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THE EFFECT OF THERMAL MODULATION ON DOUBLE DIFFUSIVE CONVECTION IN THE PRESENCE OF APPLIED MAGNETIC FIELD AND INTERNAL HEAT SOURCE

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The investigation of thermal modulation on double-diffusive stationary convection in the presence of an applied magnetic field and internal heating is carried out. A weakly nonlinear stability analysis has been performed using the finite-amplitude Ginzburg-Landau model. This finite amplitude of convection is obtained at the third order of the system. The study considers three different forms of temperature modulations. OPM-out of phase modulation, LBMO-lower boundary modulation, IPM-in phase modulation. The finite-amplitude is a function of amplitude δ_T, frequency ω and the phase difference θ. The effects of δ_T and ω on heat/mass transports have been analyzed and depicted graphically. The study shows that heat/mass transports can be controlled effectively by thermal modulation. Further, it is found that the internal Rayleigh number R_i enhances heat transfer and reduces the mass transfer in the system.

Keywords: thermal modulation, weak nonlinear analysis, internal heating, Newtonian fluid, double diffusive convection.

1. Introduction

The study of two-component thermohaline convection in porous media has found numerous applications. Due to the temperature and solutal fluctuation, there is a variation in fluid viscosity and density. Convection of two different density gradients with different rates of diffusion is known as thermohaline convection. There are many other applications related to natural convection. Some of the applications include are geology, astrophysics, and metallurgy. The basic and fundamental application of thermohaline convection is in oceanography, where heat and concentrations components exist with different gradients and diffuse at differing rates. There are other situations in which this convection takes place solidification of a binary mixture, migration of solutes in water-saturated soils, electrochemistry, crystal growth, geophysical system, the migration of moisture through air contained in fibrous insulation, earth's oceans, magma chambers, etc.

Hydrodynamic thermal instability is well documented and has been investigated, among others by Chandrasekhar [1]. Fundamental studies of double diffusion convection were mode by Huppert *et al.* [2] and Rudraiah and Shivakumara [3]. They investigated linear and nonlinear instability of double-diffusive convection in the presence of an imposed magnetic field. Stability analysis was discussed in terms of the

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critical stationary and oscillatory Rayleigh numbers. Hill [4] investigated linear and nonlinear thermal instability of double-diffusive convection using the Darcy model. It was found that oscillatory convection occurs when $\gamma_c \leq \gamma$ where γ is a measure of the internal heat source.

Double-diffusive convection in a Maxwell fluid-saturated porous layer with an internal heat source was investigated by Moli *et al*. [5]. Linear and nonlinear stability analysis was discussed using normal mode and truncated representation of the Fourier series. Baines and Gill [6] investigated double-diffusive convection and showed that direct convection can occur. Due to several applications of thermohaline convection, Huppert and Turner [7] investigated sea-going oceanography related theories. Interaction between double-diffusive convection and an imposed vertical magnetic field was studied by Rudraiah [8]. He presented the studies of linear and nonlinear theories. He also discussed the cross-diffusion effect on rotating media and chemical reaction on thermohaline magneto convection with pattern selection.

It is very important to understand the stability or instability of the Rayleigh-Benard convection under the modulation effect. It was Venezian [9] who studied the thermal modulation effect on Rayleigh-Benard convection. He used the normal mode and perturbation techniques to make stability analysis. He also obtained correction in the critical Rayleigh number as a function of wavenumber and frequency of modulation to determine the onset convection. Double-diffusive convection in a horizontal and sparsely packed porous layer was investigated by Poulikakos [10]. Similarly, Double-diffusive convection in a Maxwell fluid-saturated porous media was investigated by Wang and Tan [11]. The exchange of stability was discussed under the effect of the relaxation parameter.

Double-diffusive convection in a viscoelastic fluid-saturated porous layer using a thermal non-equilibrium model was investigated by Kumar and Bhadauria [12]. They studied linear stability using the normal mode technique and determined correction in the critical Rayleigh number. Using the truncated Fourier series method they investigated the local nonlinear theory. The effect of rotation on double-diffusive convection was investigated by Kumar and Bhadauria [13]. They presented rotational effects of double-diffusive convection following the studies of Kumar and Bhadauria [12] for porous media. The onset of double-diffusive convection in a binary Maxwell fluid-saturated porous layer with a cross-diffusion effect was investigated by Malasetty *et al.* [14].

The onset of instability of nano-fluid saturated porous media using a Galerkin method was studied by Kuznetsov and Nield [15]. The onset criteria were found through critical R_{0c} and the delay in the onset of convection due to the presence of nano-fluids was discussed. The effect of through flow and g-jitter on double-diffusive oscillatory convection in a viscoelastic fluid-saturated porous medium using complex Ginzburg-Landau model is given by Manjula *et al*. [16].

The effect of thermal modulation on double-diffusive convection was studied by Bhadauria *et al.* [17-19]. The effect of thermal modulation on double-diffusive convection [14], on magneto convection double-diffusive convection [15], on oscillatory double-diffusive convection [16], was investigated using the Ginzburg Landau model. The effect of IPM, OPM, and LBM on heat mass transport in the media was discused. The effect of OPM (out of phase modulation) is better than IPM (in phase modulation), and LBM (Lower Boundary Modulation). A similar modulation was studied by Kiran [20] where the effect of thermal modulation is discussed with nonlinear through-flow effects. Binary Maxwell fluid convection in fluid land porous layers was investigated by Narayana *et al*. [21,22]. Chaotic and oscillatory magneto convection using the Lorenz and complex GLM model was studied by Bhadauria and Kiran [23, 24]. The effect of temperature modulation on oscillatory convection was discussed by Bhadauria and Kiran [25]. This problem of convection in a viscoelastic fluid-saturated porous layer was extended to through-flow effects by Kiran *et al*. [26]. The effect of gravity modulation and nonlinear through-flow on thermal instability was investigated by Manjula *et al*. [27].

In many situations the internal heat source is very important, e.g., nuclear reactions, nuclear heat cores, nuclear energy, nuclear waste disposal, oil extractions, and crystal growth. Research on internal heat generation is very important. To form a better solid during solidification, it is very important to hold the internal energy to keep the temperature of the metal uniform. Bhadauria *et al*. [28] investigated the study of heat transport in a porous medium under gravity modulation and internal heating effects. Heat transfer results are obtained from the Ginzburg Landau equation. The same problem was extended to time-periodic thermal boundary conditions and internal heating by Bhadauria *et al*. [29].

The problem of [29] was extended to a layer of nano-fluid by Bhadauria *et al.* [30]. The eigenvalue problem is solved using the truncated Fourier series. The effect of rotational speed modulation on heat transport in a fluid layer with temperature-dependent viscosity and internal heat source was investigated by Bhadauria and Kiran [31]. The effect of temperature-dependent viscosity and internal heat source was found to enhance heat transfer. Kiran *et al*. [32,33] investigated the effect of internal heating on nanofluid convection h. The effect of internal heating on magneto-double diffusive convection in a viscoelastic fluid-saturated porous layer was studied by Altawallbeh *et al*. [34]. The study of linear and weakly nonlinear thermohaline convection in a viscoelastic fluid-saturated anisotropic porous medium with internal heat source was investigated by Srivastava and Singh [35]. The onset of thermal instability was investigated through the critical Rayleigh number. Heat and mass transfer results were obtained through the Nusselt and Sherwood numbers.

Bhadauria and Kiran [36] investigated weakly nonlinear double-diffusive convection in temperature-dependent viscous fluid-saturated porous media under temperature modulation. They derived the Ginzburg Landau equation and found the heat and mass transfer under three types of thermal modulations. The same problem was extended to gravity modulation by Bhadauria and Kiran [37]. They plotted the streamlines, isotherms, and isohalines to represent the nature of convection.

The effect of internal heating on double-diffusive convection in a couple stress fluid-saturated anisotropic porous medium was investigated by Srivastava *et al*. [38]. The linear stability analysis was mode by a critical Rayleigh number for both stationary and oscillatory convection. Through local nonlinear stability analysis, heat mass transfer was quantified. The effect of internal solutal Rayleigh number and rotation on weakly nonlinear [41-45] thermal instability was studied by Kiran [39] and Kiran and Manjula [40]. The rotation and negative internal solutal Rayleigh number reduce the mass transfer in the system (see: Malkus and Veronis [41]). Their studies gave new results and were compared with Keshri *et al*. [42]. Using the Landau model [43, 44], the effect of thermal modulation [45-48] on double-diffusive convection in the presence of a magnetic field was investigated by Bhadauria and Kiran *et al*. [49]. It was found that Chandrasekhar number Q reduces heat transfer and the Lewis number, magnetic Prandtl number enhance heat transfer in the system.

The above literature shows that no study reports the effect of the applied magnetic field on double-diffusive convection under thermal modulation [50-54]. Since the applied magnetic field with thermal modulation suppresses the heat mass transfer one needs to study thermohaline convection with an applied magnetic field. This motivates us to investigate the effect of internal heating and thermal modulation on double-diffusive convection in the presence of an applied magnetic field.

