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CHAOTIC ASSESSMENT OF THE HEAVE AND PITCH DYNAMICS MOTIONS OF AIR CUSHION VEHICLES

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In this study, a three degrees of freedom nonlinear air cushion vehicle (ACV) model is introduced to examine the dynamic behavior of the heave and pitch responses in addition to the cushion pressure of the ACV in both time and frequency domains. The model is based on the compressible flow Bernoulli's equation and the thermodynamics nonlinear isentropic relations along with the Newton second law of translation and rotation. In this study, the dynamical investigation was based on a numerical simulation using the stiff ODE solvers of the Matlab software. The chaotic investigations of the proposed model are provided using the Fast Fourier Transform (FFT), the Poincaré maps, and the regression analysis. Three control design parameters are investigated for the chaotic studies. These parameters are: ACV mass (M), the mass flow rate entering the cushion volume $(\dot{m}_{in})$, and the ACV base radius (r). Chaos behavior was observed for heave, and pitch responses as well as the cushion pressure.

Key words: air cushion vehicles, Poincaré map, Fast Fourier Transform (FFT), heave motion; pitching motion.

1. Introduction

Air Cushion Vehicles (ACV) are mainly operated by highly pressurized air which is fed to the air cushion using blowers. The flow of air is maintained due to a momentum change at the high-velocity peripheral air curtain. These blowers yield a large volume of air cushion that has a pressure a little higher than the atmospheric pressure creating a pressure difference. This pressure difference produces vehicle lift, which causes the bottom air cushion to float above the running surface. The ACVs have the capability to function in many different environments that can be coarse such as ice, water, or forests.

The hydrodynamic action of the ACV is ideally equivalent to that of a pressure distribution acting on the free surface of water. This idealization prohibits any physical contact of the lower part of the ACV with the water [1]. Abundant studies have been published regarding proper designing and dynamics of these vehicles. The rest of this section is a summary of the related study that are available in the literature.

A linear analysis of a two-dimensional section of the cushion equipped with a bag-and-finger skirt was subjected to only input heave motion in reference [2]. The skirt mass was lumped in the fingers, with the bag being modeled as a combination of massless inelastic membranes and links. The findings of the study suggested that changes in skirt geometry could not be used to radically modify an undesirable heave response, but reducing the skirt mass might be effective. The authors also found in their study that the air compressibility affected the heave response at high frequencies, with the effect becoming more prominent at low cushion-flow rates commonly used in practice.

All prior research studies combined together formed the basis of a standard physics-based dynamic simulation model for ACVs. It should be noted that the response motions computed from the previous physical simulation models of the ACVs did not display similar response when compared with the experimental results of the scaled-model tests. Lack of understanding the physical damping mechanisms in the ACV motion led to the discrepancy between the simulations and experiments. Wave generation on the free surface and deformation of the viscoelastic skirt material was used as a damping source by Chung [3] in his investigation while Graham and Sullivan [4] researched the effect of other sources of damping on the motion of the ACVs, such as the unsteady air flow throughout the fan, skirt, and cushion system.

The general configuration of ACVs including the overall dimensions, weight distribution, parametric properties, and several subsystems, was designed and optimized by T. C. Jung [5] using the expert system at the initial design phase. The skirt bag and finger systems of the ACV were further optimized for improving ride quality and stability of the vehicle using the genetic algorithm. Hence, Chung's work opened up the new avenues for designing ACVs using artificial intelligence techniques. Chung and Jung [6] also used the Genetic Algorithm to optimize the undesirable two dimensional section heave response of the ACV's bag and finger skirt system geometry. They obtained a new skirt geometry that considerably improved the resonating frequencies associated with the skirt mass at which humans were most sensitive.

An analytical model was introduced by Pollack *et al*. [8] to investigate the dynamics of an ideal air cushion cavity of an ACV. The study revealed that skirt impedance strongly affected the resonant frequencies and mode shapes of the ACV. The impedance of modern skirt systems could also dramatically alter the air cushion enclosed volume and vehicle footprint, thus influencing the system resonances and restoring moments [8].

Compressibility effect on the dynamic behavior of an ACV was investigated by Milewski *et al*. [9-10] utilizing the deformable free surface condition. In [9] the governing equations were solved on a fixed regular grid translating at the vehicle mean forward speed using the Immersed Boundary Method (IBM), whereas Milewski *et al*. in ref [10] developed a numerical program called ACVSIM that used the boundary element method with a higher order spline based model to study the skirt and the ACV dynamics. ACVSIM coupled a high-order Rankine panel method with models for air cushion and skirt dynamics to calculate the motions of the ACV.

The incompressible viscous fluid mechanics problem around the ACV near the free surface was investigated numerically using the SIMPLE algorithm and volume of fluid (VOF) method with staggered grid by Nikseresht *et al*. [11]. Many parameters were examined in the numerical procedure such as the ACV under skirt pressure distribution, initial air gap under the ACV, and effect of Froude number. A versatile and robust computational coding was done and tested in their study by applying the code on a water impacted cylindrical part of the ACV.

Hossain *et al*. [12] presented a new dynamical model for the forces on a small scale intelligent air cushion tracked vehicle (IACTV) moving over swamp peat. The air cushion system in their study partially supported 25% of vehicle's total weight making the vehicle ground contact pressure $7\ kN/m^2$ in order to make the IACTV move over the intended terrain without threats. They also mentioned that the relationships between the various vehicle parameters had been experimentally tested. Some of these parameters were: tractive efficiency, power consumption, traction coefficient, load distribution ratio, tractive effort, etc. Experimental and simulated results showed a considerable improvement in the vehicle's performance when values of 0.71 and 0.62 were used for the traction coefficient and tractive efficiency respectively.

Xie *et al*. [13] designed an air cushion system using computational fluid dynamics (CFD) simulation in which the relationships among some of the design parameters of the ACV were obtained from measurement. These parameters included vehicle's fan rotational speed, equivalent clearance height, and static air cushion pressure convert ratio. The results of their study provided good benchmarks for the design of vehicles with air-cushion system. Nan Ji [14] developed a comprehensive mathematical model for a 6-DOF motion control for ACVs. He obtained his model by superimposing the wave-making water surface deformation in calm water and wave forms in external environment. The results of this study produced in an optimized maneuverability-control system of ACVs.

The 6-DOF motion model of an ACV based on Newton's momentum and moment of momentum theorem was developed by Zhang H. *et al*. [15]. In their study, they obtained the water surface deformation under the air cushion by applying linear wave theory, the lift fan, air duct, skirt, and air cushion dynamic balancing were achieved by balancing flow across each which resulted in a stable motion of the vehicle. Earlier Sowayan and Alsaif [16] used the compressible Bernoulli's equation and the Newton's second law of translation motion to predict the vanishing oscillatory motion of an ACV. In that study, only the ACV's heave degree of freedom was considered, which was shown to have a periodic behavior.

