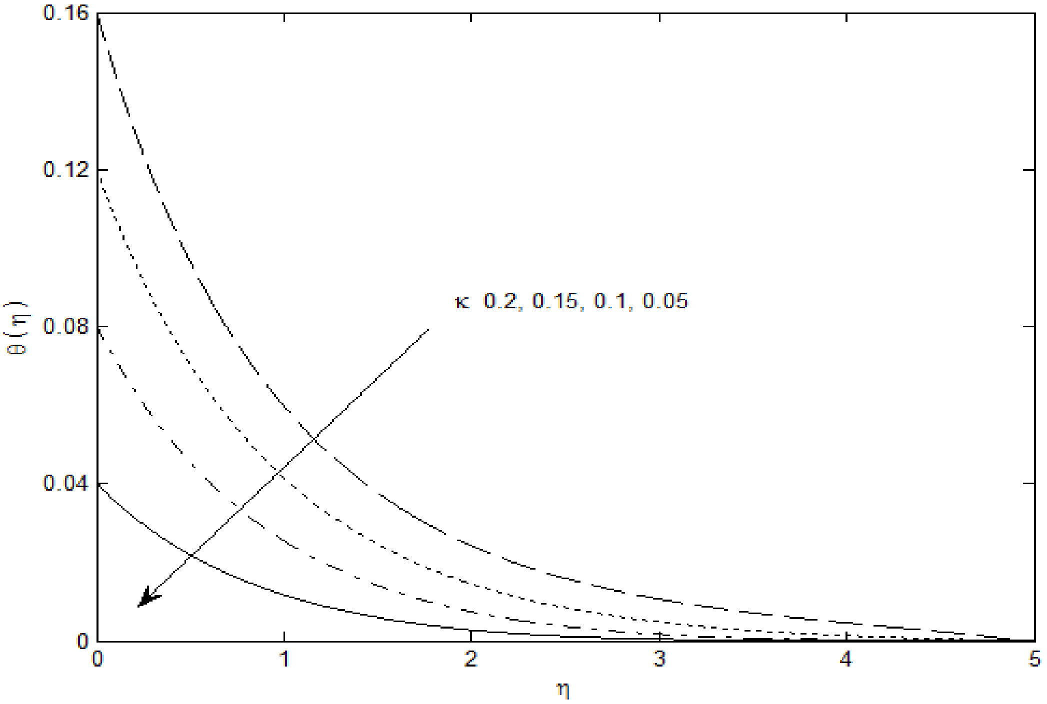


Fig.11. Energy descriptions for diverse values of the Biot number.

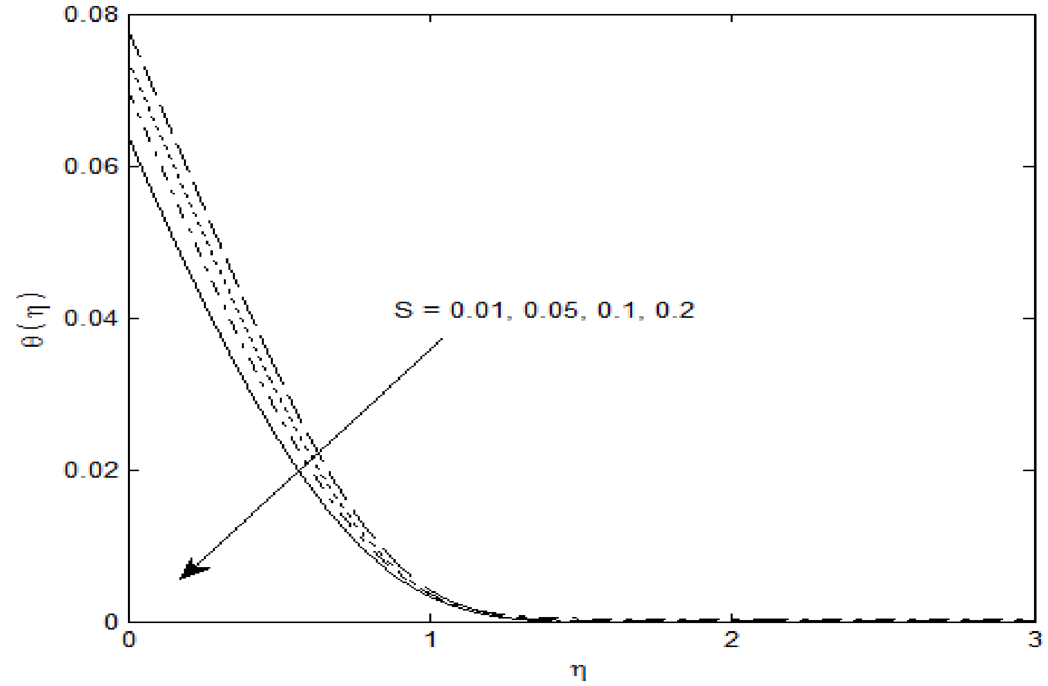


Fig.12. Energy descriptions for diverse importance of S .