2. Mathematical model

We consider a Newtonian fluid layer extended infinitely in the x-direction and confined between two parallel horizontal plates at $z = 0$ and $z = d$, a distance d apart. We consider double-diffusive convection with an upward vertical transport of heat and salt mixture in the layer. The configuration of the physical model is shown in Fig.1. A Cartesian frame of reference is chosen in which the origin lies on the lower plate and the z-axis goes vertically upward. Further, the Boussinesq approximation is employed to consider the density variations caused by gradients in the composition of the fluid. With the above assumptions, the required mathematical model is given by Kiran *et al.* [57]:

$$\nabla \cdot \vec{q} = 0 , \tag{2.1}$$

$$\frac{\partial \vec{q}}{\partial t} + (\vec{q} \cdot \nabla)\vec{q} = -\frac{1}{\rho_0}\nabla p + \frac{\rho}{\rho_0}\vec{g} - \frac{\mu}{\rho_0}\nabla^2 \vec{q} - \sigma\mu_e^2 B_0^2 \vec{q} , \tag{2.2}$$

$$\gamma\frac{\partial T}{\partial t} + (\vec{q} \cdot \nabla)T = \kappa_T \nabla^2 T + Q(T - T_0) , \tag{2.3}$$

$$\frac{\partial S}{\partial t}+(\vec{q}.\nabla)S=\kappa_s\nabla^2 S\,, \tag{2.4}$$

$$\rho=\rho_0\left[1-\alpha_T(T-T_0)+\alpha_S(S-S_0)\right] \tag{2.5}$$

where $\vec{q}$ is the velocity (u,v,w), μ is the viscosity, κ_T is the thermal diffusivity tensor, κ_s is the solutal diffusivity tensor, $\boldsymbol{T}$ is the temperature, α_T is the thermal expansion coefficient, γ is the heat capacity and Q is the internal heat source. For simplicity, γ is taken to be unity in this paper, ρ is the density, while ρ_0 is the reference density, μ_e is the magnetic permeability, B_0 is the strength of the applied magnetic field.

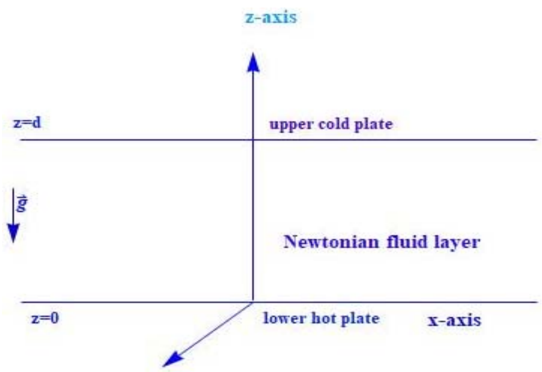


Fig.1. The physical configuration of the problem.

The externally imposed thermal boundaries are:

$$\begin{aligned} &T=T_0+\frac{\Delta T}{2}\left[1+\chi^2\delta_T\cos(\omega t)\right], && z=0\,, \\ &T=T_0-\frac{\Delta T}{2}\left[1-\chi^2\delta_T\cos(\omega t+\theta)\right], && z=d\,, \end{aligned} \tag{2.6}$$

here δ_T is the small amplitude of modulation, ω is the frequency of modulation and θ is the phase difference. The externally imposed solutal boundary conditions are:

$$\begin{aligned} &S=S_0-\frac{\Delta S}{2}, && z=0\,, \\ &S=S_0+\frac{\Delta S}{2}, && z=d \end{aligned} \tag{2.7}$$

where ΔT is the temperature difference, ΔS is the solutal difference across the fluid layer, T_0 and S_0 are reference temperature and concentration. The modulated temperature Eq.(2.6) field consists of steady and time-dependent parts given by Venezian [7]. Slow time variations are considered to prevent exponential growth of the amplitude at the steady-state.

2.1. Conduction state

In this state the fluid and concentrations are at rest, the transport of heat and mass in a conduction state only. In this state, the physical variables are of the form:

$$q_b = (0,\ 0,\ 0), \quad \rho = \rho_b(z), \quad p = p_b(z), \quad T = T_b(z), \quad S = S_b(z)\,. \tag{2.8}$$

Substituting Eq.(2.8) into Eqs (2.1)-(2.5), we get the following relations which help us to define the basic state pressure, temperature, and solute:

$$\frac{dp_b}{dz} = -\rho_b \vec{g}\,, \tag{2.9}$$

$$\kappa_T \frac{d^2(T_b - T_0)}{dz^2} + Q(T_b - T_0) = \frac{\partial T_b}{\partial t}\,, \tag{2.10}$$

$$\rho_b = \rho_0\left[1 - \alpha_T(T_b - T_0)\right], \tag{2.11}$$

$$\frac{d^2 S_b}{dz^2} = 0 \tag{2.12}$$

where b refers the basic state. Equations (2.10) and (2.12) are solved for $T_b(z)$ and $S_b(z)$ subject to the boundary condition given in Eq.(2.6) and Eq.(2.7)

$$T(z,t) = T_b(z) + \chi^2 \delta_T \operatorname{Re}\left[T_1(z,t)\right], \tag{2.13}$$

$$S_b(z) = S_0 + \Delta S\left(1 - \frac{z}{d}\right). \tag{2.14}$$

2.2. Perturbed state

At this state, the disturbances impose on the conduction state, and then the physical variables take the form:

$$\vec{q} = q_b + \vec{q}^{\,/}, \quad \rho = \rho_b + \rho^{/}, \quad p = p_b + p^{/}, \quad T = T_b + T^{/}, \quad S = S_b + S^{/}. \tag{2.15}$$

Substituting Eq.(2.15) in Eqs (2.1)-(2.4), and using the basic state solution, we get:

$$\nabla \cdot \vec{q}^{\,/} = 0\,, \tag{2.16}$$

$$\frac{\partial \vec{q}^{\,/}}{\partial t} + \left(\vec{q}^{\,/} \cdot \nabla\right)\vec{q}^{\,/} = -\frac{1}{\rho_0}\nabla p^{/} + \frac{\rho}{\rho_0}\vec{g} - \frac{\mu}{\rho_0}\nabla^2 \vec{q}^{\,/} - \sigma \mu_e^2 B_0^2 \vec{q}^{\,/}\,, \tag{2.17}$$

$$\gamma \frac{\partial T^{/}}{\partial t} + w^{/}\frac{dT_b}{dz} + \left(\vec{q}^{\,/} \cdot \nabla\right)T^{/} = \kappa_T \nabla^2 T^{/} + R_i T^{/}\,, \tag{2.18}$$

$$\frac{\partial S^{/}}{\partial t}+(\vec{q}.\nabla)S^{/}=\kappa_s\nabla^2 S^{/}. \tag{2.19}$$

For two-dimensional convection, one can introduce a stream function ψ as $u'=\frac{\partial\psi}{\partial z}$, $w'=-\frac{\partial\psi}{\partial x}$. Non-dimensional physical variables are: $(x,y,z)=d\left(x^*,y^*,z^*\right)$, $\tau\kappa_s=d^2t^*$, $d\vec{q}=\kappa_s\vec{q}^*$, $\psi=\kappa_s\psi^*$, $T'=\Delta T\,T^*$, $S'=\Delta S\,S^*$ and $\omega d^2=\kappa_s\omega^*$, then eliminating the pressure term and finally dropping the asterisk, we obtain the non-dimensional governing system as:

$$-\nabla^4\psi+Ha^2\nabla^2\psi+\left(\mathrm{Ra_T}\frac{\partial T}{\partial x}-Rs\frac{\partial S}{\partial x}\right)=-\frac{1}{Pr}\frac{\partial\nabla^2\psi}{\partial\tau}+\frac{\partial\left(\psi,\nabla^2\psi\right)}{\partial(x,z)}, \tag{2.20}$$

$$-\frac{\partial T_b}{\partial z}\frac{\partial\psi}{\partial x}-\left(\nabla^2-R_i\right)T=-\frac{\partial T}{\partial\tau}-\frac{\partial(\psi,T)}{\partial(x,z)}, \tag{2.21}$$

$$-\frac{\partial S_b}{\partial z}\frac{\partial\psi}{\partial x}-\frac{1}{Le}\nabla^2 S=-\frac{\partial S}{\partial\tau}+\frac{\partial(\psi,S)}{\partial(x,z)}. \tag{2.22}$$

The non-dimensional parameters in the above equations are:

Thermal Rayleigh number	$\mathrm{Ra}_T=\frac{\alpha_T g\Delta T d^3}{\kappa_T\nu}$	Hartmann number Ha	$\mathrm{Ha}=\mu_e B_0 d\sqrt{\frac{\sigma}{\nu}}$
Solutal Rayleigh number	$\mathrm{Rs}=\frac{\alpha_S g\Delta S d^3}{\kappa_S\nu}$,	Internal Rayleigh number R_i	$R_i=\frac{Qd^2}{\kappa_T}$
Prandtl number	$\mathrm{Pr}=\frac{\nu}{\kappa_\mathrm{T}}$,	Lewis number Le	$\mathrm{Le}=\frac{\kappa_T}{\kappa_S}$.