Han and Liu [17] assessed the trajectory tracking of ACVs using a new multivariable higher-order sliding mode (HOSM) control scheme that allowed the ACVs to be modeled as an uncertain nonlinear system with fewer degrees of freedom [17]. The ACV was modeled by a nonlinear dynamic model that was externally disturbed by certain environmental characteristics. The control approach included stabilized nominal continuous control law and super-twisting second-order sliding mode control part to reduce chattering of controlling force in surge and yaw motion. The Lyapunov exponent approach was used to judge the motion finite time stability in terms of robustness and superiority.

A mathematical model of an ACV with a ballonet type skirt was developed and solved using numerical integration by Eremeyev *et al*. [18]. Many parts of the ACV such as the rigid body, air cushion, propulsion system, ballonet, fundamental surface, etc., were dynamically incorporated in the model. The proposed model was subjected to different operating conditions and was found to be capable of accurately predicting all major characteristics of the ACV motion such as trajectory, forces, moments, air cushion pressure, etc., when subjected to different operating conditions. The featured motion calculated using the model was found to be in good agreement with the results of the model towing tests in calm water and in waves. The model was also capable of designing the preliminary design stages of the ACV's body structure to simulate both the controlled and uncontrolled dynamics of ACV's spatial movement on still and wavy water surface.

Wang and Tong [19] designed a second order nonsingular terminal sliding mode controller with extended state observer of an ACV using extended state observer and second order nonsingular terminal sliding mode heading controller to improve the safety and maneuverability of ACVs. One of the physical causes that reduce the maneuverability of ACV is chattering, which can be eliminated by introducing an extended state observer to evaluate the nonlinear uncertainties and external disturbances in the ACV. The Lyapunov analysis was performed to analyze the stability of the closed loop system of the considered ACV.

The dynamic modelling and parameter estimation of Unmanned Air Cushion Vehicle (UACV) was investigated by Zamzuri *et al*. [20]. They studied a dynamic model of the vehicle with six degrees of freedom using the Euler-Lagrange method. The six degrees of freedom consisted of the heave, sway, surge, roll, pitch and yaw motions. Solidworks® software was used to obtain and calculate the parameters of the system. The parameters that were investigated included body inertia of the system and air mass flow rate flowing into the UACV skirt. The results of the study were validated from other published experimental and simulation studies.

Fu *et al*. [21] studied a hovercraft model having four degrees of freedom with unknown curve-fitted coefficients using the constrained adaptive Lyapunov function controller in the yaw path that was varying with the speed of the hovercraft. The desired virtual yaw rate was generated using the command filter with a time-varying magnitude limit that led to a reduction in the computational complexity. Theoretical analyses indicated that the position tracking error constraint and the yaw rate constraint could be strictly guaranteed by using the proposed controller. It was also shown that, since hovercrafts normally have high surge velocities, the time variation delay could result in the measurements.

2. Problem statement and motivations

In this study, a model for the dynamic behavior of an ACV is introduced using the basics of thermodynamics and fluid mechanics. The proposed model has two degrees of freedom, which are (i) the vertical motion (heave) and (ii) the pitching motion (rotation). The air cushion pressure is also included in the investigation. The study involves the evaluation of the following dependent variables (heave, vertical velocity, cushion pressure, angular rotation motion, and angular velocity in the pitching direction).

The chaotic and fractal dynamics are also investigated for all dependent variables. The model is nonlinear, and the nonlinearity is due to many reasons, which will be explained in the successive sections of this paper. The sources of nonlinearity lead to self-excitation of the model, which is affected by several design parameters but only three of these parameters have been selected for investigation. These are (i) mass of the vehicle (M), (ii) the ACV skirt perimeter which is related to the ACV base radius (r), and (iii) the air mass flow rate into the cushion volume $(\dot{m}_{in})$.

Most ACV models including the model considered in this study are self-excited models, and therefore unwanted oscillatory motions exist in their start operations. The oscillatory motions are translated in the vertical and azimuthal directions at the transient response of the solution. These motions are undesirable as sometimes they may lead to damage or failure of the ACV parts. Therefore, one of the objectives of this study is to explore the impact of the chosen design parameters on the transient response in order to assess the oscillatory behavior of ACV, i.e. the aim is to limit or totally eliminate these oscillatory motions through proper sorting of the design parameters. This can be achieved by introducing a mathematical model that will enable one to understand the nature and physics of these vehicles. The model can be used to redesign the system so that the violent oscillatory motions can be eliminated.

3. Methodology and governing equations

The model presented in this paper is based on many physical and mathematical assumptions. There are some specific assumptions that will be stated during the derivation of the mathematical model of the problem.

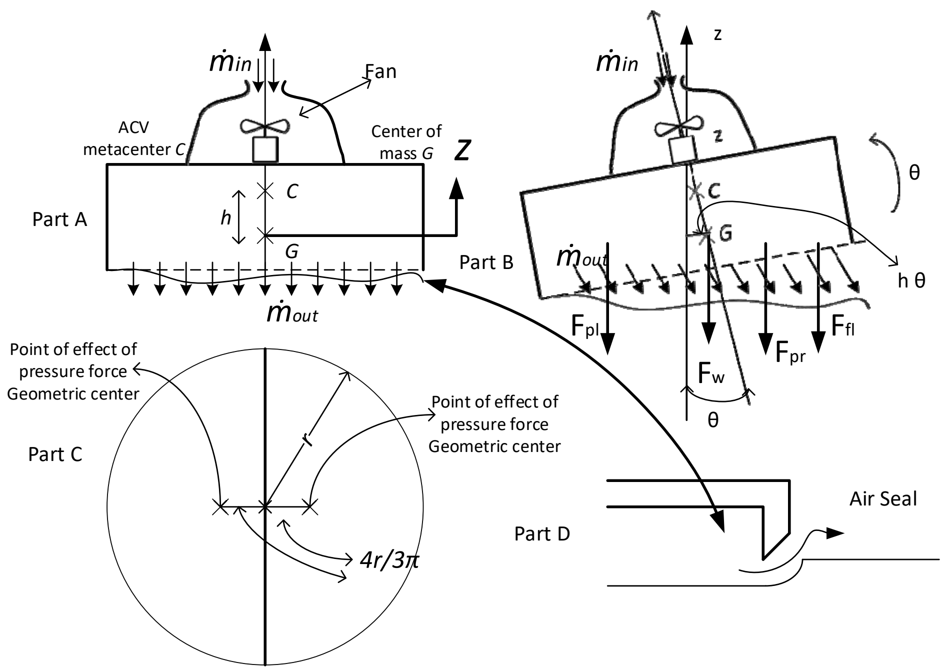

Fig.1. Air Cushion Vehicle movements - (Part A) motion in the heave direction, (Part B) pitch motion with the forces causing rotation of the ACV body, (Part C) the ACV base cross-sectional area with pressure force acting at the geometric center, (Part D) seal form of the air leakage under ACV.

