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ON A COUPLE-STRESS RIVLIN-ERICKSEN FERROMAGNETIC FLUID HEATED FROM BELOW WITH VARYING GRAVITY, ROTATION, MAGNETIC FIELD AND SUSPENDED PARTICLES FLOWING THROUGH A POROUS MEDIUM

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The thermal instability of a couple-stress Rivlin-Ericksen ferromagnetic fluid with varying gravity field, suspended particles, rotation and magnetic field flowing through a porous medium is investigated. The dispersion relation has been developed and solved analytically using the normal mode approach and linear stability theory. The effect of suspended particles, rotation, couple stress, permeability and magnetic field on the fluid layer has been investigated. For stationary conventions, it is found that suspended particles always have a destabilizing effect for $\lambda>0$ and a stabilizing effect for $\lambda<0$ and couple-stress, magnetic field and permeability of the medium have a stabilizing effect on the thermal instability under certain conditions. In the absence of the rotation couple-stress has a stabilizing effect if $\lambda>0$ and a destabilizing effect if $\lambda<0$. Rotation has a stabilizing effect if $\lambda>0$ and a destabilizing effect if $\lambda<0$. In the absence of rotation permeability has a stabilizing effect if $\lambda<0$ and a destabilizing effect if $\lambda>0$. Magnetisation always has a stabilizing effect ($\lambda>0$ or $\lambda<0$).

Key words: thermal instability, magnetic field, couple-stress fluid, ferromagnetic fluid, suspended particles.

1. Introduction

The couple-stress fluid theory was developed by Stokes [1] in 1966 and mathematically formulated by Stokes [2] in 1984. The lubrication mechanism of synovial joints is the most eminent application of couple-stress fluid in the non Newtonian fluid theory. A human joint is a dynamically loaded bearing in which the bearing is an articular cartilage and the lubricant is the synovial fluid. Squeeze-film action can provide significant protection to the cartilage surface when a fluid film is formed. The loaded-bearing synovial joints of the human body are the shoulder, hip, knee, and ankle joints, which have a low friction coefficient and are characterized by little wear. The synovial fluid is a viscous, non-Newtonian fluid that is clear or yellowish in colour. Walicki and Walicka [3] modelled the synovial fluid as a couple-stress fluid in human joints. In 1981 Chandrasekhar [4] explained the thermal instability on a bottom heating fluid layer in different hydrodyanamic conditions theoretically as well as experimentally. Sharma and Sharma [5] discussed a bottom heating couple-stress fluid in a porous medium. In the presence of a magnetic field and rotation, Kumar and Kumar [6] investigated the effect of dust particles on a bottom heating couple-stress fluid and found that dust particles have a destabilizing effect, whereas there is a stabilizing effect due to rotation. A compressible couple-stress fluid layer with suspended particles in a porous medium was examined by Sunilet *et al.* [7] who found that suspended particles had a destabilizing effect on the system. Stanly and Vasanthakumari [8] investigated the thermal instability of a dusty couple-stress fluid in the presence of rotation and magnetic field. Kumar *et al.* [9], Sharma and Thakur [10], Sunilet *et al.* [11], Mishra and Kumar [12] also studied the effect of different fields on a couple-stress fluid.

Rosenweig [13] emphasised the relevance of magnetic fluid research. The thermal instability of ferrofluids in a uniform perpendicular magnetic field was studied by Finlayson [14]. Thermal convection of a

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ferromagnetic fluid was examined by Siddheswar [15, 16] in different situations. Venkatasubramaniam and Kaloni [17], Stiles and Kagan [18] considered a horizontal ferrofluid layer and analyzed the thermos-convective instability in rotation and magnetic field respectively. The influence of the magnetic field and suspended particles on thermal convection in a ferromagnetic fluid flowing through a porous material with a varying gravity field was studied by Pant *et al.* [19]. The influence of rotation with a magnetic field on the occurrence of thermal convection in ferromagnetic liquids in a porous material was discussed by Bhagat *et al.* [20].

The effects of dust particles on thermal convection of couple-stress ferromagnetic fluids was examined by Pulkit *et al.* [21, 22]. Rahul and Naveen Sharma [23, 24, 25] considered the couple stress Rivlin Ericksen ferromagnetic fluid and studied the thermal instability in the presence of various fields. On studying the couple stress Rivlin Ericksen ferromagnetic fluid heated from the bottom with unstable gravity and suspended particles in the porous material Rahulet *et al.* [26] found that no oscillatory modes are possible and the system is not stable. Rahul and Naveen Sharma [27] examined the rotating couple stress Rivlin Ericksen ferromagnetic fluid with varying gravity and suspended particles flowing through a porous material and found that due to applying rotation field the system is stable and oscillatory modes exist.

We have extended our work and investigated the influence of suspended particles on instability of a couple-stress Rivlin-Ericksen ferromagnetic fluid with variable gravity, magnetic and rotation field flowing in a porous material .