The term $\frac{\partial T_b}{\partial z}$ is given in Eq.(2.21) and simplified:

$$\frac{\partial T_b}{\partial z}=f_1(z)+\chi^2\,\delta_T\left[f_2(z,t)\right],$$

$$f_2=Re\left[f(z)e^{-i\omega t}\right],$$

$$f_1=-\frac{\sqrt{R_i}}{2\sin\left(\sqrt{S}_i\right)}[\cos\sqrt{R}_i(1-z)+cos\sqrt{R}_i(z)], \tag{2.23}$$

$$f(z)=A(m)e^{mz}+A(-m)e^{-mz}$$

$$m=\sqrt{\lambda^2-T_i},\quad \lambda^2=-i\omega \quad \text{and} \quad A(m)=\frac{m}{2}\frac{e^{m\theta}-e^{-m}}{e^m-e^{-m}}.$$

The reader may refer to the studies of Bhadauria *et al*. [29] and Manjula *et al*. [48, 56] for results of Eq (2.23). Since we study slow variations in heat and mass transfer the time is re-scaled as $\tau = \chi^2 2t$. The nonlinear system of partial differential equations (2.20)-(2.22) is solved using the following stress free isothermal boundary conditions:

$$\psi = T = S = \frac{\partial^2 \psi}{\partial z^2} = 0 \qquad \text{at} \qquad z = 0 \qquad \text{and} \qquad z = 1 . \tag{2.24}$$

3. Finite amplitude equation and heat transport

Now consider the following asymptotic expansions, suggested by Venezian [7], Malkus and Veronis [41], Manjula *et al.* [43] and Kiran *et al.* [44], (it is a formal series of functions which has the property that truncating the series after a finite number of terms provides an approximation to a given function) introduced in Eqs (2.20)-(2.22) to resolve the nonlinearity:

$$\left(\psi, T, S, Ra_T\right) = \chi^0 \left(0, 0, 0, R_{0c}\right) + \chi^1 (\psi_1, T_1, S_1, 0) + \chi^2 (\psi_2, T_2, S_2, R_2) + \ldots \tag{3.1}$$

where R_{0c} is the critical Rayleigh number at which instability takes place. The system equations (2.20)-(2.22) will be solved for different orders of χ .

3.1. Lowest order case $\left(\chi^1\right)$

This order is just like a linear problem because no nonlinear term appears in this case:

$$\begin{pmatrix} \nabla^2 Ha^2 - \nabla^4 & -R_{0c}\dfrac{\partial}{\partial \mathrm{x}} & Rs\dfrac{\partial}{\partial \mathrm{x}} \\ -\dfrac{\partial \mathrm{T_b}}{\partial \mathrm{z}}\dfrac{\partial}{\partial \mathrm{x}} & -\nabla^2 - R_i & 0 \\ \dfrac{\partial}{\partial \mathrm{x}} & 0 & -\dfrac{1}{\mathrm{Le}}\nabla^2 \end{pmatrix} \begin{pmatrix} \psi_1 \\ T_1 \\ S_1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} . \tag{3.2}$$

The following solutions are assumed according to the boundary conditions (given in Eq.(2.24)):

$$\psi_1 = A(\tau)\sin\left(k_c x\right)\sin\left(\pi z\right) , \tag{3.3}$$

$$T_1 = \frac{-4\pi^2 k_c}{\delta_R^2 \left(4\pi^2 - R_i\right)} A(\tau)\cos\left(k_c x\right)\sin\left(\pi z\right) , \tag{3.4}$$

$$S_1 = -\frac{k_c Le}{\beta^2} A(\tau)\sin\left(k_c x\right)\cos\left(\pi z\right) , \tag{3.5}$$

where $\delta_R^2 = \beta^2 - R_i$ and $\beta^2 = k_c^2 + \pi^2$. The critical value of the Rayleigh number for the onset of stationary convection is given by:

$$R_{0c} = \frac{\delta_R^2\left(\beta^4 + Ha^2\beta^2\right)}{I_1 k_c^2} - \frac{\delta_R^2 Rs\, Le}{I_1 \beta^2} \tag{3.6}$$

where $I_1 = \int_0^{\frac{2\pi}{k_c}} \frac{dT_b}{dz} \sin^2 \pi z dz$ is the thermal modulation coefficient.

3.2. Second order case $\left(\chi^2\right)$

In this order, the nonlinear effects enter the system through the Jacobian term of Eqs (2.20)-(2.22). We get the following relation:

$$\begin{pmatrix} \nabla^2 Ha^2 - \nabla^4 & -R_{0c}\frac{\partial}{\partial \mathrm{x}} & Rs\frac{\partial}{\partial \mathrm{x}} \\ -\frac{\partial \mathrm{T_b}}{\partial \mathrm{z}}\frac{\partial}{\partial \mathrm{x}} & -\nabla^2 - R_i & 0 \\ \frac{\partial}{\partial \mathrm{x}} & 0 & -\frac{1}{\mathrm{Le}}\nabla^2 \end{pmatrix} \begin{pmatrix} \psi_2 \\ T_2 \\ S_2 \end{pmatrix} = \begin{pmatrix} R_{21} \\ R_{22} \\ R_{23} \end{pmatrix} \tag{3.7}$$

where

$$R_{21} = 0, \tag{3.8}$$

$$R_{22} = \frac{\partial\left(\psi_1, T_1\right)}{\partial\left(x, z\right)}, \tag{3.9}$$

$$R_{23} = \frac{\partial\left(\psi_1, S_1\right)}{\partial\left(x, z\right)}. \tag{3.10}$$

The second order solutions subjected to the boundary conditions Eq.(2.24) are obtained as:

$$\psi_2 = 0, \tag{3.11}$$

$$T_2 = \frac{-2\pi^3 k_c^2}{\delta_R^2\left(4\pi^2 - R_i\right)^2} A(\tau)^2 \sin\left(2\pi z\right), \tag{3.12}$$

$$S_2 = \frac{-k_c^2 Le^2}{8\pi\beta^2} A(\tau)^2 \sin\left(2\pi z\right). \tag{3.13}$$

The horizontally averaged Nusselt number for the stationary mode of convection is given by:

$$\mathrm{Nu} = \frac{heat\,transfer\,due\,to\left(conduction + convection\right)}{heat\,transfer\,due\,to\left(conduction\right)}. \tag{3.14}$$

Substituting the expressions of T_2 and T_b in the Eq.(3.14) and simplifying, we get:

$$\mathrm{Nu} = 1 + \frac{8\pi^4 k_c^2 \sin\sqrt{R_i}}{\delta_R^2 \left(4\pi^2 - R_i\right)^2 \sqrt{R_i}\left(\cos\sqrt{R_i} + 1\right)} A(\tau)^2 . \tag{3.15}$$

The horizontally averaged Sherwood number for the stationary mode of convection is given by:

$$\mathrm{Sh} = \frac{mass\,transfer\,due\,to\left(conduction + convection\right)}{mass\,transfer\,due\,to\left(conduction\right)} . \tag{3.16}$$

Substituting the expressions of S_2 and S_b in Eq.(3.16) and simplifying, we get:

$$\mathrm{Sh} = 1 + \frac{k_c^2\,\mathrm{Le}^2}{4\beta^2} A(\tau)^2 . \tag{3.17}$$

3.3. Third order case $\left(\chi^3\right)$

At this stage, the following system is obtained involving many terms in RHS:

$$\begin{pmatrix} \nabla^2 Ha^2 - \nabla^4 & -R_{0c}\dfrac{\partial}{\partial \mathrm{x}} & Rs\dfrac{\partial}{\partial \mathrm{x}} \\ -\dfrac{\partial \mathrm{T_b}}{\partial \mathrm{z}}\dfrac{\partial}{\partial \mathrm{x}} & -\nabla^2 - R_i & 0 \\ \dfrac{\partial}{\partial \mathrm{x}} & 0 & -\dfrac{1}{\mathrm{Le}}\nabla^2 \end{pmatrix} \begin{pmatrix} \psi_3 \\ S_3 \\ V_3 \end{pmatrix} = \begin{pmatrix} R_{31} \\ R_{32} \\ R_{33} \end{pmatrix}, \tag{3.18}$$

$$R_{31} = -\frac{1}{Pr}\frac{\partial \nabla^2 \psi_1}{\partial \tau} - \mathrm{R}_2 \frac{\partial T_1}{\partial \mathrm{x}} - \mathrm{R}_{0\mathrm{c}} \frac{\partial T_2}{\partial \mathrm{x}} + \mathrm{Rs}\frac{\partial S_2}{\partial \mathrm{x}} , \tag{3.19}$$

$$R_{32} = -\frac{\partial T}{\partial \tau} + \frac{\partial T_2}{\partial \mathrm{z}}\frac{\partial \psi_1}{\partial \mathrm{x}} - \frac{\partial T_1}{\partial \mathrm{z}}\frac{\partial \psi_2}{\partial \mathrm{x}} - \delta_T f_2 \frac{\partial \psi_1}{\partial \mathrm{x}} , \tag{3.20}$$

$$R_{33} = -\frac{\partial S}{\partial \tau} + \frac{\partial S_2}{\partial \mathrm{z}}\frac{\partial \psi_1}{\partial \mathrm{x}} . \tag{3.21}$$

Substituting the first and second order solutions in Eqs (3.19)-(3.21), R_{31}, R_{32} and R_{33} will be simplified. Using the following solvability condition [27, 29, 39, 43-44] we derive the Ginzburg-Landau equation:

$$\int_0^1 \int_0^{\frac{2\pi}{k_c}} \left(R_{31}\widehat{\psi_1} + R_{32}\widehat{T_1} + R_{33}\widehat{S_1} \right) dxdz = 0 \tag{3.22}$$

where $\left(\widehat{\psi_1}, \widehat{T_1}, \widehat{S_1}\right)$ is the solutions set of first order adjoint system;

$$\Delta_1 \frac{\partial A(\tau)}{\partial \tau} + F(\tau) A(\tau) - \Delta_2 A(\tau)^3 = 0 ,$$

$$\Delta_1 = \frac{\beta^2}{Pr} + \frac{\left(\beta^4 + Ha^2\beta^2\right)k_c^2}{\delta_R^2} - \frac{k_c^2 Rs\, Le^2}{\beta^4}, \qquad F(\tau) = \frac{R_{0c} k_c^2}{\delta_R^2}\left(1 + \frac{4\pi^2}{\left(4\pi^2 - R_i\right)} 2\mathrm{I}_2\right), \tag{3.23}$$

$$\Delta_2 = \frac{\pi^2\left(\beta^4 + Ha^2\beta^2\right)k_c^2}{2\delta_R^2\left(4\pi^2 - R_i\right)} - \frac{k_c^2 Rs\, Le^3}{8\beta^4}, \qquad I_2 = \int_0^1 f_2 \sin^2 \pi z dz .$$

Equation (3.23) is a Ginzburg-Landau equation (GLE) also known as the Bernoulli equation. Obtaining its analytical solution is difficult due to its non-autonomous nature. So, it has been solved numerically using NDSolve of Mathematica 17 with suitable initial amplitude $A(0) = \alpha_0$. In our calculations, we use $R_2 = R_{0c}$, to keep the parameters to a minimum. The Nusselt number (3.15) and Sherwood number (3.17) is evaluated numerically using the value of $A(\tau)$ while solving Eq.(3.23).

