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EFFECT OF MAGNETIC FIELD ON THERMOSOLUTAL INSTABILITY OF ROTATING FERROMAGNETIC FLUID UNDER VARYING GRAVITY FIELD

S. K. PUNDIR, P. K. NADIAN*, R. PUNDIR
Department of Mathematics
S.D. (P.G.) College, Muzaffarnagar
Uttar Pradesh-251001, INDIA
E-mail: pknadian2204@gmail.com

This paper deals with the theoretical investigation of the effect of a magnetic field, rotation and magnetization on a ferromagnetic fluid under varying gravity field. To find the exact solution for a ferromagnetic fluid layer contained between two free boundaries, we have used a linear stability analysis and normal mode analysis method. For the case of stationary convection, a stable solute gradient has a stabilizing effect, while rotation has a stabilizing effect if $\lambda > 0$ and destabilizing effect if $\lambda < 0$. Further, the magnetic field is discovered to have both a stabilizing and destabilizing effect for both $\lambda > 0$ and $\lambda < 0$. It is likewise discovered that magnetization has a stabilizing effect for both $\lambda > 0$ and $\lambda < 0$ in the absence of the stable solute gradient. Graphs have been plotted by giving numerical values of various parameters. In the absence of rotation, magnetic field and stable solute gradient, the principle of exchange of stabilities is found to hold true for certain conditions.

Keywords: thermosolutal instability, ferromagnetic fluid, rotation, magnetic field, magnetization.

1. Introduction

A ferromagnetic fluid (additionally referred to as a magnetic fluid) is an electrically non-conducting colloidal suspension of solid ferromagnetic particles in a non-electrically conducting carrier fluid like water, kerosene, organic solvent etc. These colloidal particles are covered with stabilizing surfactants which forbid particle agglomeration even if a strong uniform magnetic field gradient is applied to the ferromagnetic fluid. Rosensweig [1] discussed this difficulty in his monograph. There are numerous stability issues on ferromagnetic fluids. Bénard convection (Chandrasekhar [2]), double-diffusive convection (thermosolutal instability) are a few instability issues in ferromagnetic fluids. Finlayson [3] studied the convective instability of a ferromagnetic fluid for a fluid layer heated from below in the presence of a uniform vertical magnetic field. Sekar and Vaidyanathan [4] studied the convective instability of a magnetized ferrofluid in a rotating porous medium. Also, the convective instability of a layer of a ferromagnetic fluid rotating about a vertical axis was discussed by Gupta and Gupta [5]. The thermosolutal convection in a ferromagnetic fluid was studied by Sharma *et al.* [6]. Sunil *et al.* [7] discussed the effect of rotation on a ferromagnetic fluid heated and soluted from below saturating a porous medium. Also, the effect of the magnetic field dependent viscosity on the thermosolutal convection in a ferromagnetic fluid saturating a porous medium was studied by Sunil *et al.* [8]. The effect of rotation on the double-diffusive convection in a magnetized ferrofluid with internal angular momentum was studied by Mahajan *et al.* [9]. The Bénard convection in ferromagnetic fluids was mentioned by many authors (Siddheswar [10, 11], Venkatasubramaniam and Kaloni [12]). The thermosolutal convection in a ferromagnetic fluid saturating a porous medium was investigated with the aid of using the results of Sunil *et al.* [13]. Also, Sunil *et al.* [14] studied the effect of the magnetic-field-dependent viscosity on the thermosolutal convection of a rotating ferromagnetic fluid saturating a porous medium. The magneto-rotational convection for ferromagnetic fluids in the presence of compressibility and heat source via a porous medium was studied with the aid of using the results of Sharma *et al.* [15]. Recently, Nadian *et al.* [16] discussed the effect of rotation on the thermal instability of a couple-stress ferromagnetic fluid in the presence of a variable gravity field. The thermal instability of a couple-stress ferromagnetic fluid in the presence of a variable gravity field, rotation and magnetic field was discussed by Nadian *et al.* [17]. Also, the effect of rotation on a couple-stress ferromagnetic fluid heated and soluted from below in the presence of a variable gravity field was studied by Nadian *et al.* [18].

The present study is planned to examine the effect of rotation and a magnetic field on the thermosolutal instability of a ferromagnetic fluid in the presence of a variable gravity field.We have assumed that gravity is varying as $g=\lambda g_0$, where g_0 is the value of g at the Earth's surface, which is always positive and λ can be positive or negative as gravity increases or decreases upwards from its value g_0.

2. Mathematical formulation of the problem

Here, we consider an infinite, horizontal layer of thickness d of an electrically non-conducting incompressible ferromagnetic fluid heated and soluted from below bounded with the aid of using planes $z=0$ and $z=d$ as shown in Fig.1. The fluid is acted upon by a uniform rotation $\boldsymbol{\Omega}(0,0,\Omega)$, uniform magnetic field $\boldsymbol{H}(0,0,H)$ and variable gravity field $\boldsymbol{g}(0,0,-g)$, where $g=\lambda g_0$. The ferromagnetic fluid layer is heated and soluted from below leading to an adverse temperature gradient $\beta=\dfrac{T_0-T_1}{d}$ with $T_0>T_1$ and $\beta'=\dfrac{C_0-C_1}{d}$, where C_0 and C_1 are the constant concentrations of the lower and upper boundaries with $C_0>C_1$.