The other assumptions are presented in this section as follows:

1. The working fluid in this study is pressurized air inside the cushion volume underneath the ACV.
2. The flow of air from the cushion as shown in Fig.1. is assumed to be subsonic and compressed isentropically.
3. Also, thermodynamic equilibrium is assumed inside the cushion volume. Furthermore, the ACV geometry is assumed to be symmetrical about the heave motion axis.
4. Lumped capacitance of the cushion volume and the extended skirt are assumed which allows clearance surface contact that enables air flow to escape through a uniform hover gap as displayed in Fig.1.
5. Motion of the ACV is only in the heave and pitch directions.
6. Only three design parameters as mentioned in the previous section were chosen for studying the dynamics behavior of the model in this paper.

7. The following model's parameters were fixed. These parameters are as follows: skirt length (L), metacenter distance (h), ACV height from ground (d), air temperature (T), and the correction coefficient factor (c_0).

ACVs have been found to experience a fierce oscillatory heave motion that is coupled with pitch motion. This is due to the interaction of its mechanical motion and the air behavior of the cushion volume. Therefore, the governing equation needs to be coupled to correctly predict this interaction. Moreover, the mathematical model that is going to be introduced in this study will enable one to understand the nature of ACV's response. It will help ACV designers to redesign the ACV's system response in a way that can eliminate violent oscillatory motion. The equations of motion for the ACV's heave and pitching motion are derived in the next subsections.

3.1. Heave motion governing equations

Using Newton's second law of motion, the balance forces on the vehicle in the *z*-direction lead to the vertical equation of motion as:

$$M\ddot{z} = (p - p_a)A - Mg - F_M \tag{3.1}$$

where M is the mass of the vehicle, $\ddot{z}$ is the acceleration in the vertical direction, p is the inside cushion pressure, p_a is the atmospheric pressure, g is the gravitational acceleration, A is the ACV base cross-sectional area of the cushion, which is assumed to be circular having a radius r, and F_M is the change in momentum force due to the flow of the air mass. F_M represents some kind of thrust force which moves the ACV in the air. This force is produced by the air mass flow rate originated from the blowers which are connected to the ACV engine. It is basically generated through the reaction of accelerating the mass of the air in and out of the ACV.

It should be noted that the cushion pressure is assumed homogenous throughout the base of the ACV. The change of the cushion pressure with time is derived in a previous work [22] of to be:

$$\dot{p} = \frac{\gamma RT}{V}\left[\dot{m}_{in} - \dot{m}_{out} - \frac{pA\dot{z}}{RT}\right] \tag{3.2}$$

where γ is the specific heat ratio, R is the air gas constant, T the air flowing temperature out of the cushion volume, and $\dot{m}_{in}$ is a fixed mass flow rate entering the cushion volume. The mass flow rate $\dot{m}_{in}$ to the cushion volume depends on the fan power that is attached to the ACV's engine. It is assumed to be a fixed value due to its dependency on the kinetic energy of the air which is generated by the blower fan power. One can use the first law of thermodynamics to show that $\dot{m}_{in} = \dot{W} / ke$, where $\dot{W}$ is the fan power, and ke is the kinetic energy of the air before and after the fan. This is an acceptable approximation since the flow is assumed to be an isothermal flow.

The cushion volume V is defined as $V = A\times(z+d)$, where d is the ACV vertical distance from the ground. The mass flow rate exiting in the cushion $\dot{m}_{\text{out}}$ is derived by applying the compressible Bernoulli's equation, and is given by [22] as:

$$\dot{m}_{out} = \frac{c_0 pLz}{\sqrt{RT}} \left\{ \frac{2\gamma}{\gamma - 1} \left[\left(\frac{p_a}{p} \right)^{\frac{2}{\gamma}} - \left(\frac{p_a}{p} \right)^{\frac{(\gamma+1)}{\gamma}} \right] \right\}^{\frac{1}{2}} . \tag{3.3}$$

The momentum force associated with the air mass flow is evaluated as follows:

$$F_M = \dot{m} \times VehicleVelocity \tag{3.4}$$

where $\dot{m}$ is the air mass flow rate and is given to be $(\dot{m}_{in} - \dot{m}_{out})$. The vehicle velocity has two components: one component is in the heave direction ($\dot{z}$) and the other component is in the pitch coordinate (θ) which is equal to $r\dot{\theta}$. The vehicle velocity is calculated for the two components using the cosine law as follows:

$$VehicleVelocity = \sqrt{(\dot{z})^2 + (r\dot{\theta})^2 - 2r\dot{z}\dot{\theta}\cos\theta} \tag{3.5}$$

where r is the radius of the ACV base, θ is the pitch angular displacement, and $\dot{\theta}$ is the angular velocity of the ACV. Equation (3.5) can be simplified using the fact that the pitch angular displacement θ is small [24], therefore, $\cos\theta \approx 1$, which yields the vehicle velocity to be:

$$VehicleVelocity = \sqrt{(\dot{z})^2 + (r\dot{\theta})^2 - 2r\dot{z}\dot{\theta}} = (\dot{z} - r\dot{\theta}) . \tag{3.6}$$

Therefore, by using Eq.(3.4) the force due to momentum flow can be written as:

$$F_M = (\dot{m}_{in} - \dot{m}_{out}) \times (\dot{z} - r\dot{\theta}) . \tag{3.7}$$

It should be noted that $\dot{m}_{out}$ is evaluated using Eq.(3.3). Lastly, Eq.(3.7) is substituted into Eq.(3.1), which yields the equation for the vertical motion of the vehicle given as:

$$M\ddot{z} = (p - p_a)A - Mg - (\dot{m}_{in} - \dot{m}_{out}) \times (\dot{z} - r\dot{\theta}) . \tag{3.8}$$

3.2. Pitch motion governing equations

The cushion geometry of the ACV is the most relevant factor for pitch restoring motion. The ACV is considered to be operating in the cases of fixed surface and free surface together as shown in Fig.2. When the ACV is in heave motion only, the metacenter C of the ACV due to the cushion pressure is aligned with the center of mass G, an equilibrium state will be created. When the vehicle is disturbed with pitch motion by an angle θ, the center of mass G moves to the right through a distance $(h\theta)$. This indicates a positive moment for G because the center of mass G cannot be negative for an air cushion vehicle.

For an ACV with two symmetric chambers divided into left and right parts the force caused by the cushion pressure is distributed to the two parts; the right pressure force F_{pr} which will decrease the cushion pressure by an amount $(p - p_a)\theta$. Conversely, the left pressure force F_{pl} will increase the cushion pressure by an amount $(p - p_a)\theta$. Eventually, oscillation occurs in the θ-direction due to the physical tendency of the vehicle to return to equilibrium. The point of action of these two forces is at the geometric center of the ACV base, which is located at the half circle on both sides of the ACV base. This is depicted in Fig.2. Consequently, the moment arm of the pressure forces is $\frac{4}{\pi}r$ where r is the radius of the ACV base. Therefore, the cushion pressure moment for the right pressure force F_{pr} and left pressure force F_{pl} is evaluated as:

$$\mathcal{M}_p = (p - p_a)\times(1+\theta)\times\frac{4}{3\pi}r\times\frac{A}{2} - (p - p_a)\times(1-\theta)\times\frac{4}{3\pi}r\times\frac{A}{2} \tag{3.9}$$