4. Results and discussions

We have investigated the effects of thermal modulation [50-55] and internal heating on double-diffusive stationary convection. The layer is confined with electrically conducting Newtonian fluid. The Ginzburg-Landau model was employed to study finite-amplitude convection. Our aim is to analyze heat/mass transfer in the system. The non-autonomous amplitude equation (3.23) is solved numerically using the Runge-Kutta 4th order method. The numerical results for Nu/ Sh and the effect of each parameter on heat/mass transport are reported in Figs 2-12. It is found that the values of Nu/ Sh start with 1 showing the conduction state. Also, Nu/ Sh increases with time showing the convection state and further in time the system achieves its uniform nature. Now we discuss the effect of different parameters on Nu/ Sh .

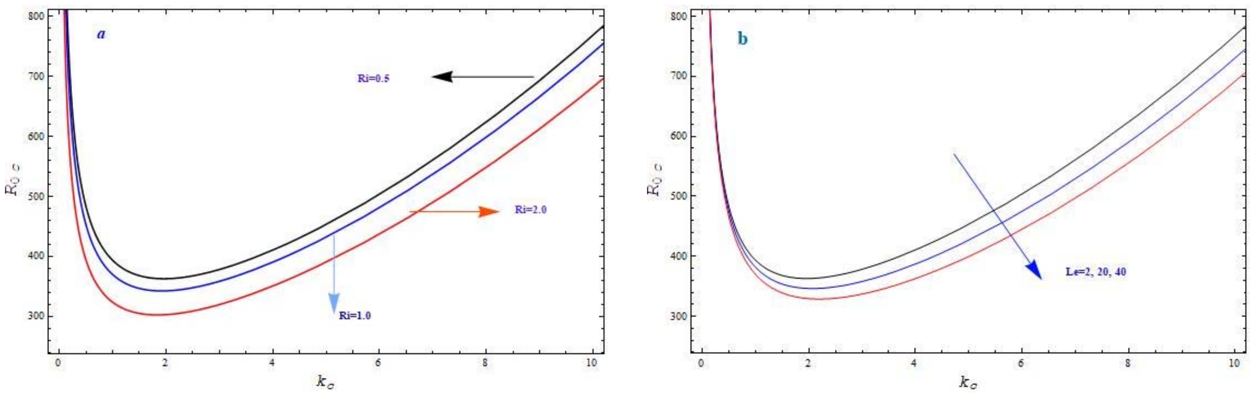


Fig.2. Variation of the thermal Rayleigh number for various values of the internal heat parameter R_i .

We discuss the results in two parts (i) marginal stability curves (ii) heat and mass transfer. The graphs of neutral stability curves for various parameters are shown in Figs 2 and 3. The parameter values are fixed at $R_i = 0.4$, $\Pr = 1$, $\mathrm{Ha} = 2$, $\mathrm{Le} = 1.5$, $\mathrm{Rs} = 10$, $\delta_1 = 0.1$, and $\omega = 4$. Marginal stability curves are drawn based on the critical Rayleigh number R_{0c} (defined in Eq.(3.6))at which the onset of convection takes place.

In Fig.2 the stability curves have been depicted. Upon increasing the value of the internal Rayleigh number R_i and Lewis number Le the thermal Rayleigh number decreased, resulting in advance of the onset of convection. The corresponding stability curves are presented in Figs 2a and 2b. These are the results were obtained by Srivastava *et al.* [35] and Manjula *et al.* [48,56,57] for convection in porous media. The effect of the Hartmann number and solutal Rayleigh number on R_{0c} versus wavenumber is presented in Fig. 3.

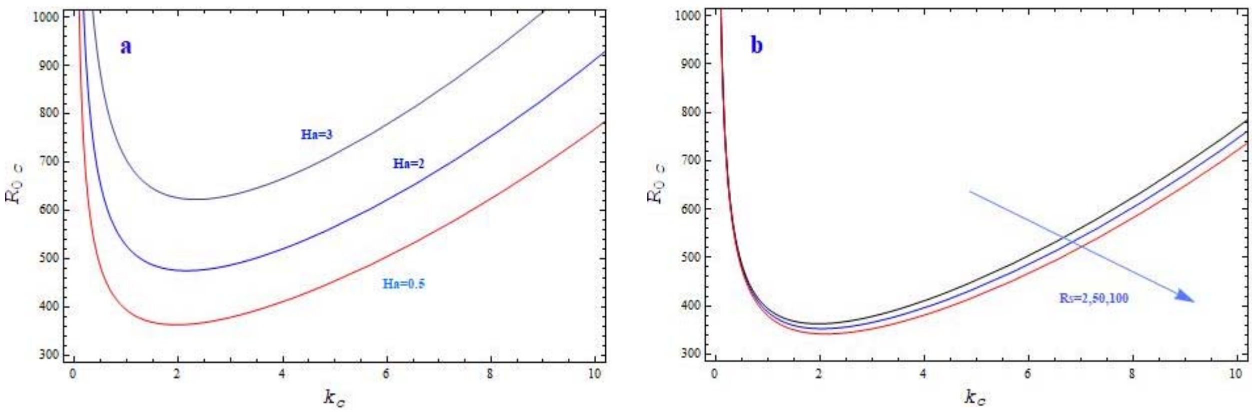


Fig.3. The stability curves under the effect of the magnetic parameter Ha .

Upon increasing the value of Ha, the critical Rayleigh number R_{0c} increases, thus the effect of Ha is to stabilize the system (see Fig.3a). This indicates that the applied magnetic field is to suppress the stability. The trend is reversed for the solutal Rayleigh number Rs , where R_{0c} decreases upon increasing the values of Rs showing the destabilizing effect (see Fig.3b).

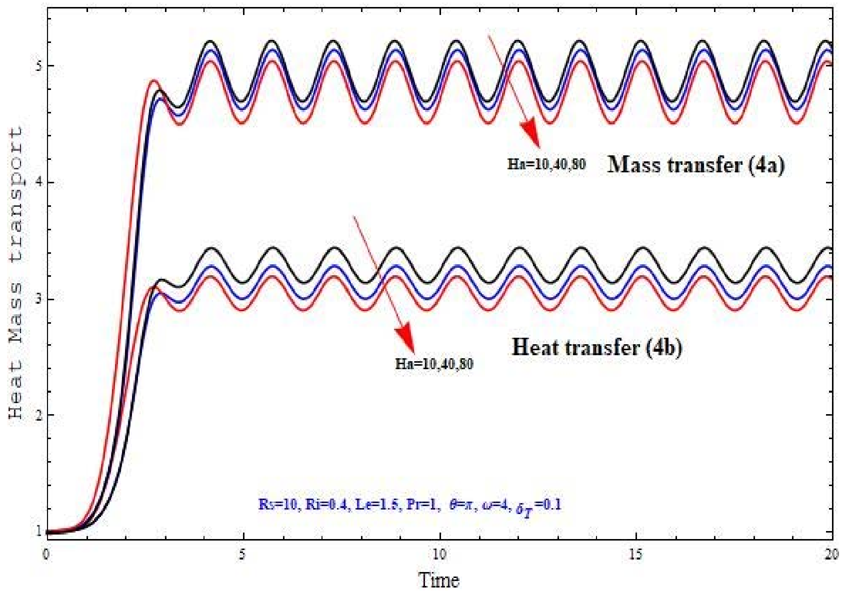


Fig.4. The effect of the Hartmann number on Sh and Nu .

Now we present our results corresponding to a weakly nonlinear theory. Using a weakly nonlinear theory the Ginzburg-Landau equation [27, 29, 39, 43-45, 60-65] (given in Eq.(3.23)) is derived. The modulation effect is considered at third order, where the solvability condition is used for the existence of GLE. Using this equation heat and mass transfer results are presented in terms of the Nusselt and Sherwood numbers given in Eqs (3.15 and 3.17). Since the results are almost similar for both Nu and Sh, first we present the results for Nu. In Fig.4 (a), the effect of the Hartmann number (see Keshri *et al*. [42]) on Sh is presented. It is a ratio of electromagnetic force to the viscous force and measures the strength of the magnetic force to the viscous force. An increment in Ha decreases the value of Sh showing mass transfer reduction. The results corresponding to heat transfer are presented in Fig.4 (b). The same effect of Ha can be seen, but mass transfer levels are more than these of heat transfer rates. To see the results clearly the same results repeated in Fig.5(b). These results are compared with Kiran *et al*. [58, 59].