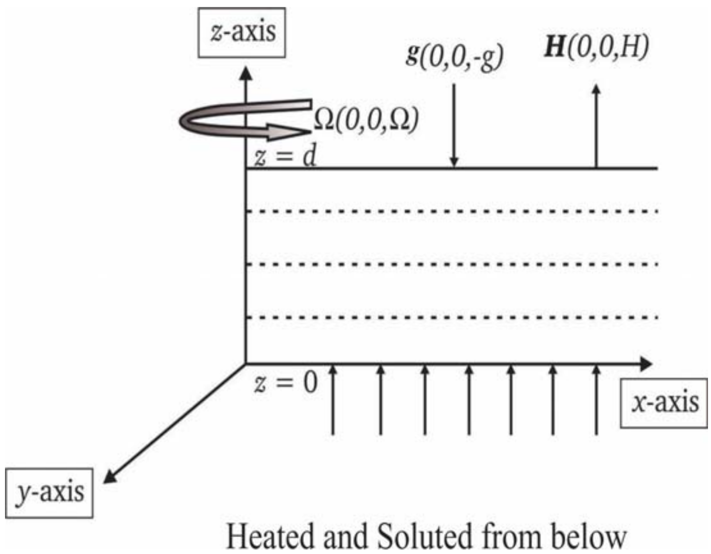

Fig.1. Geometrical Configuration.

Since ferromagnetic fluids react quickly to a magnetic torque, so we expect the subsequent situation to hold

$$\boldsymbol{M}\times\boldsymbol{H}=0. \tag{2.1}$$

Now, assuming that the fluid is electrically non-conducting and that displacement current is negligible, Maxwell's equations become

$$\nabla.\boldsymbol{B}=0, \qquad \nabla\times\boldsymbol{H}=0. \tag{2.2}$$

In Chu formulation of electrodynamics, the relation between the magnetic field $\boldsymbol{H}$, magnetization $\boldsymbol{M}$ and magnetic induction $\boldsymbol{B}$ is

$$\boldsymbol{B}=\mu_0\left(\boldsymbol{H}+\boldsymbol{M}\right). \tag{2.3}$$

We expect that magnetization is aligned with the magnetic field, however there is a dependence the magnitude of the magnetic field, temperature and salinity, so that

$$\boldsymbol{M}=\frac{\boldsymbol{H}}{H}M\left(H,T,C\right). \tag{2.4}$$

Let p, ρ, T, C, α, α', κ_T, κ_S and $\boldsymbol{q}\left(u_1,u_2,u_3\right)$ denote, respectively, the pressure, density, temperature, solute concentration, thermal expansion coefficient, an analogous solvent coefficient of expansion, thermal conductivity, solute conductivity and velocity of the fluid. The equations of conservation of momentum, continuity, conservation of temperature and solute concentration are

$$\left[\frac{\partial\boldsymbol{q}}{\partial t}+\left(\boldsymbol{q}.\nabla\right)\boldsymbol{q}\right]=-\frac{1}{\rho_0}\nabla p+\boldsymbol{g}\left(1+\frac{\delta\rho}{\rho_0}\right)+\frac{1}{\rho_0}M.\nabla H+2\left(\boldsymbol{q}\times\boldsymbol{\Omega}\right)+\frac{\mu_e}{4\pi\rho_0}\left(\nabla\times\boldsymbol{H}\right)\times\boldsymbol{H}, \tag{2.5}$$

$$\nabla.\boldsymbol{q}=0, \tag{2.6}$$

$$\frac{\partial T}{\partial t}+\left(\boldsymbol{q}.\nabla\right)T=\kappa_T\nabla^2 T, \tag{2.7}$$

$$\frac{\partial C}{\partial t}+\left(\boldsymbol{q}.\nabla\right)C=\kappa_S\nabla^2\,\mathrm{C}. \tag{2.8}$$

Maxwell's equations [19] of electromagnetism are given by

$$\frac{\partial\boldsymbol{H}}{\partial t}=\left(\boldsymbol{H}.\nabla\right)\boldsymbol{q}+\eta\nabla^2\boldsymbol{H}, \tag{2.9}$$

$$\nabla.\boldsymbol{H}=0. \tag{2.10}$$

The density equation of state is

$$\rho=\rho_0\left[1-\alpha\left(T-T_0\right)+\alpha'\left(C-C_0\right)\right] \tag{2.11}$$

where the suffix zero refers to the value at the reference level $z=0$. ∇H is the magnetic field gradient. Also,

$$\nabla^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}, \qquad H=\left|\boldsymbol{H}\right|, \qquad B=\left|\boldsymbol{B}\right|, \qquad M=\left|\boldsymbol{M}\right|.$$

In the present study, we assume that magnetization depends only on temperature T i.e. $M = M(T)$. Thus, as the first approximation, we consider that

$$M = M_0 \left[1 - \gamma' (T - T_0) \right] \tag{2.12}$$

where, M_0 denotes the magnetization at $T = T_0$ with T_0 being the reference temperature and $\gamma' = \dfrac{1}{M_0} \left(\dfrac{\partial M}{\partial T} \right)_H$.