where A is the base cross-sectional area of the ACV and it is equal to πr^2. One can simplify this equation as:

$$\mathcal{M}_\mathrm{p} = \frac{4}{3}r^3(p - p_\mathrm{a})\theta. \tag{3.10}$$

The weight force (Mg) has a moment arm of $(h\theta)$, and the force associated with the air momentum mass flow rate (F_M) has a moment arm $r\cos\theta$. However, for a small pitch angle θ the moment arm of F_M is simply equal to r. To express the motion of the ACV in the pitch direction, one can use Newton's second law of rotation $\left(\sum\mathcal{M}_G = I\ddot{\theta}\right)$ as follows:

$$I\ddot{\theta} = \frac{4}{3}r^3(p - p_\mathrm{a})\theta - Mg(h\theta) - (\dot{m}_\mathrm{in} - \dot{m}_\mathrm{out})\times(\dot{z} - r\dot{\theta})\times r \tag{3.11}$$

or

$$I\ddot{\theta} = \frac{4}{3}r^3(p - p_\mathrm{a})\theta - Mg(h\theta) + \\ -\left(\dot{m}_\mathrm{in} - \frac{c_0 pLz}{\sqrt{RT}}\left\{\frac{2\gamma}{\gamma - 1}\left[\left(\frac{p_\mathrm{a}}{p}\right)^{\frac{2}{\gamma}} - \left(\frac{p_\mathrm{a}}{p}\right)^{\frac{(\gamma+1)}{\gamma}}\right]\right\}^{\frac{1}{2}}\right)\times(\dot{z} - r\dot{\theta})\times r \tag{3.12}$$

where I is the ACV moment of inertia about its center of mass. Equations (3.1)-(3.3), (3.8) and (3.12) are solved numerically in the following section.

4. Results and discussion

The governing nonlinear stiff system of ordinary differential equations (3.1)-(3.3), (3.8) and (3.12) are solved numerically using the implicit Runge-Kutta numerical integration method. This can be performed by converting the system of governing equations into a system of first order ordinary differential equation by state space analysis equations (4.1) to (4.5). This solution method is adopted in this study because it has the necessary numerical stability that is essential to solve the current system of differential equations

$$\dot{z}_1 = z_2, \tag{4.1}$$

$$\dot{\theta}_1 = \theta_2, \tag{4.2}$$

$$\dot{z}_2 = \frac{A}{M}(p - p_a) - g - \frac{1}{M}(\dot{m}_{in} - \dot{m}_{out}) \times (z_2 - r\theta_2), \tag{4.3}$$

$$\dot{\theta}_2 = \frac{1}{I}\left\{\frac{4}{3}r^3 (p - p_a)\theta_1 - Mg(h\theta_1) - (\dot{m}_{in} - \dot{m}_{out}) \times (z_2 - r\theta_2) \times r\right\} \tag{4.4}$$

and

$$\dot{p} = \frac{\gamma RT}{A(d + z_1)}\left[\dot{m}_{in} - \dot{m}_{out} - \frac{pAz_2}{RT}\right]. \tag{4.5}$$

To solve this system of first order ordinary nonlinear differential equations numerically, a Matlab M-file program is written that uses a fixed time step Runge-Kutta method. The accuracy of the results depends on the type of solver used in the Matlab. There are some terms which can lead to quick changes in the solution. Therefore, this system is considered a stiff system. The Matlab simulation uses the following initial conditions:

$$z(t = 0) = 0.001\,\text{m}, \dot{z}(t = 0) = 0.001\,\text{m/s}, \tag{4.6}$$

$$p(t = 0) = 101000\ \text{Pa}, \tag{4.7}$$

$$\theta(t = 0) = 0.01\,\text{rad}, \qquad \dot{\theta}(t = 0) = 0.001\,rad/sec. \tag{4.8}$$

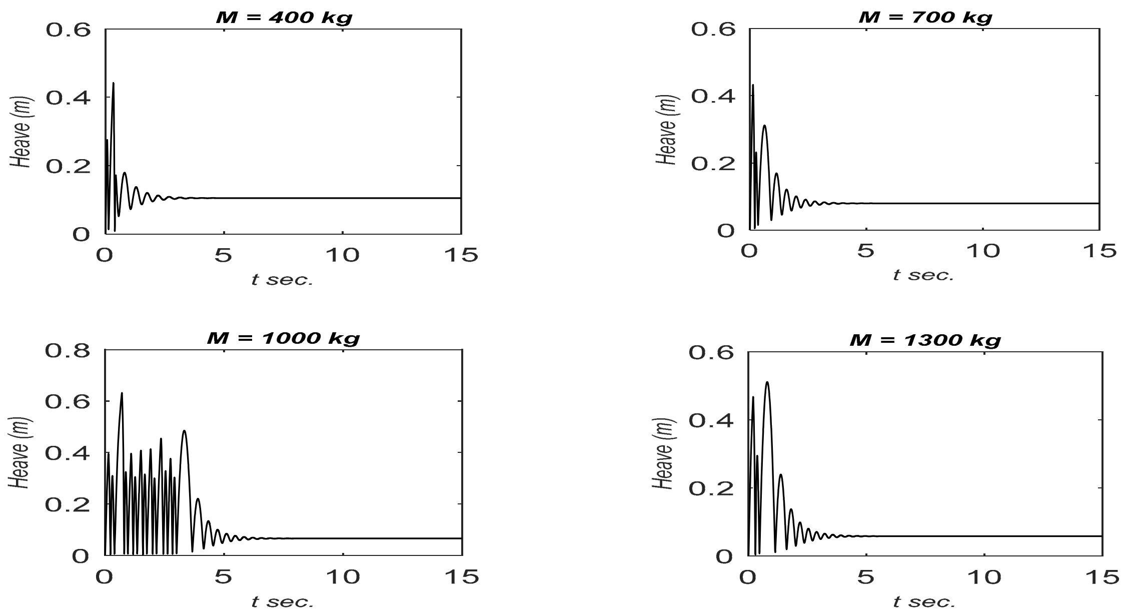

Fig.2. Time history of the vertical motion (heave) for $\dot{m}_{in} = 25\,kg/s$ and $r = 0.85m$.

Since the system comprises highly nonlinear and stiff equations, the most appropriate Matlab solver to handle such equations is *ode15s* [25]. This solver is capable of finishing the integration with the least number of steps and is considered fastest among other stiff solvers because the constant Jacobian is identified at each time step. To assess the behavior of this model and examine the presence of oscillatory motions in the ACV, some parameters that substantially affect the solution response have been identified. Three of the identified parameters that include the ACV mass (*M*), the entering mass flow rate in the cushion ($\dot{m}_{in}$) , and ACV base radius (*r*) have been chosen to vary as they significantly affect the oscillatory motion of the vehicle. The other parameters are given fixed values for the sake of computations in the Matlab software.