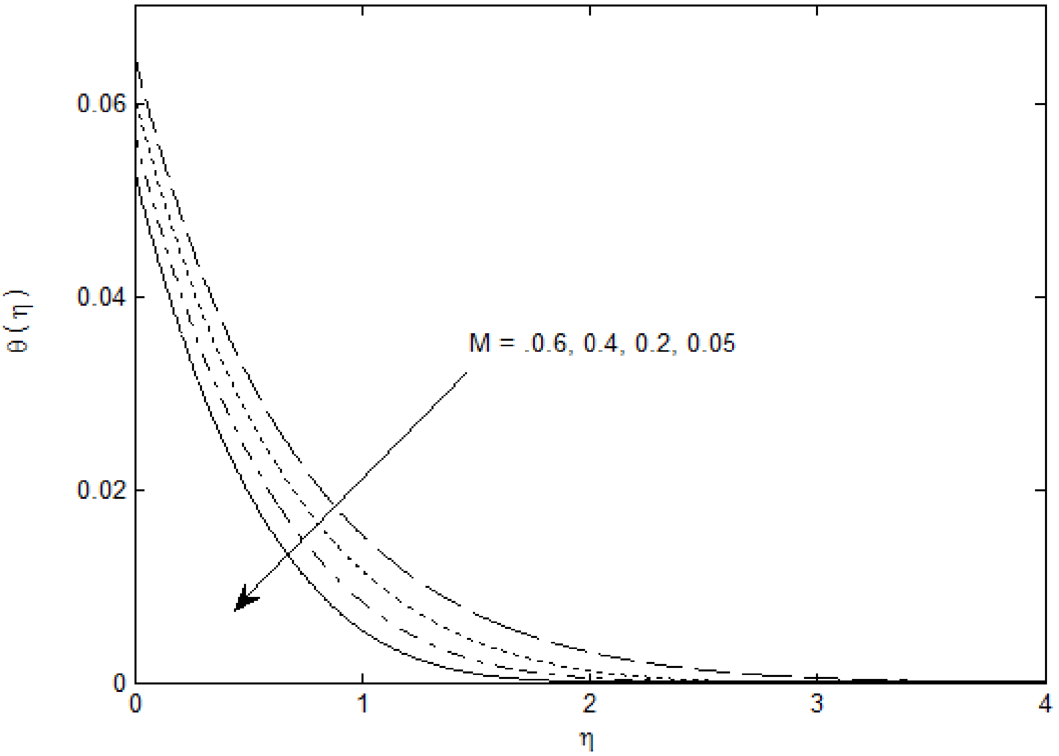


Fig.13. Energy descriptions for diverse values of the magnetic parameter.

Table 2. Comparison of values of $\mathrm{Nu}\left(\mathrm{Re}_x\right)^{-\frac{1}{2}}$ for $\varsigma = 0.5, m = 2, \mathrm{Pr} = 6.2, S = 0.8$ and $\delta = -0.1$ when N_R and Q are not considered i.e. $\left(N_R = 1, Q = 1\right)$.

M	φ	γ	$-M_3\theta'(0)$ Hayat *et al.* [26] HAM solution	$-M_3\theta'(0)$ Present study (Shooting method)
0.5	0.05	0.3	0.336178	0.330998
0.7			0.336187	0.331033
1.0			0.336201	0.331119
2.0			0.336996	0.332016
0.2	0.01		0.301109	0.298495
	0.05		0.334288	0.329618
	0.07		0.355511	0.348101
	0.1		0.379977	0.376500
	0.05	0.1	0.113946	0.113439
		0.3	0.332611	0.329963
		0.5	0.537019	0.539584
		0.7	0.767305	0.745753

5. Conclusions

In the present study, an analysis was made in order to investigate an MHD three dimensional motion of a nanofluid over a contracting surface with N_R and Q through permeable medium. The physical impact of different parameters for instance the magnetic parameter, permeability variable, shrinking variable, mass transfer parameter, Biot number, radiation parameter and heat source/sink parameter on momentum and energy descriptions are depicted and discussed in this paper. The main observations are listed below.

- The effects of the magnetic parameter *M*, nanoparticle volume fraction ϕ, shrinking parameter λ, mass transfer parameter *S* and Biot number γ on the velocity description f' are similar.
- The velocity description f' reduces while the nanoparticle volume fraction ϕ enhances.
- The effects of the magnetic parameter *M*, radiation parameter N_R, porosity parameter *d*, shrinking parameter λ, and mass transfer parameter *S* on the temperature description θ are similar.
- The energy profile θ reduces when the radiation parameter N_R enhances.
- The energy description θ enhances as the nanoparticle volume fraction ϕ, heat source/sink parameter δ and Biot number γ increase.
- The temperature profile θ in the case of heat source $(\delta > 0)$ is superior than in the case of sink $(\delta < 0)$.

Nomenclature

B_0 – magnetic flux density
C – concentration of species
Cp – specific heat at constant pressure
c – shrinking rate
h_1 – convective heat transfer coefficient
K – permeability parameter
k^* – absorption coefficient

k_{nf} – thermal conductivity
M – magnetic parameter
M_1, M_2, M_3 and k_2 – constant related to properties of the nanofluids
N_R – radiation parameter
Pr – Prandtl number
Q – heat source parameter
q_r – radiative heat flux
Re_x – local Reynolds number
S – suction/injection parameter
T – the fluid temperature
T_∞ – the fluid temperature at infinity
W – suction parameter
α_{nf} – thermal diffusivity of the nanofluid
δ – shrinking parameter
ς – non-dimensional Permeability parameter
η – similarity variable
θ – dimensionless temperature
κ – Biot number parameter
μ_{nf} – dynamic viscosity of the nanofluid
ν_{nf} – kinematic viscosity of nanofluid
ρ_{nf} – density of the nanofluid
$(\rho Cp)_{nf}$ – heat capacitance of the nanofluid
σ^* – Stefan-Boltzmann constant

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