3. Basic state and perturbation equations

The basic motionless solution is

$$\begin{gathered} \boldsymbol{q} = (0,0,0), \quad p = p(z), \quad \rho = \rho(z) = \rho_0 [1 - \alpha\beta z + \alpha'\beta' z], \quad T = T(z) = T_0 - \beta z, \\ C = C_0 - \beta' z, \quad \boldsymbol{\Omega} = (0,0,\Omega), \quad \boldsymbol{H} = (0,0,H), \quad M = M_0 (1 + \gamma'\beta z), \quad \boldsymbol{M} = \boldsymbol{M}(z). \end{gathered} \tag{3.1}$$

The perturbed flow may be represented as

$$\begin{gathered} \boldsymbol{q} = (0,0,0) + (u_1, u_2, u_3), \quad \boldsymbol{h} = (0,0,H) + (h_x, h_y, h_z), \quad T = T(z) + \theta, \\ C = C(z) + \gamma, \quad \rho = \rho(z) + \delta\rho, \quad p = p(z) + \delta p, \quad M = M(z) + \delta M \end{gathered} \tag{3.2}$$

where, $\boldsymbol{q}(u_1, u_2, u_3)$, $\boldsymbol{h}(h_x, h_y, h_z)$, θ, γ, $\delta\rho$, δp, δM are, respectively, the perturbation in fluid velocity $\boldsymbol{q}(0,0,0)$, magnetic field $\boldsymbol{H}$, temperature $\boldsymbol{T}$, solute gradient C, density ρ, pressure p and magnetization $\boldsymbol{M}$.

Linearizing the equation of perturbation and analyzing the perturbation into normal modes, we assume that the perturbation quantities are of the form,

$$(u_3, \theta, \xi, \zeta, h_z, \gamma) = \left[W(z), \Theta(z), X(z), Z(z), K(z), \Gamma(z) \right] . e^{\left(ik_x x + ik_y y + nt\right)} \tag{3.3}$$

where, k_x and k_y are wave numbers in the x and y directions, respectively, and $k = \left({k_x}^2 + {k_y}^2 \right)^{1/2}$ is the resultant wave number of the disturbance and n is the frequency of any arbitrary disturbance (which is generally a complex constant).

Now, eliminate the physical quantities using the non-dimensional parameters $a = kd$, $\sigma = \left(nd^2 / \nu \right)$, $p_1 = (\nu / \kappa_T)$, $p_2 = (\nu / \eta)$, $q = (\nu / \kappa_S)$, $D^* = dD$ and dropping $(*)$ for convenience, we obtain,

$$\sigma\left(D^2-a^2\right)W+\frac{\lambda\alpha a^2 d^2}{\nu}\left(g_0-\frac{\gamma' M_0\nabla H}{\rho_0\alpha\lambda}\right)\theta-\frac{\lambda g_0\alpha' a^2 d^2}{\nu}\Gamma+$$
$$+\frac{2\Omega d^3}{\nu}DZ-\frac{\mu_e Hd}{4\pi\rho_0\nu}\left(D^2-a^2\right)DK=0, \tag{3.4}$$

$$\sigma Z=\frac{2\Omega d}{\nu}DW+\frac{\mu_e Hd}{4\pi\rho_0\nu}DX, \tag{3.5}$$

$$\left(D^2-a^2-\sigma p_2\right)X=-\frac{Hd}{\eta}DZ, \tag{3.6}$$

$$\left(D^2-a^2-\sigma p_2\right)K=-\frac{Hd}{\eta}DW, \tag{3.7}$$

$$\left(D^2-a^2-\sigma p_1\right)\Theta=-\frac{\beta d^2}{\kappa_T}\mathrm{W}, \tag{3.8}$$

$$\left(D^2-a^2-\sigma q\right)\Gamma=-\frac{\beta' d^2}{\kappa_S}\mathrm{W}. \tag{3.9}$$

Now eliminating X, Θ, K, Z and Γ among Eqs (3.4)-(3.9), we obtain the stability governing equation,

$$\lambda a^2 R_f.\frac{1}{\left(D^2-a^2-\sigma p_1\right)}W=\sigma\left(D^2-a^2\right)W+\lambda a^2 S.\frac{1}{\left(D^2-a^2-\sigma q\right)}W+$$
$$+T_A\left\{\frac{\left(D^2-a^2-\sigma p_2\right)}{\left[\sigma\left(D^2-a^2-\sigma p_2\right)+QD^2\right]}\right\}D^2W+Q\left[\frac{\left(D^2-a^2\right)}{\left(D^2-a^2-\sigma p_2\right)}\right]D^2W \tag{3.10}$$

where, $R_f=\left(\alpha\beta d^4/\nu\kappa_T\right)\left[g_0-\left(\gamma' M_0\nabla H/\rho_0\alpha\lambda\right)\right]$ is the Rayleigh number for ferromagnetic fluids, $S=\left(g_0\alpha'\beta' d^4/\nu\kappa_S\right)$ is the solute Rayleigh number, $T_A=\left[\left(2\Omega d^2/\nu\right)\right]^2$ is the modified Taylor number and $Q=\left(\mu_e H^2 d^2/4\pi\rho_0\nu\eta\right)$ is the Chandrasekhar number.

If $\lambda>0$, $g_0>\left(\gamma' M_0\nabla H/\rho_0\alpha\lambda\right)$, then $R_f<R$, which implies that convection begins in a ferromagnetic fluid at a higher thermal Rayleigh number and if $\lambda<0$, then $R_f>R$, which means that convection begins in a ferromagnetic fluid at a lower thermal Rayleigh number.