2. Mathematical formulation

Consider a thin layer of an infinite in compressible, electrically non conducting couple-stress Rivlin-Ericksen ferromagnetic fluid with suspended particles bounded between two infinite horizontal planes situated at $z=0$ and $z=d$. A uniform magnetic field $\boldsymbol{H}(0,0,H)$, a uniform rotation $\boldsymbol{\Omega}(0,0,\Omega)$ and variable gravity field $\boldsymbol{g}(0,0,-g)$ are applied along the vertical direction which is taken as the z- axis, where $g=\lambda g_o$, g_o is the value of g at $z=0$, which is always positive and λ can be positive or negative as gravity increases or decreases upwards from its value at $z=0(g_o)$.

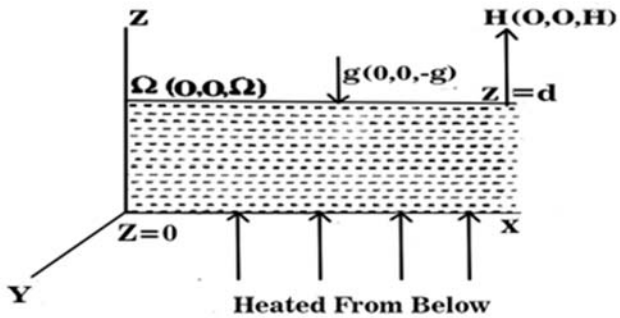

Fig.1. Geometrical problem.

The considered fluid layer is heated from below so that the uniform temperature gradient $\beta=\left(\left|\frac{dT}{dz}\right|\right)$ is maintained.

A ferromagnetic fluid reacts so quickly to a magnetic torque that we can simulate the following conditions to maintain,

$$\boldsymbol{M}\times\boldsymbol{H}=0 \tag{2.1}$$

where M represents magnetization and H represents the magnetic field intensity.

Maxwell's equation is also satisfied by the ferromagnetic fluid. Here, we examine an electrically non conducting fluid, therefore the displacement current is negligible. Maxwell's equation becomes,

$$\nabla \cdot \boldsymbol{B} = 0 , \qquad \nabla \times \boldsymbol{H} = 0 . \tag{2.2}$$

The magnetic field intensity H, magnetization M and magnetic induction B are correlated in Chu formulation of electro dynamics such that,

$$\boldsymbol{B} = \mu_0 \left(\boldsymbol{H} + \boldsymbol{M} \right) . \tag{2.3}$$

Let us assume that magnetization depends on the amplitude of magnetic field and temperature. Let us also assume that magnetization is aligned with the magnetic field so that,

$$\boldsymbol{M} = \frac{\boldsymbol{H}}{H} M\left(H,T\right) \tag{2.4}$$

Pressure, density, temperature, coefficient of thermal expansion, kinematic viscosity, kinematic viscous elasticity, couple-stress viscosity, medium porosity, medium permeability, magnetic permeability, thermal diffusivity, resistivity, velocity of fluid, velocity of suspended particles, and number density of suspended particles are denoted by $p, \rho,\ T, \alpha, \nu, \varepsilon,\ K_1, \mu_e, \kappa_T, \eta,\ \boldsymbol{q} = (u_1,u_2,u_3),\ \boldsymbol{q_s}\left(x,t\right)$ and $N_s\left(x,t\right)$ respectively. $K = 6\pi\mu\eta'$ is the Stokes drag coefficient. η' is the radius of the suspended particle.

In the presence of rotation, the magnetic field and suspended particles the equation of motion, continuity and heat conduction of the couple-stress Rivlin-Ericksen ferromagnetic fluid flowing through a porous medium are

$$\begin{aligned} \frac{1}{\varepsilon}\left[\frac{\partial \boldsymbol{q}}{\partial t} + \frac{1}{\varepsilon}(\boldsymbol{q}\cdot\nabla)q\right] = -\frac{1}{\rho_0}\nabla p + g\left(1 + \frac{\delta\rho}{\rho_0}\right) + \frac{1}{\rho_0}\boldsymbol{M}.\nabla\boldsymbol{H} + \left(\nu - \frac{\mu'}{\rho_0}\nabla^2\right)\nabla^2\boldsymbol{q} + \\ -\frac{1}{K_1}\left(\nu + \nu'\frac{\partial \boldsymbol{q}}{\partial t}\right) + \frac{KN_S}{\varepsilon\rho_0}(\boldsymbol{q_S} - \boldsymbol{q}) + \frac{2}{\varepsilon}(\boldsymbol{q}\times\boldsymbol{\Omega}) + \frac{\mu e}{4\pi\rho_0}\left[(\nabla\times\boldsymbol{H})\times\boldsymbol{H}\right], \end{aligned} \tag{2.5}$$

$$\nabla \cdot \boldsymbol{q} = 0 , \tag{2.6}$$

$$\frac{\partial T}{\partial t} + (\boldsymbol{q}\cdot\nabla)T = \kappa_T\ \nabla^2\ T . \tag{2.7}$$

Maxwell's equation of electromagnetism is

$$\varepsilon\frac{\partial \boldsymbol{H}}{\partial t} = (\boldsymbol{H}\cdot\nabla)\boldsymbol{q} + \varepsilon\eta\nabla^2\boldsymbol{H} , \tag{2.8}$$

$$\nabla \cdot \boldsymbol{H} = 0 . \tag{2.9}$$

The equation of state is

$$\rho = \rho_0\left[1 - \alpha\left(T - T_0\right)\right] \tag{2.10}$$

where T_0 and ρ_0 represent the value of temperature and density at $z=0$, respectively.