Figure 5(a) illustrates that the effect of the internal heat parameter R_i on Nu, it is clear from the figure that heat transfer increases upon increasing the value of R_i from 0.4 to 0.9. The reason behind this is to decrease the value of the critical thermal number R_{0c} with an increment of R_i for stationary case (see Fig.(2)a). Therefore, the effect of internal heat parameter is to destabilize the system (also see the studies of [60-65]). These results confirm the studies of Kiran *et al.* [32,33] and are comparable with the results of Kiran *et al.* [40,45,57] for solutal internal Rayleigh number S_i. Figure 5(b) depicts the effect of the internal heat parameter R_i on Sh, it is clear from the figure that mass transfer decreases upon increasing the value R_i from 0.4 to 0.9. Therefore, the effect of the internal heat parameter is to stabilize the system.

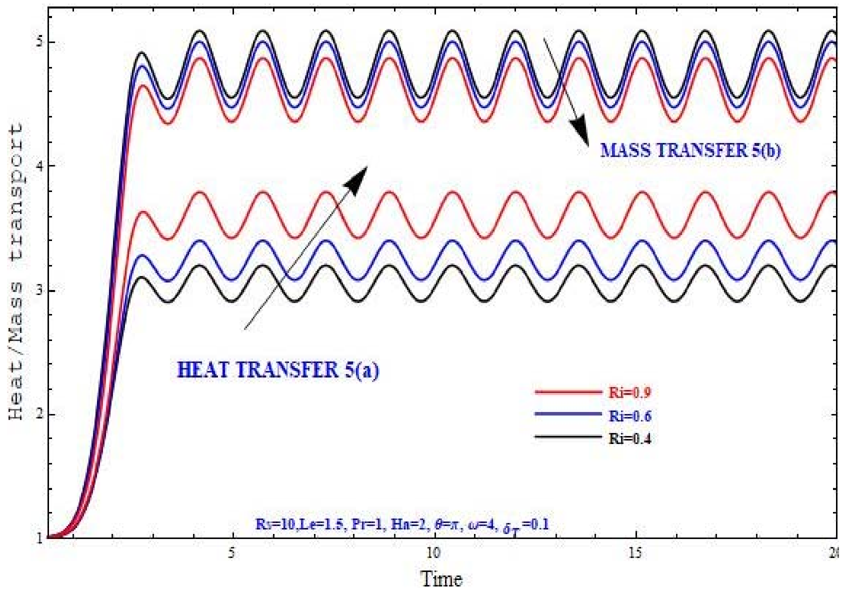
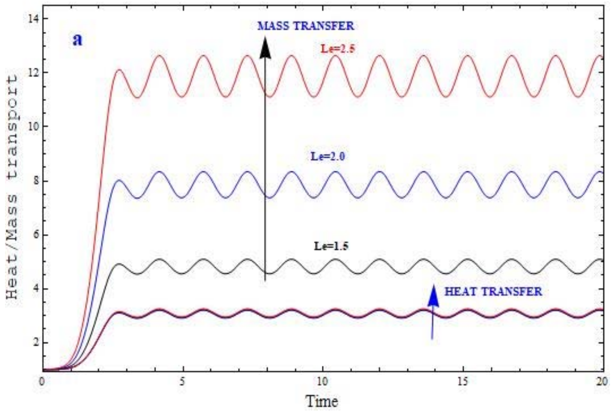


Fig.5. The effect of the internal Rayleigh number on Sh and Nu.

In Fig.6(a) the effect of the Lewis number (Le) on Sh is depicted. This Lewis number is used to characterize the flows of fluids where there is a simultaneous momentum and diffusion transfer. It is found that mass transfers enhance in the system drastically for lower values of Le. A similar offer of Le on Nu can be seen but there is a very slow increment therein (see Fig.6(b)). The effect of the Prandtl number Pr on Sh is presented in Fig.7(a). It defines the ratio of momentum diffusivity to thermal diffusivity. When Pr is near to 1 then both momentum and heat dissipate through the fluid at about the same rate. And when $Pr << 1$, thermal diffusivity dominates. For large values, $Pr >> 1$, the momentum diffusivity dominates. In this paper, we consider Pr value around *1*, and Fig.7(a) and 7(b) show the enhancement in mass and heat transfer upon increasing Pr . These results confirm the results of Bhadauria and Kiran [37] and Manjula *et al*. [43,55, 56].



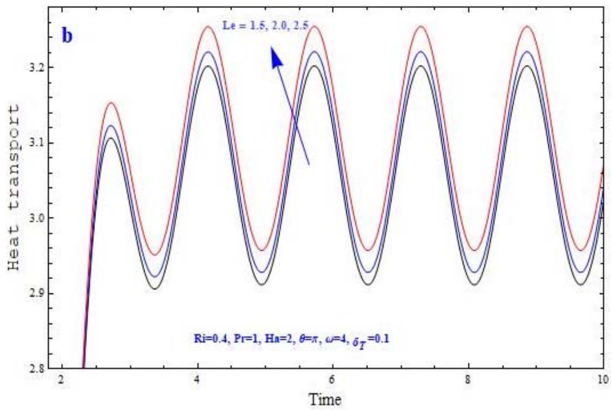
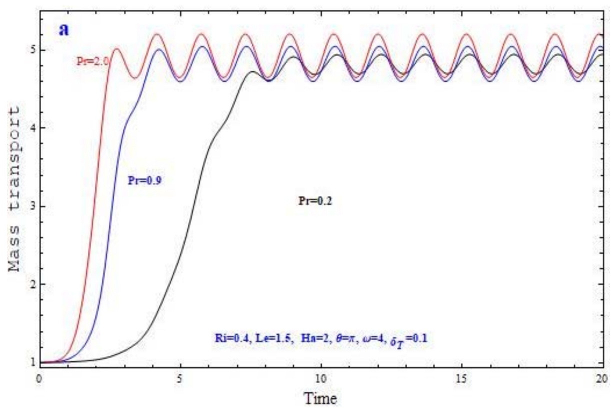


Fig.6. The effect of the Lewis number Le on mass transport (a) and heat transport (b).



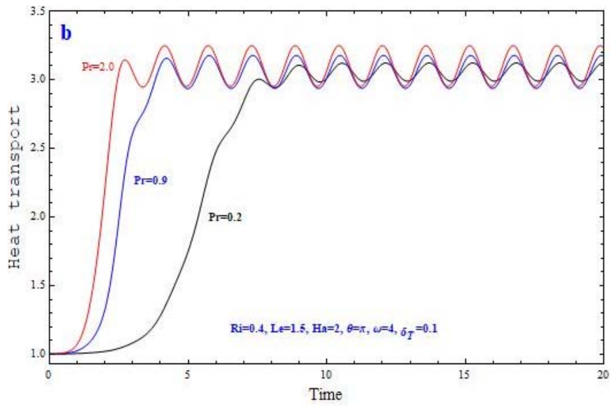


Fig.7. The effect of the Prandtl number on mass transport (a) and heat transport (b).

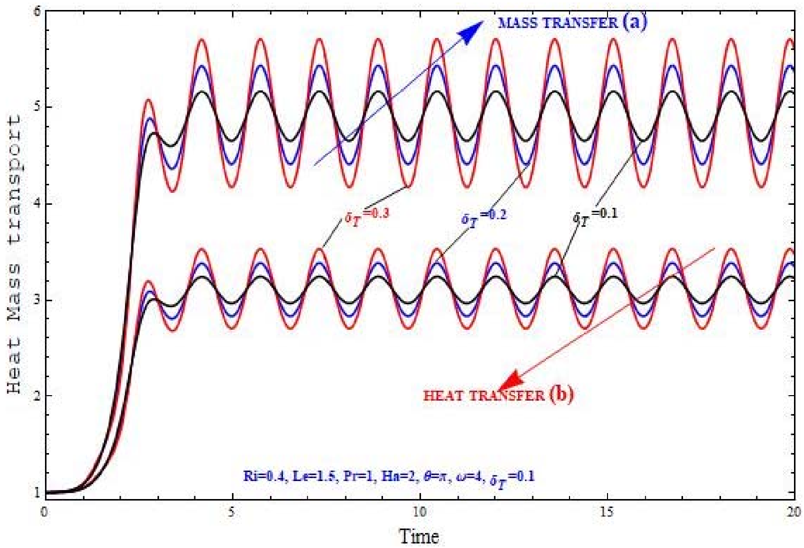


Fig.8. The effect of amplitude of modulation on mass and heat transfer.

It is found that incremental values of Rs enhance Nu/ Sh showing that the rate of heat/mass transfer increases. In general, Rs effect must reduce heat/mass transfer, due to the effect of R_i and nonlinearity there is a reverse trend of Rs . The corresponding results may be observed in the following studies [35-38].