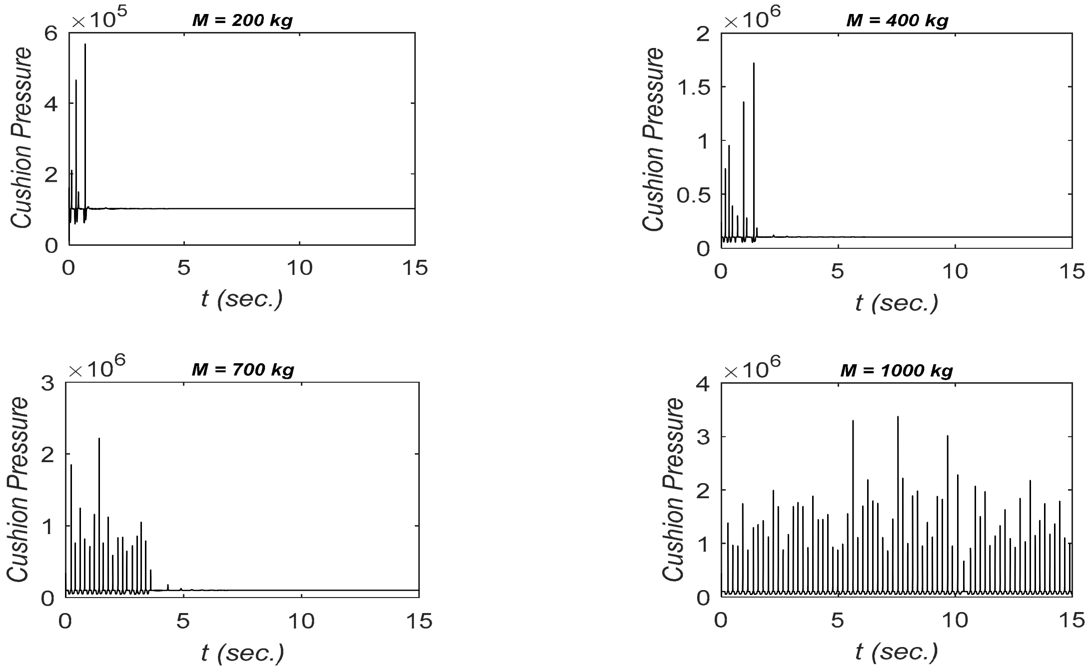

Fig.3. Cushion pressure time history for $\dot{m}_{in} = 25\,kg/s$ and $r = 0.85m$.

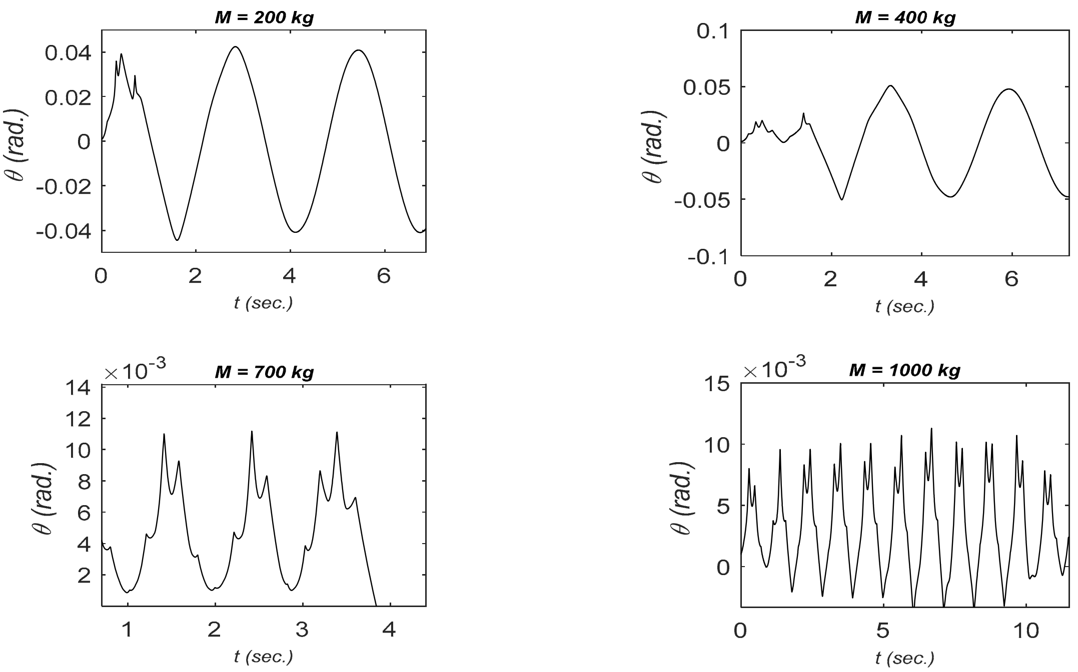

Fig.4. Zoomed-out pitch motion time history for $\dot{m}_{in} = 25\,kg/s$ and $r = 0.85m$.

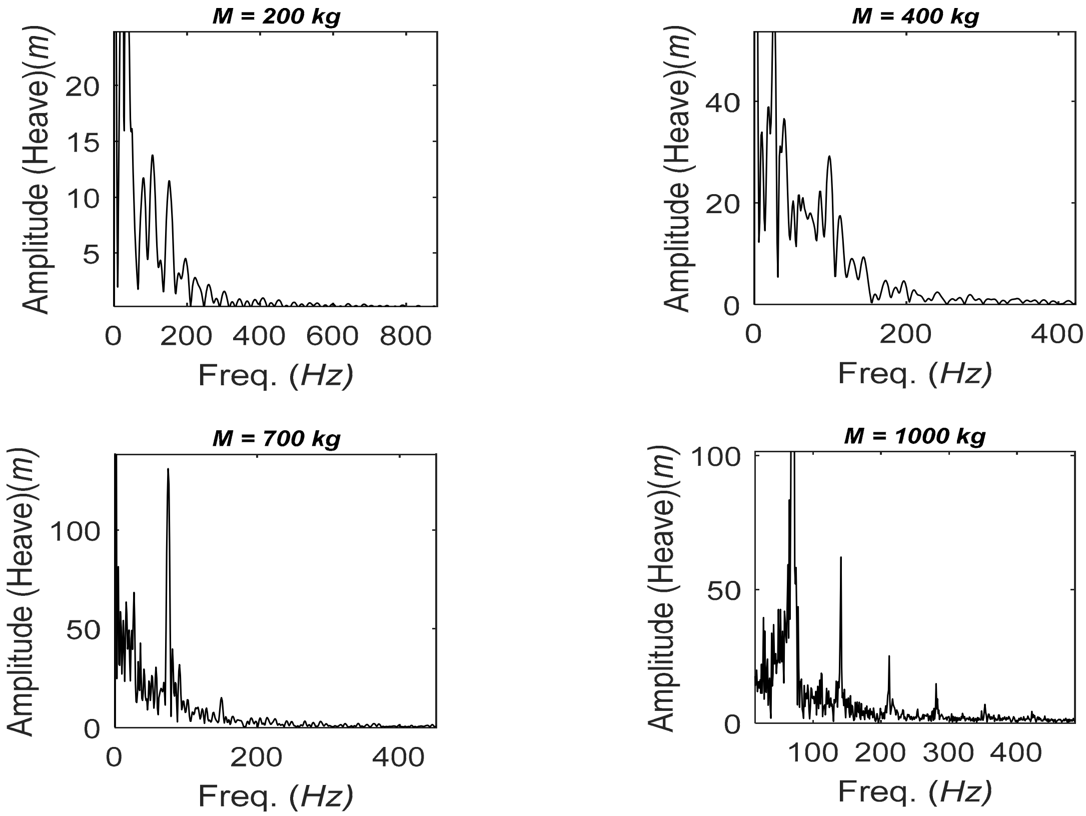

Fig.5. Zoomed-in spectrum of the transient heave response for $\dot{m}_{in} = 25\,kg/s$ and $r = 0.85m$.