On boundaries, the perturbation in the temperature is zero due to the fact that each of the boundaries is maintained at constant temperature. The suitable boundary conditions are,

$$W=0,\quad Z=0,\quad \Theta=0,\quad X=0,\quad \Gamma=0 \quad \text{at} \quad z=0 \quad \text{and} \quad z=1,$$

also (3.11)

$$DZ=DX=D^2W=D^4W=0 \quad \text{at} \quad z=0 \quad \text{and} \quad z=1.$$

From Eq.(3.11) it is apparent that each the even order derivatives of W vanishes at the boundaries. Therefore the proper solution of Eq.(3.10) characterizing the lowest mode is

$$W = W_0 \sin \pi z \tag{3.12}$$

where W_0 is a constant.

Now using Eq.(3.12) in Eq.(3.10), we get

$$\begin{aligned} R_1 = i\sigma_1 \frac{(1+x_w)(1+x_w+i\sigma_1 p_1)}{\lambda x_w} + S_1 \frac{(1+x_w+i\sigma_1 p_1)}{(1+x_w+i\sigma_1 q)} + \\ + \frac{T_{A_1}}{\lambda x_w} \frac{(1+x_w+i\sigma_1 p_1)(1+x_w+i\sigma_1 p_2)}{\left[i\sigma_1(1+x_w+i\sigma_1 p_2)+Q_1\right]} + \frac{Q_1}{\lambda x_w} \frac{(1+x_w)(1+x_w+i\sigma_1 p_1)}{(1+x_w+i\sigma_1 p_2)} \end{aligned} \tag{3.13}$$

where, $x_w = \left(a^2/\pi^2\right)$, $i\sigma_1 = \left(\sigma/\pi^2\right)$, $R_1 = \left(R_f/\pi^4\right)$, $S_1 = \left(S/\pi^4\right)$, $T_{A_1} = \left(T_A/\pi^4\right)$, $Q_1 = \left(Q/\pi^2\right)$.

4. Analytical discussion

4.1. Stationary convection

When stability sets in as stationary convection, the marginal state will be characterized by $\sigma_1 = 0$. So puting $\sigma_1 = 0$ in Eq.(3.13), we get

$$R_1 = S_1 + \frac{T_{A_1}(1+x_w)^2}{Q_1 \lambda x_w} + \frac{Q_1(1+x_w)}{\lambda x_w}. \tag{4.1}$$

Equation (4.1) expresses the modified Rayleigh number R_1 as a function of the parameters S_1, T_{A_1}, Q_1 and dimensionless wave number x_w. To examine the effect of the solute gradient, rotation and magnetic field, we have to study the behavior of (dR_1/dS_1), $\left(dR_1/dT_{A_1}\right)$, (dR_1/dQ_1) analytically.

Now by Eq.(4.1), we have

$$\frac{dR_1}{dS_1} = 1 \text{(which is positive)} \tag{4.2}$$

Clearly, the solute gradient has a stabilizing effect on the system

$$\frac{dR_1}{dT_{A_1}} = \frac{(1+x_w)^2}{Q_1 \lambda x_w}, \tag{4.3}$$

which shows that rotation has a stabilizing effect on the system if $\lambda > 0$ and a destabilizing effect if $\lambda < 0$

$$\frac{dR_1}{dQ_1} = \frac{(1+x_w)}{\lambda x_w}\left[1 - \frac{T_{A_1}(1+x_w)}{Q_1^2}\right], \tag{4.4}$$

which shows that magnetic field has a stabilizing effect on the system if

$$\lambda > 0, \quad Q_1^2 > T_{A_1}(1+x_w) \text{ and } \lambda < 0, \quad Q_1^2 < T_{A_1}(1+x_w).$$

Also, the magnetic field has a destabilizing effect on the system if

$$\lambda > 0, \quad Q_1^2 < T_{A_1}(1+x_w) \text{ and } \lambda < 0, \quad Q_1^2 > T_{A_1}(1+x_w).$$

In the absence of rotation, Eq.(4.4) becomes

$$\frac{dR_1}{dQ_1} = \frac{(1+x_w)}{\lambda x_w},$$

which shows that the magnetic field has a stabilizing effect on the system if $\lambda > 0$ and a destabilizing effect if $\lambda < 0$.
Now to see the effect of magnetization, we examine (dR/dM_0) analytically.

$$\frac{dR}{dM_0} = \left[S_1\pi^4 + \frac{T_{A_1}\pi^4(1+x_w)^2}{Q_1\lambda x_w} + \frac{Q_1\pi^4(1+x_w)}{\lambda x_w}\right]\left(1 - \frac{\gamma' M_0 \nabla H}{\rho_0\alpha\lambda g_0}\right)^{-2}\left(\frac{\gamma'\nabla H}{\rho_0\alpha\lambda g_0}\right), \tag{4.5}$$

which shows that in the absence of the solute gradient; magnetization has a stabilizing effect for both $\lambda > 0$ and $\lambda < 0$.

4.2. Stability of the system and oscillatory modes

Multiplying Eq.(3.4) by W^* (conjugate of W) and integrating over the range of z and making use of Eqs (3.5)-(3.9) together with boundary conditions (3.11), we get

$$\begin{aligned} &\sigma I_1 + d^2\left[\sigma^* I_2 + \frac{\mu_e\eta}{4\pi\rho_0\nu}(I_3 + p_2\sigma I_4)\right] + \frac{\mu_e\eta}{4\pi\rho_0\nu}\left(I_5 + p_2\sigma^* I_6\right) + \\ &-\frac{\lambda\alpha a^2\kappa_T}{\nu\beta}\left(g_0 - \frac{\gamma' M_0\nabla H}{\rho_0\alpha\lambda}\right)\left(I_7 + p_1\sigma^* I_8\right) + \frac{\lambda g_0\alpha' a^2\kappa_S}{\nu\beta'}\left(I_9 + q\sigma^* I_{10}\right) = 0 \end{aligned} \tag{4.6}$$