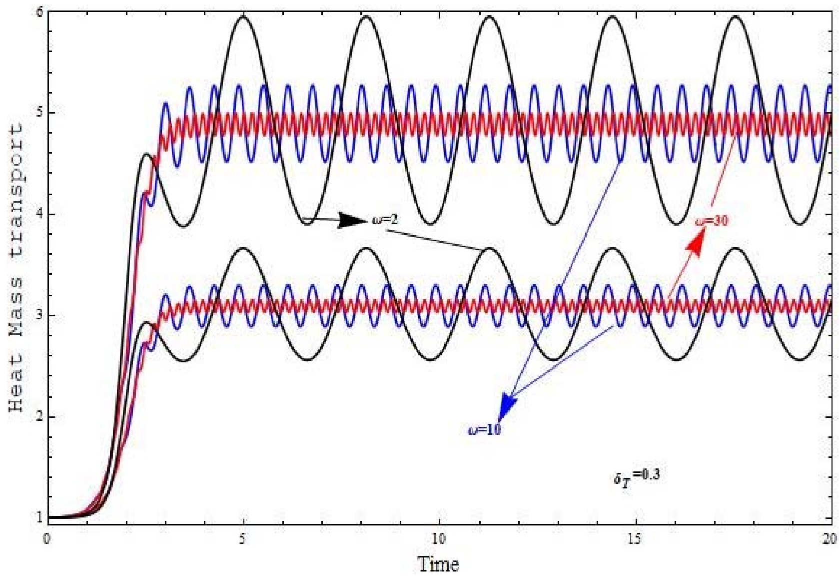


Fig.9. The effect of frequency of modulation on mass and heat transfer.

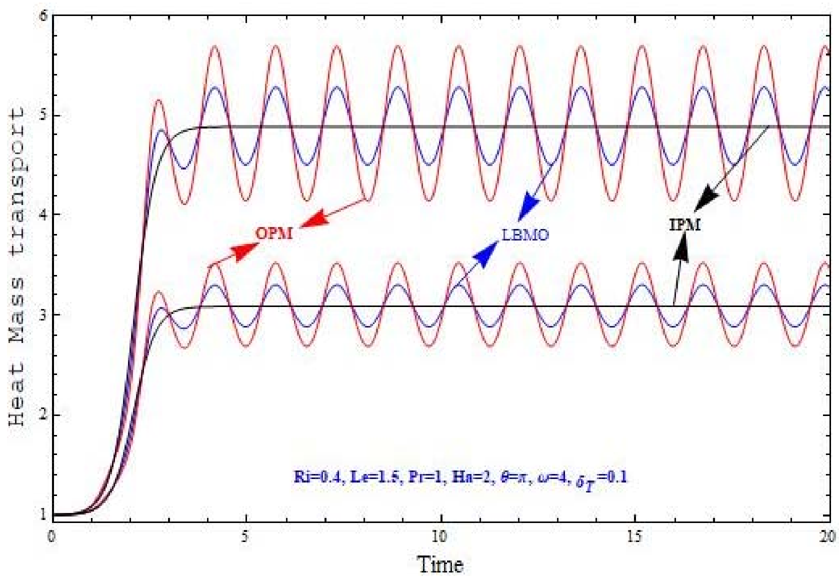


Fig.10. Three different profiles of thermal modulation.

The effect of amplitude and frequency of modulation is presented in Figs 8 and 9. The effect of amplitude of modulation is to increase Nu/ Sh , and leads to heat/mass transport (see Figs 8). The effect of frequency of modulation is to decrease Nu/ Sh (given in Fig.9). For high frequency of thermal modulation, the effect of modulation disappears altogether. It is observed that the amplitude of modulation enhances the heat transfer but an opposite effect is observed for ω. These results agree with the linear theory of Venezian [9], Malashetty *et al*. [14] and other studies [46-52] for gravity modulation [61-65]. The effect of three different temperature profiles OPM, LBMO and IPM have been compared in Fig.10. It is clear that the rate of mass and heat transfer follows the following order:

$$[Sh/Nu]_{OPM} > [Sh/Nu]_{LBMO} > [Sh/Nu]_{IPM}.$$

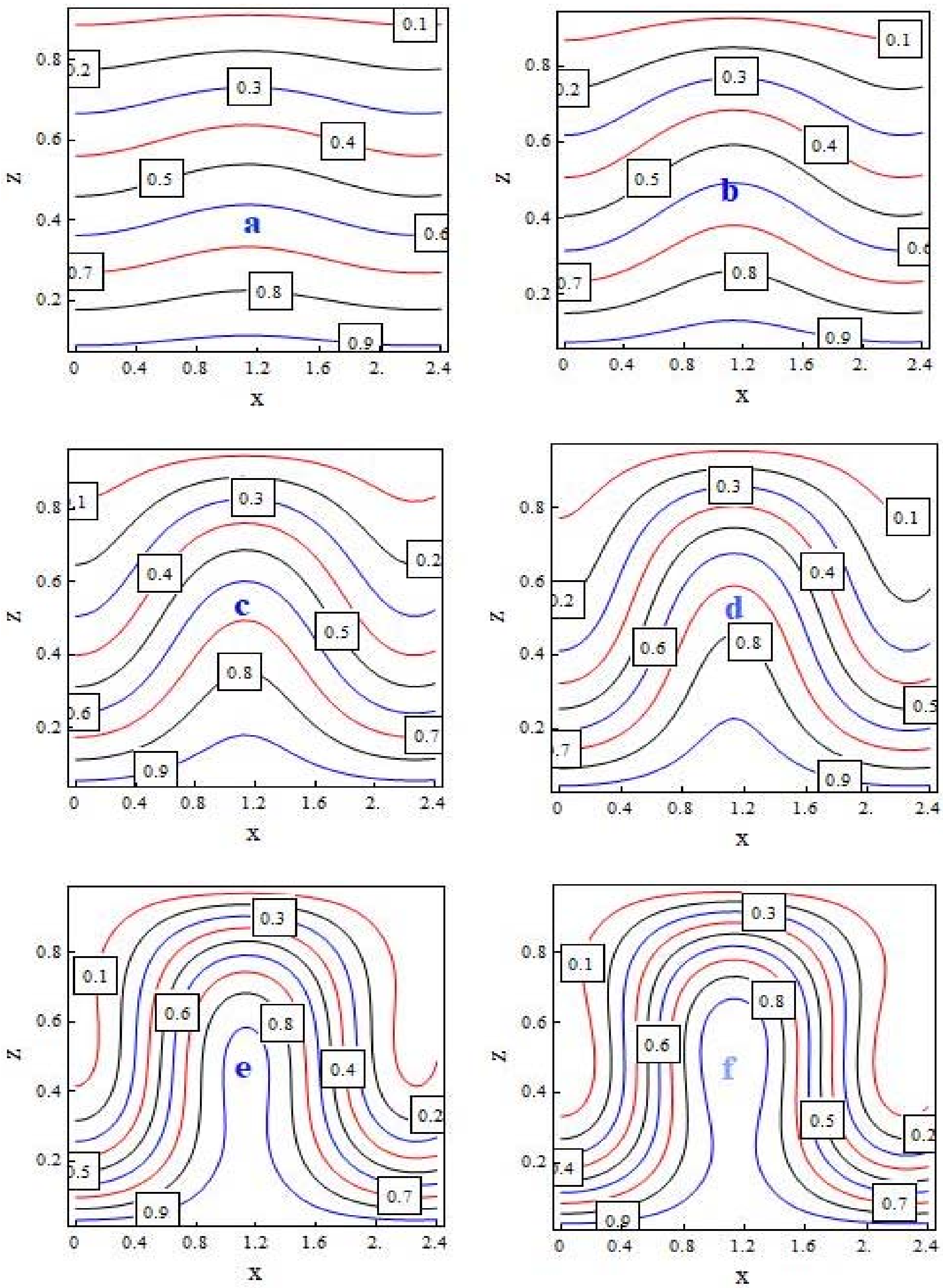


Fig.11. Isohalines for various values of Le at instances $\tau = 1.0$ (a) $\mathrm{Le} = 0.5$; (b) $\mathrm{Le} = 1.5$; (c) $\mathrm{Le} = 2.5$; (d) $\mathrm{Le} = 3.5$; (e) $\mathrm{Le} = 4.0$; (f) $\mathrm{Le} = 5.0$.

In Fig. 11, isotherms are depicted respectively at slow time $s = 0.0$, 0.5, 1.0, 1.5, 2.0, 2.5, 3.0 at $\chi = 0.5$. The figures shows that when the time $\tau = 1.0$ isotherms are straight lines (see Figs.11a,b) showing the conduction state. However, as time passes the isotherms lose their evenness. This shows that convection is taking place in the system, and becomes faster on further increasing the time τ. However, the system achieves its steady state beyond $\tau = 1.5$ as there is no change in isotherms (see Figs 11d-f.). These results of isotherms may be compared with the studies of Manjula *et al*. [56], Kiran *et al*. [32, 45, 57] and Bhadauria and Kiran [37].