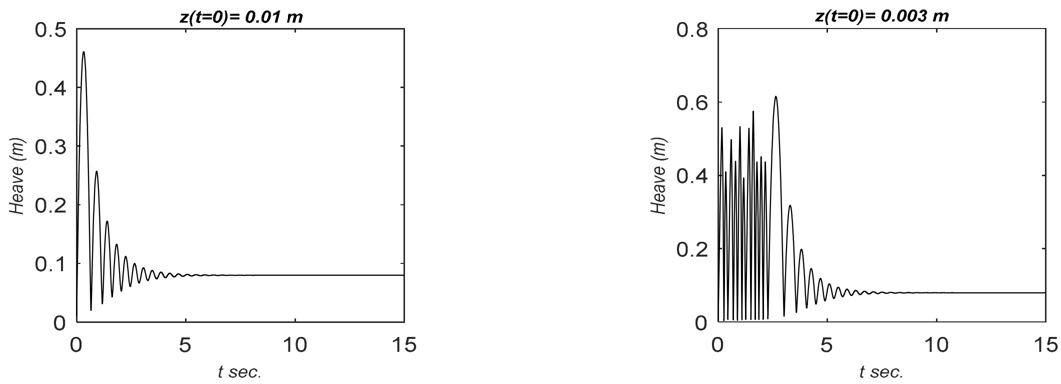

Fig.6. Heave motion sensitivity to the initial conditions for $\dot{m}_{in} = 25\,kg/s, M = 700kg$ and $r = 0.85m$.

Without loss of generality, specific ranges are assigned to the above three design parameters. These ranges are: the vehicle mass (M) $[100kg$ to $1300kg]$, the entering mass flow rate $(\dot{m}_{in})$ $[10kg/s$ to $30kg/s]$, and the vehicle base radius range is $[0.8m$ to $1.0m]$. The transient response of the ACV is obtained by varying the design parameters. This has led to generating a substantial amount of data, a summary of which is presented below.

Samples results are presented in Figs 2-8. Results for the heave motion are shown in Figs 2, 5 and 6. The heave figures show the presence of oscillatory motions. The violent oscillatory motions in the heave response in the transient stage are present and with a good selection of the design parameters, these oscillations can be limited.

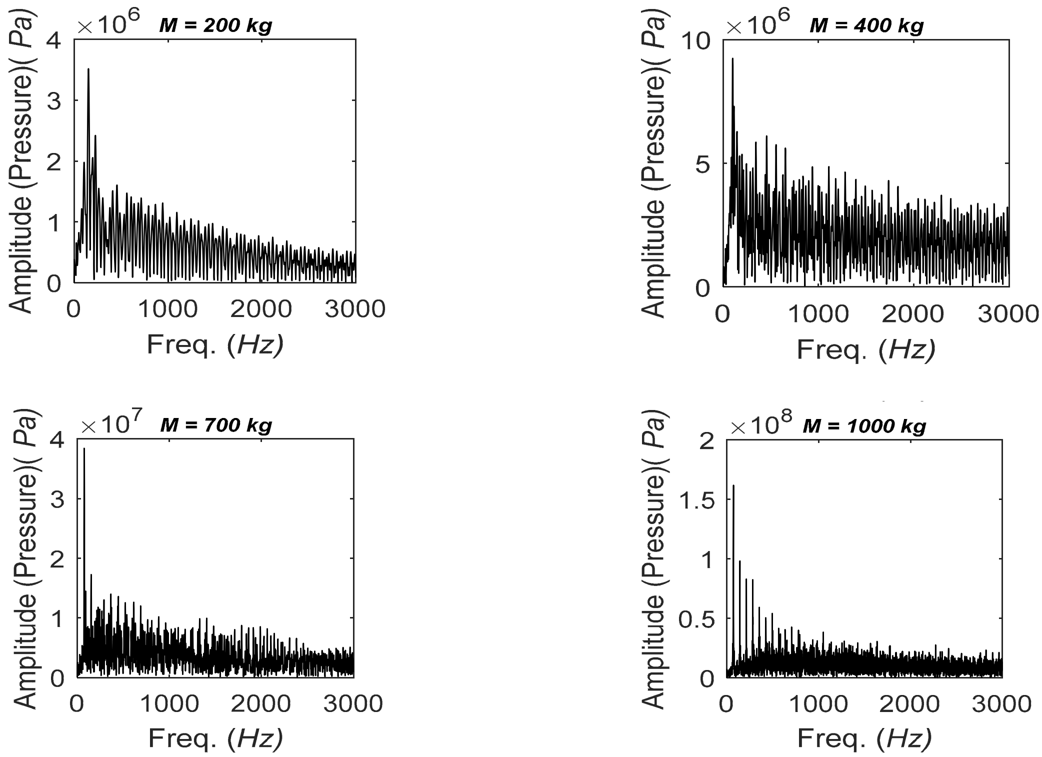

Fig.7. Spectrum of the cushion pressure for $\dot{m}_{in} = 25\,kg/s$ and $r = 0.85m$.