where

$$I_1 = \int\left(|DW|^2 + a^2|W|^2\right)dz,\ I_2 = \int\left(|Z|^2\right)dz,$$

$$I_3 = \int\left(|DX|^2 + a^2|X|^2\right)dz, I_4 = \int|X|^2 dz,$$

$$I_5 = \int\left(\left|D^2K\right|^2 + 2a^2|DK|^2 + a^4|K|^2\right)dz, I_6 = \int\left(|DK|^2 + a^2|K|^2\right)dz,$$

$$I_7 = \int\left(|D\Theta|^2 + a^2|\Theta|^2\right)dz, I_8 = \int\left(|\Theta|^2\right)dz,$$

$$I_9 = \int\left(\left|D\Gamma\right|^2 + a^2\left|\Gamma\right|^2\right)dz, I_{10} = \int\left(\left|\Gamma\right|^2\right)dz.$$

All these integrals from $I_1 - I_{10}$ are positive definite.

Now putting $\sigma = \sigma_r + i\sigma_i$ in Eq.(4.6) and equating real and imaginary parts, we get

$$\begin{aligned}\sigma_r\Bigg[I_1 + d^2 I_2 + \frac{\mu_e \eta d^2}{4\pi\rho_0\nu} p_2 I_4 + \frac{\mu_e\eta}{4\pi\rho_0\nu} p_2 I_6 - \frac{\lambda\alpha a^2\kappa_T}{\nu\beta}\left(g_0 - \frac{\gamma' M_0 \nabla H}{\rho_0\alpha\lambda}\right) p_1 I_8 + \\ + \frac{\lambda g_0 \alpha' a^2 \kappa_S}{\nu\beta'} q I_{10}\Bigg] = -\Bigg[\frac{\mu_e\eta d^2}{4\pi\rho_0\nu} I_3 + \frac{\mu_e\eta}{4\pi\rho_0\nu} I_5 + \\ - \frac{\lambda\alpha a^2\kappa_T}{\nu\beta}\left(g_0 - \frac{\gamma' M_0\nabla H}{\rho_0\alpha\lambda}\right) I_7 + \frac{\lambda g_0\alpha' a^2\kappa_S}{\nu\beta'} I_9\Bigg]\end{aligned} \tag{4.7}$$

and

$$\begin{aligned}\sigma_i\Bigg[I_1 - d^2 I_2 + \frac{\mu_e\eta d^2}{4\pi\rho_0\nu} p_2 I_4 - \frac{\mu_e\eta}{4\pi\rho_0\nu} p_2 I_6 + \\ + \frac{\lambda\alpha a^2\kappa_T}{\nu\beta}\left(g_0 - \frac{\gamma' M_0\nabla H}{\rho_0\alpha\lambda}\right) p_1 I_8 - \frac{\lambda g_0\alpha' a^2\kappa_S}{\nu\beta'} q I_{10}\Bigg] = 0.\end{aligned} \tag{4.8}$$

It is obvious form Eq.(4.7) that σ_r may be positive or negative which means that the system may be stable or unstable.

In the absence of rotation, magnetic field and stable solute gradient, Eq.(4.8) becomes

$$\sigma_i\left[I_1 + \frac{\mu_e\eta d^2}{4\pi\rho_0\nu} p_2 I_4 + \frac{\lambda\alpha a^2\kappa_T}{\nu\beta}\left(g_0 - \frac{\gamma' M_0\nabla H}{\rho_0\alpha\lambda}\right) p_1 I_8\right] = 0.$$

If $\lambda g_0 \geq \dfrac{\gamma' M_0 \nabla H}{\rho_0\alpha}$, then the terms in the bracket are positive definite, which means that $\sigma_i = 0$. Therefore, oscillatory modes are not allowed and the principle of exchange of stabilities is satisfied if $\lambda g_0 \geq \dfrac{\gamma' M_0\nabla H}{\rho_0\alpha}$.

5. Numerical computations

The dispersion relation (4.1) is analyzed numerically also. The numerical values of the thermal Rayleigh number R_1 are determined for numerous values of the solute gradient S_1, rotation T_{A_1}, magnetic field Q_1 and magnetization M_0. Also, graphs are plotted between R_1 and S_1, R_1 and T_{A_1}, R_1 and Q_1, R_1 and M_0 as shown in Figures (2)-(10).

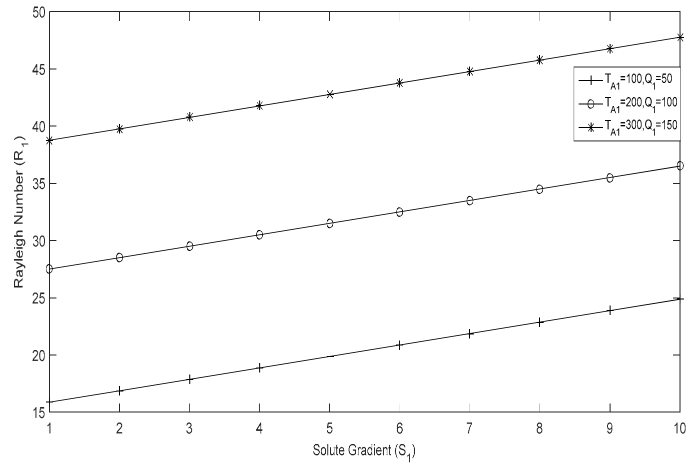

Fig.2. Variation of R_1 with S_1 for fixed $\lambda = 5$, $T_{A_1} = 100, 200, 300$ and $Q_1 = 50, 100, 150$.