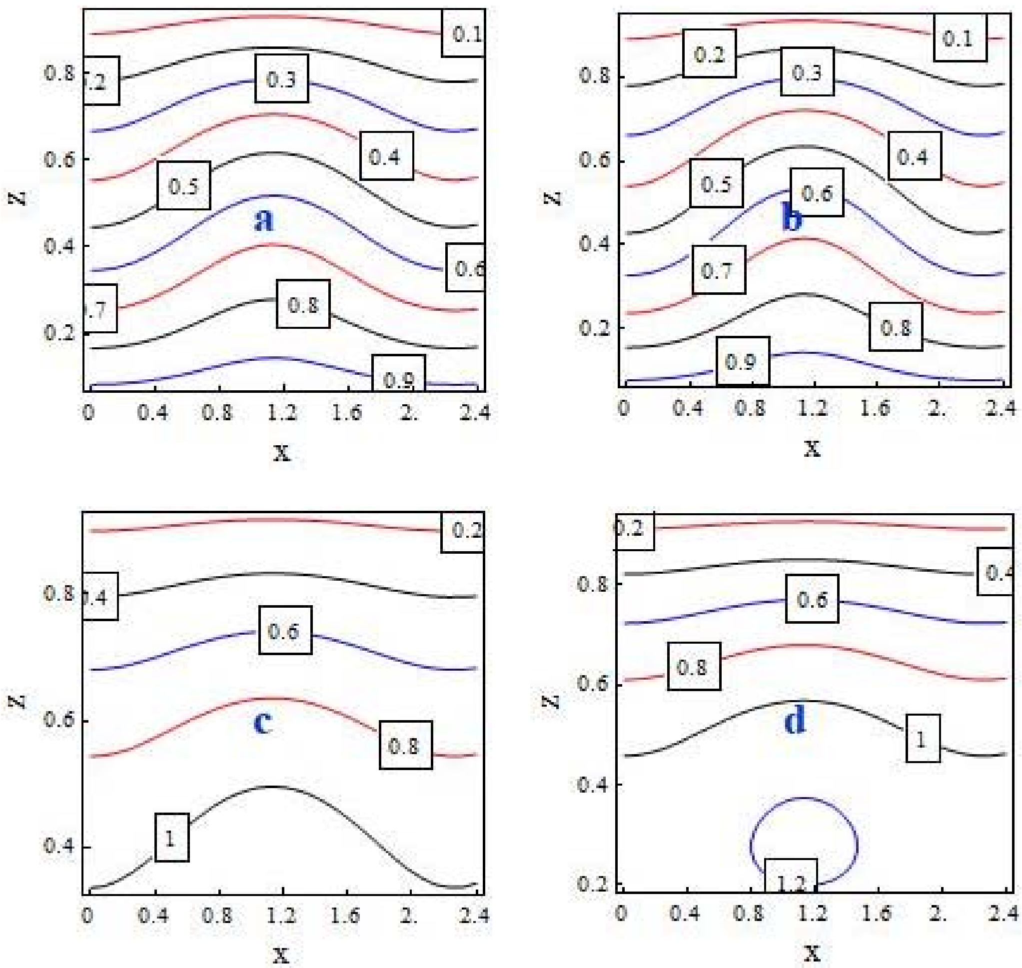


Fig.12. Isotherms for various values of R_i at instances $s = 1.0$ (a) $R_i = 0.5$ (b) $R_i = 1.0$; (c) $R_i = 1.5$; (d) $R_i = 2.0$.

The effect of R_i on isotherms is presented in Fig. 12. It is clear that the effect of R_i is to change the evenness of isotherms R_i. This indicates that R_i destabilizes the system and enhances the heat transfer. Similarly, the effect of Le on isohalines can be seen in Fig.12. As the value of Le varies from 0 to 5 for a fixed instant of time $\tau = 1.0$, isohalines lose their cuteness and show the instability of the flow with mass transport.

5. Conclusions

The effect of thermal modulation and internal heating on double diffusive convection has been investigated by performing a weakly nonlinear stability analysis resulting in the Ginzburg-Landau equation [46-52]. The following conclusions are made from the study.

1. The effect of Pr, Rs and Le is to enhance heat and mass transfer.
2. The effect of R_i is to enhance heat transfer and diminish mass transfer.
3. The effect of the Hartmann number (Ha) is to decrease heat and mass transfer.
4. The effect of amplitude δ_T of modulation is to enhance heat and mass transfer.
5. Lower values of ω enhance heat and mass transfer. But, for higher values of ω the trend is reversing [48-52].
6. Two cases OPM and LBMO are effective modes for heat and mass transfer.
7. The order of heat and mass transfer for three types of modulation is given

$$[\text{heat and mass}]_{OPM} > [\text{heat and mass}]_{LBMO} > [\text{heat and mass}]_{IPM} .$$

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Nomenclature

A – amplitude of convection
D – depth of the fluid layer
b – basic state
c – critical
g – acceleration due to gravity
Ha – Harman number
k – wave number
k – vertical unit vector
Le – Lewis number
p – reduced pressure
Pr – Prandtl number
q – fluid velocity
Ra_T – thermal Rayleigh number
R_i – internal Rayleigh number
Rs – solutal Rayleigh number
R_{0c} – critical Rayleigh-number
T – temperature
t – time
α_T – coefficient of thermal expansion
α_S – solute expansion coefficient
δ_T – amplitude of thermal modulation
θ – phase angle
κ_T – effective thermal diffusivity

μ – dynamic viscosity of the fluid
μ_e – magnetic permeability
ν – kinematic viscosity
ρ – fluid density
τ – slow time (dimensionless)
χ – perturbation parameter
Ψ – stream function
ω – thermal modulation frequency
$^{/}$ – perturbed quantity
$*$ – dimensionless quantity
0 – reference value

REFERENCES

[1] Chandrasekhar S. (1961): *Hydrodynamic and Hydromagnetic Stability*.– Oxford University Press, London, UK.

[2] Huppert H.E. and Sparks R.S.J. (1984): *Double-diffusive convection due to crystallization in magmas*.– Annual Review of Earth and Planetary Sciences, vol.12, pp.11-37.

[3] Rudraiah N and Shivakumara I.S. (1984): *Double-diffusive convection with an imposed magnetic field*.– Int. J of Heat and Mass Transfer, vol.27, No.10, pp.1825-1836.

[4] Hill A. (2005): *Double-diffusive convection in a porous medium with a concentration based internal heat source*.– Proc. R. Soc. A vol.461, pp.561-574.

[5] Zhao M., Zhang Q. and Wang S. (2014): *Linear and nonlinear stability analysis of double diffusive convection in a maxwell fluid saturated porous layer with internal heat source*.– Journal of Applied Mathematics, Article ID 489279, pp.01-12.

[6] Baines P. G. and Gill A. E. (1969): *On thermohaline convection with linear gradients*.– J. Fluid Mech. vol.37, pp.289-306.

[7] Huppert H. E. and Turner, J.S. (1981): *Double diffusive convection*.– J. Fluid Mech., vol.106, pp.299-329.

[8] Rudraiah N. (1986): *Double-diffusive magnetoconvection*.– J. Phys., vol.27, pp.233-266.

[9] Venezian G.(1969): *Effect of modulation on the onset of thermal convection*.– J. of Fluid Mech., vol.35, pp.243-254.

[10] Poulikakos D. (1986): *Double diffusive convection in a horizontally sparsely packed porous layer*.– Int. Commun. Heat Mass Transf., vol.13, pp.587-598.

[11] Wang S. and Tan W. (2008): *Stability analysis of double-diffusive convection of Maxwell fluid in a porous medium heated from below*.– Phys. Lett. A., vol.372, pp.3046-3050.

[12] Kumar A. and Bhadauria B.S. (2011): *Double diffusive convection in a porous layer saturated with viscoelastic fluid using a thermal non-equilibrium model*.– Phys. Fluids, vol.23, p.054101.

[13] Kumar A. and Bhadauria B.S. (2011): *Nonlinear two dimensional double diffusive convection in a rotating porous layer saturated by a viscoelastic fluid*.– Transp. Porous Media., vol.87, pp.229-250.

[14] Malashetty M. S., W. Tan and Swamy M. (2009): *The onset of double diffusive convection in a binary viscoelastic fluid saturated anisotropic porous layer*.– Phys. Fluids, vol.21, pp.084101.

[15] Kuznetsov A.V.and Nield D.A. (2010): *The onset of double-diffusive nanofluid convection in a layer of a saturated porous medium*.– Transport in Porous Media, vol.85, pp.941-951.

[16] Manjula S.H. and Kiran P.(2019): *Throughflow and gravity modulation effects on double diffusive oscillatory convection in a viscoelastic fluid saturated porous medium*.– Adv. Sci. Eng. Med., vol.12, No.3, pp.01-10. doi:10.1166/asem.2020.2565

[17] Bhadauria B.S. (2007): *Double diffusive convection in a porous medium with modulated temperature on the boundaries*.– Transport in Porous Media, vol.70, pp.191-211.

[18] Bhadauria B.S. and Kiran P. (2014): *Weak nonlinear double diffusive magneto-convection in a Newtonian liquid under temperature modulation*.– Int. J. Eng. Math., Article ID 296216, pp.01-14.

[19] Bhadauria B.S. and Kiran P. (2014): *Heat and mass transfer for oscillatory convection in a binary viscoelastic fluid layer subjected to temperature modulation at the boundaries*.– Int. Communi in Heat and Mass Transf., vol.58, pp.166-175.

[20] Kiran P. (2016): *Throughflow and non-uniform heating effects on double diffusive oscillatory convection in a porous medium*.– Ain Shams Engineering J., vol.7, No.1, pp.453-462.

[21] Narayana M., Sibanda P., Motsa S. and Narayana P.A.L. (2012): *Linear and nonlinear stability analysis of binary Maxwell fluid convection in a porous medium*.– Heat Mass Transfer, vol.48, pp.863-874.

[22] Narayana M., Gaikwad S., Sibanda P. and Malge. R. (2013): *Double diffusive magneto-convection in viscoelastic fluids*.– Int. J. of Heat and Mass Transfer, vol.67, pp.194-201.

[23] Bhadauria B.S. and Kiran P. (2014): *Chaotic and oscillatory magneto-convection in a binary viscoelastic fluid under G-jitter*.– Int. J Heat Mass Transf., vol.84, pp.610-624.

[24] Bhadauria B.S. and Kiran P. (2014): *Weak nonlinear oscillatory convection in a viscoelastic fluid layer under gravity modulation*.– Int. J. Non-linear Mech., vol.65, pp.133-140.