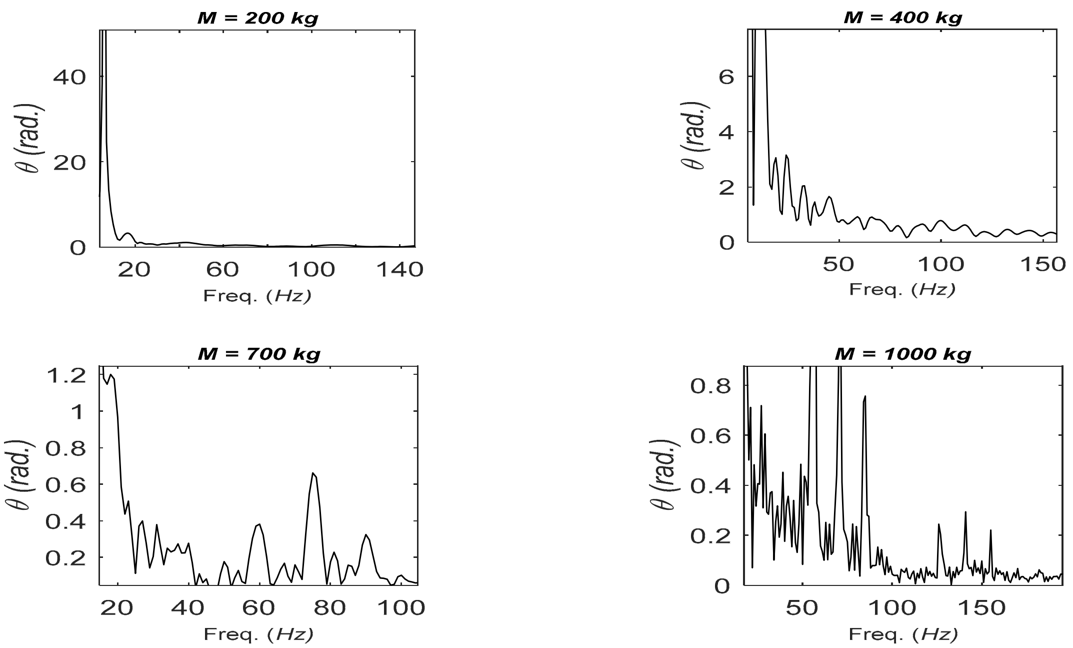

Fig.8. Zoomed-in pitch motion spectrum for $\dot{m}_{in} = 25\,kg/s$ and $r = 0.85m$.

The steady state spectrum of the heave motion as an oscillatory motion with small amplitude is shown in Fig.5, which indicates a clear instability in the motion of the vehicle. Figure 3 indicates the time history of the cushion pressure. The values of the cushion pressure fluctuate about a certain value that is higher than the atmospheric pressure at each time step. Then its value returns to the atmospheric pressure value, which makes the mass flow rate out of the cushion volume $\dot{m}_{out}$ is equal to zero. Figures 4 and 8 illustrate the oscillating pitch response for different values of the design parameters. One can conclude from these figures that the pitch response is not periodic when a high value of the ACV mass (M) is considered.

The frequency spectrums for the heave, cushion pressure, and the pitch motion are presented in Figs 5,7 and 8. It is evident from these figures that the self-excited response yields an excessive amount of noise. This noise which appears in the above mentioned figures represent many distinct frequency components. Figure 5 indicates the steady state spectrum of the heave motion, which shows excessive amount of damped noise of small amplitude.

4.1. Chaos investigation

The chaotic behavior of the model is investigated for the ACV's heave motion, cushion pressure, and the pitch motion. In order to observe the sensitivity of the model to the initial conditions, the chaos study requires simulations of the model for different initial conditions [26, 27]. Two methods have been considered to identify the chaotic behavior. These methods are: the popular Fast Fourier Transform (FFT) and the Poincaré maps.

The FFT of the frequency spectrum such as the one shown in Fig.5, allows us to identify the frequency components from a noisy signal such as heave response. For example, the steady state heave response reveals a number of distinct frequencies for some design conditions in the frequency spectrum of Fig.5. The presence of large noise was noticed also at the early transient stage for some design conditions such as those shown in Figs 2 and 6. Figure 6 indicates how sensitive the heave motion is to the initial conditions. It is clear that the heave motion is not sensitive to the initial conditions, except in the transient part of the solution. It can be concluded that the chaotic behavior of the heave motion does not exist for the range considered. Therefore, a slow damped periodic motion exists in the heave motion of the model. This will be confirmed later through the Poincaré maps as well.

The spectrum (Fig.7) for the cushion pressure indicated a tremendous amount of fundamental frequency (noise), confirming a definite chaotic behavior of the cushion pressure of the ACV in this model. The amplitude of the cushion pressure increases as the mass of the vehicle (M) increases. Figures 8 indicates the chaotic behavior of the pitch motion. This will be confirmed more using the Poincaré maps.

The other method for chaos investigation is the Poincaré maps. The Poincaré maps represent the meeting of a definite lower-dimensional subspace with a periodic orbit in the state variable space of the ACV continuous dynamical conduct. By considering different values of the design parameters in the Matlab simulation, many Poincaré maps were obtained for the heave motion, cushion pressure, and the pitch motion.

In order to construct the Poincaré maps, one needs to adopt certain sampling strategy for evaluating the Poincaré section. Many sampling strategies were introduced in the literature, an accepted sampling rule is to choose $t_i = \dfrac{i}{\omega_n} + t_0$, where t_i is the time at the i^{th} section, t_0 is the initial time, ω_n is the first fundamental frequency which can be achieved from the spectrum response, and i is the number of sampling points [26, 27].

Samples of the Poincaré maps are presented in Figs 9-11. After reviewing the presented Poincaré maps, the chaotic behavior was confirmed for both the cushion pressure and the pitch motion for some values of the design parameters. This happens mostly when the design parameters have non-uniform values relative to each other. For example, in the range considered in this study, when the vehicle mass (M) is high, the vehicle base radius r is low and the mass flow rate $\dot{m}_{in}$ is also high, the chaotic behavior is present for the cushion pressure as confirmed by the spectrum figures. The chaotic behavior arising from the pitch motion is confirmed from Fig.11.

The heave dynamics showed a periodic damped response for some design parameters. At the start of the transient response some irregular harmonics are observed in the heave response. Figure 2 indicates some large noise for the high vehicle mass (M) in the early stage of the transient heave response. This may allow one to conclude the lack of presence of chaos in the heave motion. This is confirmed by having a different Poincaré map by slightly altering the initial conditions, a closed orbit of regular motion is present as shown in Fig.9.

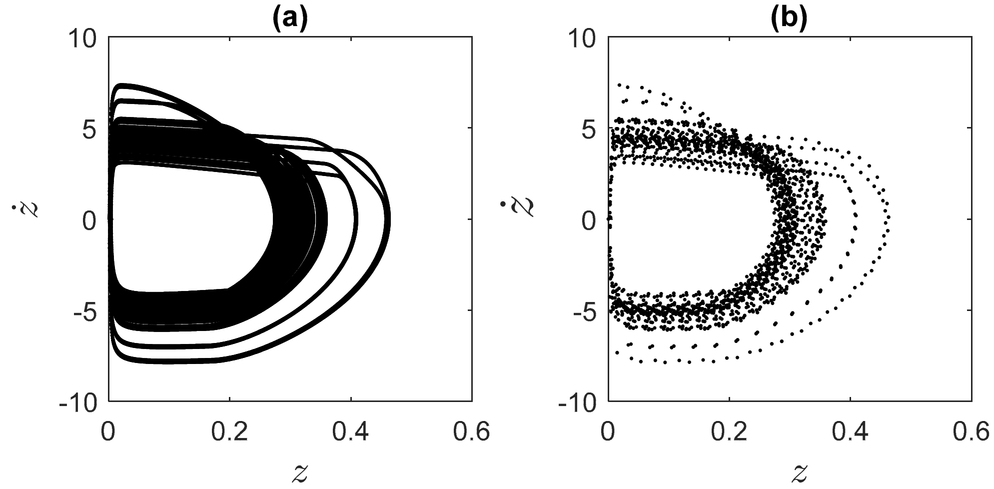

Fig.9. Heave motion: (a) Full Poincaré map; (b) Sampled Poincaré map, for $\dot{m}_{in} = 15\,kg/s, M = 700\ kg$ and $r = 0.75\ m, z(t=0) = 0.001m$.