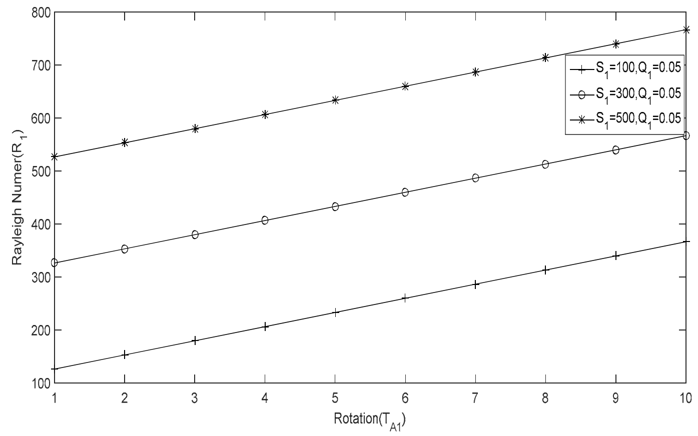

Fig.3. Variation of R_1 with T_{A_1} for fixed $\lambda = 3$, $S_1 = 100, 300, 500$ and $Q_1 = 0.05$.

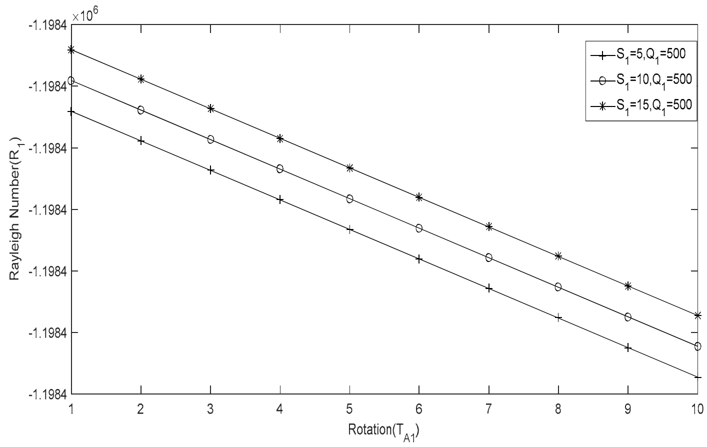

Fig.4. Variation of R_1 with T_{A_1} for fixed $\lambda = -7$, $S_1 = 5, 10, 15$ and $Q_1 = 500$.

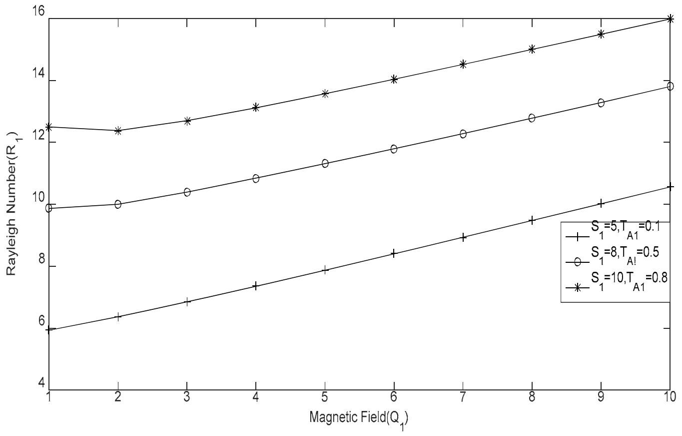

Fig.5. Variation of R_1 with Q_1 for fixed $\lambda = 2$, $S_1 = 5,8,10$ and $T_{A_1} = 0.1,0.5,0.8$.

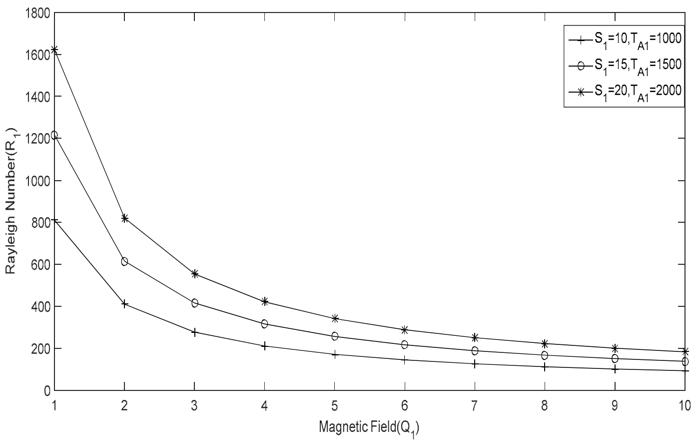

Fig.6. Variation of R_1 with Q_1 for fixed $\lambda = 5$, $S_1 = 10,15,20$ and $T_{A_1} = 1000,1500,2000$.