[25] Bhadauria B.S. and Kiran P. (2014): *Weakly nonlinear oscillatory convection in a viscoelastic fluid saturating porous medium under temperature modulation*.– Int. J. Heat Mass Transf., vol.77, pp.843-851.

[26] Kiran P., Manjula SH. and Narasimhulu Y. (2018): *Weakly nonlinear oscillatory convection in a viscoelastic fluid saturated porous medium with throughflow and temperature modulation*.– Int. J. of Applied Mechanics and Eng., vol.23, No.3, pp.01-28.

[27] Manjula S.H. and Kiran P. and Bhadauria B.S.(2020): *Throughflow and G-jitter effects on oscillatory convection in a rotating porous medium*.– Adv. Sci. Eng. Med., vol.12, pp.01-10.

[28] Bhadauria B.S., Hashim I. and Siddheshwar PG. (2013): *Study of heat transport in a porous medium under G-jitter and internal heating effects*.– Transport in Porous Media, vol.96, pp.21-37.

[29] Bhadauria B.S., Hashim I. and Siddheshwar P.G. (2013): *Effects of time-periodic thermal boundary conditions and internal heating on heat transport in a porous medium*.– Transport in Porous Media, vol.97, pp.185-200.

[30] Bhadauria B.S, Kiran P. and Belhaq M. (2014): *Nonlinear thermal convection in a layer of nanofluid under g-jitter and internal heating effects*.– MATEC Web of Conferences, vol.16, Article Number 0900309003, p.7.

[31] Bhadauria, B.S, Kiran., P. (2014): *Effect of rotational speed modulation on heat transport in a fluid layer with temperature dependent viscosity and internal heat source*.– Ain Shams Eng J., vol.5, No.4, pp.1287-1297.

[32] Kiran P., Bhadauria B.S. and Kumar V.(2016): *Thermal convection in a nanofluid saturated porous medium with internal heating and gravity modulation*.– J. of Nanofluids, vol.5, No.3, pp.328-339.

[33] Kiran P. and Narasimhulu Y. (2018): *Internal heating and thermal modulation effects on chaotic convection in a porous medium*.– J. of Nanofluids, vol.7, No.3, pp.544-555

[34] Altawallbeh AA., Hashim I. and Bhadauria BS. (2019): *Magneto-double diffusive convection in a viscoelastic fluid saturated porous layer with internal heat source*.– AIP Conference Proceedings, vol.2116, No.1, Article number 030015.

[35] Srivastava A. and Singh A. K. (2018): *Linear and weak nonlinear double diffusive convection in a viscoelastic fluid saturated anisotropic porous medium with internal heat source*.– J. of Applied Fluid Mechanics, vol.11, No.1, pp.65-77.

[36] Bhadauria B.S. and Kiran P. (2014): *Weakly nonlinear double diffusive convection in a temperature dependent viscosity fluid saturated porous medium under temperature modulation*.– Int. J. Eng Trends and Tech, pp.146-153

[37] Bhadauria B.S. and Kiran P. (2015): *Weak nonlinear double diffusive magneto convection in a newtonian liquid under gravity modulation*.– J. of Applied Fluid Mechanics, vol.8, No.4, pp.735-746.

[38] Srivastava A., Bhadauria B.S. and Hashim I. (2014): *Effect of internal heating on double diffusive convection in a couple stress fluid saturated anisotropic porous medium*.– Advances in Materials Science &Appl., vol.3, pp.24-45.

[39] Kiran P. (2020): *Concentration modulation effect on weakly nonlinear thermal instability in a rotating porous medium*.– J. of Applied Fluid Mechanics, vol.13, No.5, pp.01-13.

[40] Kiran P. and Manjula S.H.(2019): *Weakly nonlinear mass transfer in an internally soluted and modulated porous layer*.– Adv. Sci. Eng. Med., vol.12, No.3, pp.1-10, doi:10.1166/asem.2020.2566.

[41] Malkus W.V.R. and Veronis G. (1958): *Finite amplitude cellular convection*.– J. of Fluid Mech., vol.4, pp.225-260.

[42] Keshri Om.P., Gupta V.K. and Kumar A. (2018): *Study of weakly nonlinear mass transport in Newtonian fluid with applied magnetic field under concentration/gravity modulation*.– Nonlinear Engineering. 2018, pp.2-10, https://doi.org/10.1515/nleng-2018-0058.

[43] Manjula S.H., Kiran P., Reddy R. and Bhadauria B.S. (2020): *The complex Ginzburg Landau model for an oscillatory convection in a rotating fluid layer*.– Int. J. of Applied Math. and Mech., vol.25, pp.75-92.

[44] Kiran P., Bhadauria B.S. and Roslon R. (2020): *The effect of throughflow on weakly nonlinear convection in a viscoelastic saturated porous medium*.– J. of Nanofluid, vol.8, pp.01-11.

[45] Kiran P. and Bhadauria B.S (2015): *Chaotic convection in a porous medium under temperature modulation*.– Transport in Porous Media, vol.107, pp.745-763.

[46] Bhadauria B.S nad Kiran. P. (2013): *Heat transport in an anisotropic porous medium saturated with variable viscosity liquid under temperature modulation*.– Transport in Porous Media.vol.100, pp.279-295.

[47] Kiran P and Bhadauria B.S (2015): *Nonlinear throughout effects on thermally modulated porous medium*.– Ain Shams Eng J., vol.7, No.1, pp.473-482.

[48] Manjula S.H., Kiran P. and Narasimhulu Y. (2018): *Heat transport in a porous medium saturated with variable viscosity under the effects of thermal modulation and internal heating*.– Int. J. of Emerging Tech. and Innovative Res., vol.5, pp.59-75.

[49] Bhadauria B.S. and Kiran P. (2014): *Weak nonlinear double-diffusive magneto-convection in a newtonian liquid under temperature modulation*.– Int. J. of Engg Mathematics, Article ID 296216, pp.01-14.

[50] Kiran P. and Bhadauria B.S. and Narasimhulu Y. (2016): *Nonlinear throughflow effects on thermally modulated rotating porous medium*.– J. of Applied Nonlinear Dynamics, vol.6, pp.27-44.

[51] Kiran P. and Bhadauria B.S (2016): *Throughflow and rotational effects on oscillatory convection with modulation*.– Nonlinear Studies, vol.23, No.3, pp.439-455.

[52] Kiran P. and Bhadauria B.S. and Narasimhulu Y. (2018): *Oscillatory magneto-convection under magnetic field modulation*.– Alexandria Engg J., vol.57, pp.445-453.

[53] Kiran P. and Bhadauria B.S. (2016): *Weakly nonlinear oscillatory convection in a rotating fluid layer under temperature modulation*.– J. of Heat Transf., vol.138, No.5, p.10.

[54] Kiran P. and Narasimhulu Y. (2018): *Weak nonlinear thermal instability in a Dielectric fluid layer under temperature modulation*.– Int. J. of Advanced Research Trends in Eng and Tech., vol.5, pp.470-476.

[55] Manjula S.H. and Kiran P. (2020): *Throughflow and gravity modulation effects on double diffusive oscillatory convection in a viscoelastic fluid saturated porous medium*.– Adv. Sci. Eng. Med., vol.12, pp.612-621.

[56] Manjula S.H., Kiran P. and Bhadauria B.S. (2020): *Throughflow and g-jitter effects on oscillatory convection in a rotating porous medium*.– Adv. Sci. Eng. Med., vol.12, pp.781-791.

[57] Kiran P., Manjula S.H. and Roslan R. (2019): *The effect of gravity modulation on double diffusive convection in the presence of applied magnetic field and internal heat source*.– Adv. Sci. Eng. Med., vol.12, pp.792-805.

[58] Kiran P., Manjula S.H., Narasimlu G. and Roslan R. (2019): *The effect of modulation on heat transport by a weakly nonlinear thermal instability in the presence of applied magnetic field and internal heating*.– Int J of Applied Mathematics and Mechanics., vol.25, No.4, pp.96-115.

[59] Manjula S.H., Kiran P. and Narayanamoorthy S. (2020): *The effect of gravity driven thermal instability in the presence of applied magnetic field and internal heating*.– AIP Conference Proceedings, vol.2261, pp.030042.

[60] Kiran P., Manjula S.H., Suresh P. and Raj Reddy P. (2020): *The time periodic solutal effect on oscillatory convection in an electrically conducting fluid layer*.– AIP Conference Proceedings, vol.2261, pp.030004.

[61] Kiran P. (2016): *Nonlinear throughflow and internal heating effects on vibrating porous medium.*– Alexandria Eng. J., vol.55, No.2, pp.757-767.

[62] Kiran P. (2015): *Throughow and g-jitter effects on binary fluid saturated porous medium*.– Applied Math and Mech., vol.36, No.10, pp.1285-1304.

[63] Kiran. P. (2015): *Nonlinear thermal convection in a viscoelactic nanofluid saturated porous medium under gravity modulation*.– Ain Shams Eng J., vol.7, pp.639-651.

[64] Kiran P. and Narasimhulu Y. (2017): *Weakly nonlinear oscillatory convection in an electrically conduction fluid layer under gravity modulation*.– Int J Appl. Comput. Math., vol.3, No.3, pp.1969-1983.

[65] Bhadauria B.S., Singh M.K., Singh A., Singh B.K. and Kiran P. (2016): *Stability analysis and internal heating effect on oscillatory convection in a viscoelastic fluid saturated porous medium under gravity modulation*.– Int. J of Applied Mechanics and Engg., vol.21, No.4, pp.785-803.

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