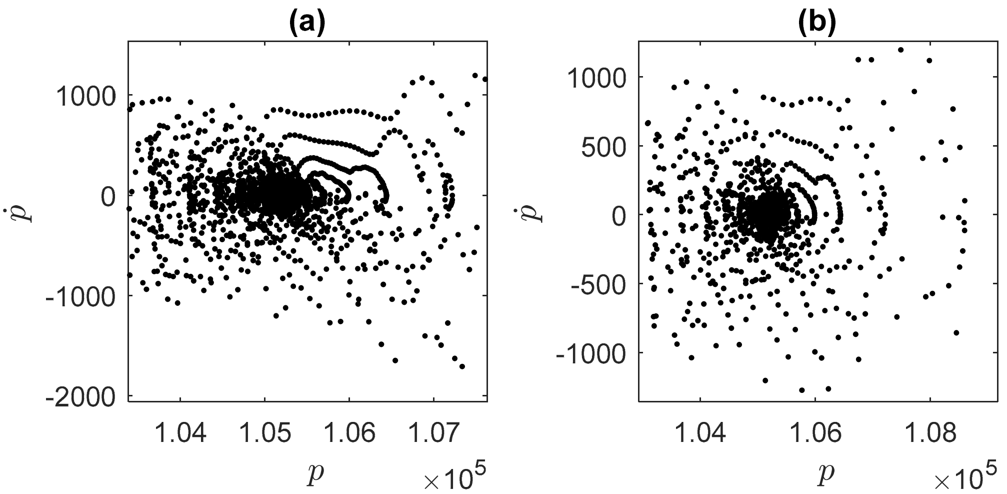

Fig.10. Cushion pressure: (a) Poincaré map (b) Poincaré section, for $\dot{m}_{in} = 15\,kg/s$, $M = 700kg$, and $r = 0.75m$, $z(t=0) = 0.001m$.

This study is a continuation of the study in references [16, 22], where only heave motion was considered in modelling the ACV. Obviously, adding the pitch degree of freedom to the current study increases the complexity of the problem. Therefore, in the current study a strong interaction between the pressure forces and the pitching motion were encountered. Also, the dynamics behavior of the model in this study becomes less stable and chaotic motion is observed.

The heave Poincaré maps in Fig. 9 show a closed curve with a quasi-periodic heave motion indicating a large amount of oscillatory motion, which is not preferable for the ACV heave motion. The chaotic behavior of the cushion pressure may accelerate the failure of the cushion volume due to the inconsistency of the magnitudes of the body forces inside the vehicle cushion volume. The cushion pressure portrait on the Poincaré map (Fig. 10) shows a clear irregularity in the trajectory of the pitch motion.

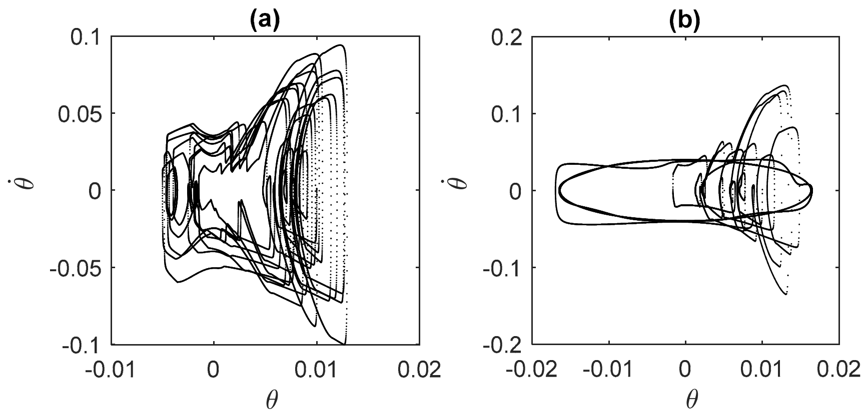

Fig.11. Pitch motion: (a) Poincaré map (b) Poincaré section, for $\dot{m}_{in} = 15\,kg/s, M = 700kg$ and $r = 0.75m$, $z(t=0) = 0.001m$.

It may be seen in Fig. 11 that the phase trajectories of the pitch motion are complicated, irregular, and the structure of its points has no rules. These Poincaré maps of the pitch motion reveal chaotic movements which result in unstable azimuthal motion of the ACV. This can cause a body discomfort for the rider of these vehicles.

5. Conclusions

A highly nonlinear model for the dynamic behavior of the heave and pitch motions of an ACV is considered in this study. The model is based on the Newton's second law of translation and rotation, and the basics of thermodynamics and fluid mechanics. The nonlinearity of the model arises due to the presence of the polytropic exponent in the governing equations, which results in making the model self-excited. The dynamic behavior of the ACV is numerically studied using the ode stiff solvers provided in the Matlab software. Results obtained from the Matlab numerical simulations include the estimation of the heave motion, vertical velocity, cushion pressure, angular rotation, and angular velocity in the pitching direction.

Also, the chaotic motion of this model is investigated for all dependent variables, and for some other design parameters. Three of the design parameters have been found to have a significant effect on the chaotic behavior of the model. These are: the mass of the vehicle *M*, the vehicle base radius *r*, and air mass flow rate into the cushion volume $\dot{m}_{\text{in}}$. The model is tested for the sensitivity of the initial conditions, and it is found to be sensitive to the cushion pressure, and the pitch motion. Damped periodic harsh oscillatory motion was encountered for the ACV heave motion. The FFT spectrum and the Poincaré maps indicated a chaotic behavior of the model for the cushion pressure, and the pitch motion. Uninterrupted oscillations in the cushion pressure and pitch motion are observed.

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Nomenclature

A – ACV base area $\left(m^2\right)$
A_e – exit area $\left(m^2\right)$
c_0 – correction coefficient factor $(=0.7)$
d – ACV height from the ground $(0.008\ m)$
F_M – force due to the momentum flow (N)
h – metacenter distance $(=0.15m)$
I – ACV moment of inertia $(kg\ m^2)$
L – skirt length (m)
M – mass of the vehicle (kg)
m – cushion air mass (kg)
$\dot{m}$ – mass flow rate of air (kg / s)
$\mathcal{M}_G$ – moment about center of mass (Nm)
$\dot{m}_{in}$ – mass flow rate of air inside the cushion volume (kg / s)
$\dot{m}_{out}$ – mass flow rate of air outside the cushion volume (kg / s)
$\dot{p}$ – cushion pressure (Pa)
p_a – atmospheric pressure $(101000Pa)$
r – radius of the ACV base (m)
T – temperature of air $(298\ K)$
V – cushion volume $\left(m^3\right)$
z – heave displacement (m)
$\ddot{z}$ – vertical acceleration $\left(m / s^2\right)$
γ – specific heat ratio $\left(c_p / c_v\right)$
θ – pitch angular displacement (radian)
$\dot{\theta}$ – angular velocity of the ACV (rad / sec)
ρ – air density $\left(kg / m^3\right)$
ρ_a – atmospheric air density $\left(1.189\ kg / m^3\right)$

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