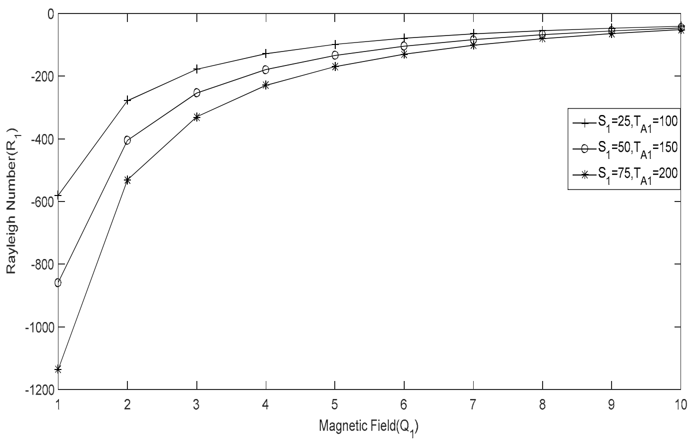

Fig.7. Variation of R_1 with Q_1 for fixed $\lambda = -2$, $S_1 = 25,50,75$ and $T_{A_1} = 100,150,200$.

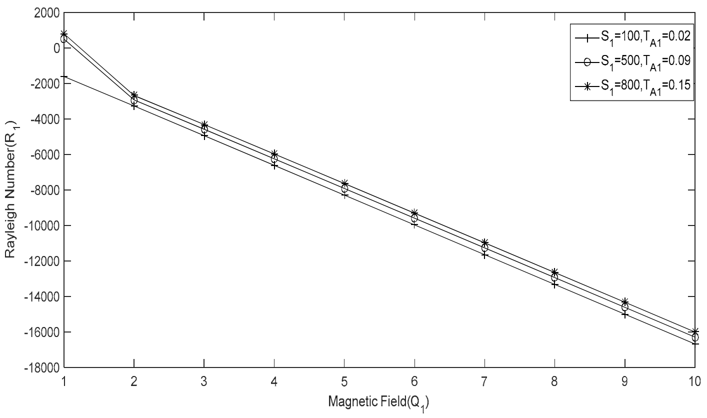

Fig.8. Variation of R_1 with Q_1 for fixed $\lambda = -10$, $S_1 = 100, 500, 800$ and $T_{A_1} = 0.02, 0.09, 0.15$.

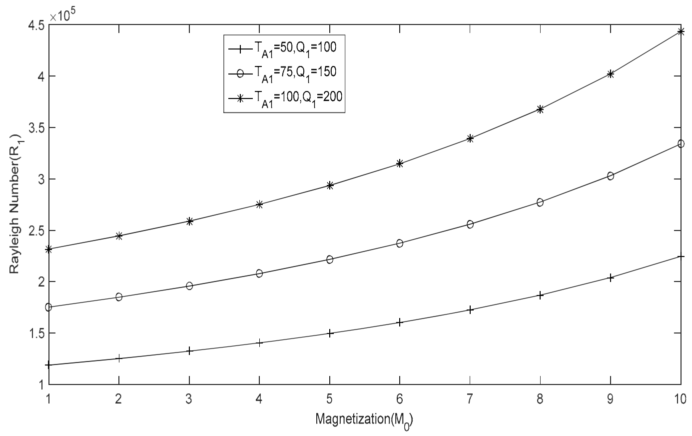

Fig.9. Variation of R_1 with M_0 for fixed $\lambda = 0.1$, $\rho_0 = 10$, $\alpha = 10$, $\gamma = 0.5$, $\nabla H = 10$, $T_{A_1} = 50, 75, 100$ and $Q_1 = 100, 150, 200$.

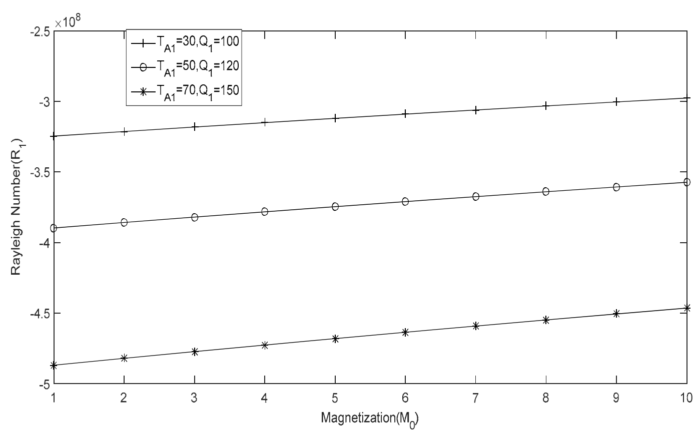

Fig.10. Variation of R_1 with M_0 for fixed $\lambda = -0.5$, $\rho_0 = 10$, $\alpha = 10$, $\gamma = 0.5$, $\nabla H = 10$, $T_{A_1} = 30, 50, 70$ and $Q_1 = 100, 120, 150$.

In Fig.2, critical Rayleigh number R_1 is plotted in opposition to the solute gradient parameter S_1 for $T_{A_1}=100,200,300$; $Q_1=50,100,150$ and $\lambda>0(\lambda=5)$, which suggests that the critical Rayleigh number R_1 increases with an increase in the solute gradient parameter S_1. So, the solute gradient has a stabilizing effect on the system.

In Fig.3, the critical Rayleigh number R_1 is plotted in opposition to the rotation parameter T_{A_1} for $S_1=100,300,500$; $Q_1=0.05$ and $\lambda>0(\lambda=3)$, which indicates that the critical Rayleigh number R_1 increases with an increase in the rotation parameter T_{A_1}. So, rotation has a stabilizing effect on the system.

In Fig.4, the critical Rayleigh number R_1 is plotted in opposition to the rotation parameter T_{A_1} for $S_1=5,10,15;Q_1=500$ and $\lambda<0(\lambda=-7)$, which shows that the critical Rayleigh number R_1 decreases with an increase in the rotation parameter T_{A_1}. So, rotation has a destabilizing effect on the system.

In Fig.5, the critical Rayleigh number R_1 is plotted in opposition to the magnetic field parameter Q_1 for $S_1=5,8,10$; $\mathrm{T}_{A_1}=0.1,0.5,0.8$ and $\lambda>0(\lambda=2)$, which indicates that the critical Rayleigh number R_1 increases with an increase in the magnetic field parameter Q_1. So, the magnetic field has a stabilizing effect on the system.

In Fig.6, the critical Rayleigh number R_1 is plotted in opposition to the magnetic field parameter Q_1 for $S_1=10,15,20$; $\mathrm{T}_{A_1}=1000,1500,2000$ and $\lambda>0(\lambda=5)$, which suggests that the critical Rayleigh number R_1 decreases with an increase in the magnetic field parameter Q_1. So, the magnetic field has a destabilizing effect on the system.

In Fig.7, the critical Rayleigh number R_1 is plotted in opposition to the magnetic field parameter Q_1 for $S_1=25,50,75$; $\mathrm{T}_{A_1}=100,150,200$ and $\lambda<0(\lambda=-2)$, which indicates that the critical Rayleigh number R_1 increases with an increase in the magnetic field parameter Q_1. So, the magnetic field has a stabilizing effect on the system.

In Fig.8, the critical Rayleigh number R_1 is plotted in opposition to the magnetic field parameter Q_1 for $S_1=100,500,800$; $\mathrm{T}_{A_1}=0.02,0.09,0.15$ and $\lambda<0(\lambda=-10)$, which indicates that the critical Rayleigh number R_1 decreases with an increase in magnetic field parameter Q_1. So, the magnetic field has a destabilizing effect on the system.

In Fig.9, the critical Rayleigh number R_1 is plotted in opposition to the magnetization parameter M_0 for $T_{A_1}=50,75,100$; $Q_1=100,150,200$ and $\lambda>0(\lambda=0.1)$, which indicates thatthe critical Rayleigh number R_1 increases with an increase in magnetization parameter M_0. So, the magnetization has a stabilizing effect on the system.

In Fig.10, the critical Rayleigh number R_1 is plotted in opposition to the magnetization parameter M_0 for $T_{A_1}=30,50,70$; $Q_1=100,120,150$ and $\lambda<0(\lambda=-0.5)$, which indicates that the critical Rayleigh number R_1 increases with an increase in the magnetization parameter M_0. So, magnetization has a stabilizing effect on the system.

7. Conclusions

To the best of our knowledge, the problem of the thermosolutal instability of a ferromagnetic fluid in the presence of a variable gravity field, rotation and horizontal magnetic field has not been investigated so far. So, in the present paper, we have discussed the effect of the magnetic field on a rotating ferromagnetic fluid heated and soluted from below in the presence of a variable gravity field. To obtain the dispersion relation, we have used the linearized perturbation theory and normal mode technique. The main results from the evaluation of this present problem are as follows.

- For stationary convection,
  I. the solute gradient has a stabilizing effect on the system.
  II. rotation has a stabilizing effect on the system if $\lambda > 0$ and a destabilizing effect if $\lambda < 0$.
  III. the magnetic field has a stabilizing effect on the system if $\lambda > 0,\ Q_1^2 > T_{A_1}(1+x_w)$ and $\lambda < 0,\ Q_1^2 < T_{A_1}(1+x_w)$. Also, the magnetic field has a destabilizing effect on the system if $\lambda > 0,\ Q_1^2 < T_{A_1}(1+x_w)$ and $\lambda < 0,\ Q_1^2 > T_{A_1}(1+x_w)$. In the absence of rotation; the magnetic field has a stabilizing effect on the system if $\lambda > 0$ and a destabilizing effect if $\lambda < 0$.
  IV. in the absence of the solute gradient; magnetization has a stabilizing effect for both $\lambda > 0$ and $\lambda < 0$.
- The principle of exchange of stabilities is not valid for the present problem under consideration, whereas in the absence of rotation, magnetic field and stable solute gradient, the principle of exchange of stabilities (PES) is valid for the present problem if $\lambda g_0 \geq \dfrac{\gamma' M_0 \nabla H}{\rho_0 \alpha}$.

Nomenclature

d	− depth of layer [m]
a	− dimensionless wave number
$\mathbf{g}$	− acceleration due to gravity [m/s^2]
g	− gravity field [m/s^2]
k	− wave number [1/m]
k_x, k_y	− horizontal wave numbers [1/m]
n	− growth rate [1/s]
p	− fluid pressure [pa]
Q	− Chandrasekhar number
T_A	− Taylor number
R	− Rayleigh number
S	− solute Rayleigh number
T	− temperature [K]
t	− time [s]
$\mathbf{\Omega}$	− rotation vector having components $(0,0,\Omega)$
(u_1, u_2, u_3)	− component of velocity after perturbation
α	− coefficient of thermal expansion [1/K]
β	− uniform temperature gradient [K/m]
β'	− uniform solute gradient [K/m]
θ	− perturbation in temperature [K]
γ	− perturbation in concentration [K]
κ_T	− thermal diffusivity [m^2/s]
κ_S	− solute diffusivity [m^2/s]
ν	− kinematic viscosity [m^2/s]
ρ	− density [Kg/m^3]
∇	− del operator
∂, D	− curly operators and derivative with respect to $z(d/dz)$

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