$\nabla \boldsymbol{H}$ denotes the magnetic field gradient.

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2},$$

$$H = |\boldsymbol{H}|, \quad B = |\boldsymbol{B}|, \quad M = |\boldsymbol{M}|.$$

Due to suspended particles in the fluid a force gets generated which is proportional to the difference of velocity of the fluid and velocity of suspended particles. Since the force applied by the suspended particles on the fluid and the force applied by the fluid on suspended particles are equal, there must be an another force, equal in magnitude but opposite in sign in equation of motion for the suspended particles. The distance between particles is taken to be relatively large compared to their diameter so inter particles reactions are not considered.
The equations of motion, continuity for suspended particles for the above assumptions, are

$$mN_S\left[\frac{\partial \boldsymbol{q_S}}{\partial t} + \frac{1}{\varepsilon}(\boldsymbol{q_S}\cdot\nabla)\boldsymbol{q_S}\right] = KN_S(\boldsymbol{q} - \boldsymbol{q_S}), \tag{2.11}$$

$$\varepsilon\frac{\partial N_S}{\partial t} + \nabla\cdot(N_S\cdot\boldsymbol{q_S}) = 0 \tag{2.12}$$

where mN_S is the density of suspended particles.

Let C_V, C_S, ρ_p and C_P denote the capacity of the fluid at constant volume, the heat capacity of suspended particles, density of porous material and heat capacity of porous material, respectively. If there is thermal equilibrium, then the equation of heat conduction is

$$E.\frac{\partial T}{\partial t} + (\boldsymbol{q}\cdot\nabla)T + \frac{mN_S C_S}{\rho_0 C_V}\left(\varepsilon\frac{\partial}{\partial t} + \boldsymbol{q_S}\cdot\nabla\right)T = \kappa_T\nabla^2\ \mathrm{T} \tag{2.13}$$

where

$$E = \varepsilon + (1-\varepsilon)\frac{\rho_{PC_P}}{\rho_{0C_V}} \text{ is a constant,}$$

$$\kappa_T = \frac{q'}{\rho_0 C_V}$$

here q' is the effective thermal conductivity of pure fluids.

In general, to complete the system a state equation is needed, which satisfies M in two thermodynamic variables H and T. In the present paper we assume magnetization to be independent of the magnetic field and depends on temperature only. Therefore,

$$M = M(T).$$

As the first approximation, we consider that

$$M = M_0\left[1-\gamma\left(T-T_0\right)\right] \tag{2.14}$$

where M_0 is the magnetization at $T=T_0$ and

$$\gamma = \frac{1}{M_0}\left(\frac{\partial M}{\partial T}\right)H .$$

3. Basic state and perturbation equations

The rest state of the fluid is given by

$$\boldsymbol{q} = (0,0,0), \quad p = p(z), \quad \rho = \rho(z) = \rho_0\left(1+\alpha\beta z\right),$$

$$\boldsymbol{\Omega} = (0,0,\Omega), \quad \boldsymbol{M} = M(z) = M_0\left(1+\gamma\beta z\right), \quad T = T(z) = T_0 - \beta z, \tag{3.1}$$

$$\boldsymbol{H} = (0,0,H), \quad q_S = (0,0,0), \quad N_S = N_0.$$

To study the character of equilibrium let us apply a little perturbation on the layer of the fluid due to which some disturbances take place in the system. Now we consider that these small disturbances are the function of time and space variable. The perturbed flow may be represented as

$$\boldsymbol{q} = (0,0,0) + \left(u_1, u_2, u_3\right), \quad T = T(z) + \theta, \quad \rho = \rho(z) + \delta\rho,$$

$$p = p(z) + \delta p, \quad \boldsymbol{H} = (0,0,H) + \left(h_x, h_y, h_z\right), \tag{3.2}$$

$$\boldsymbol{q_S} = (0,0,0) + (l,r,s) \quad M = M(z) + M$$

where $\boldsymbol{q}$ is the perturbation in fluid velocity, $\boldsymbol{q_S}$ is the velocity of suspended particles, ρ is the density, p is the pressure, $\boldsymbol{H}$ is the magnetic field, $\boldsymbol{T}$ is the temperature and $\boldsymbol{M}$ is the magnetization are $\boldsymbol{q}(u_1,\ u_2,\ u_3)$, $\boldsymbol{q_S}(l,r,s)$ $\delta\rho, \delta p, \boldsymbol{h}\left(h_x,\ h_y,\ h_z\right), \theta, \delta M$ respectively.

To decompose the disturbances we use the normal mode method. Let us take perturbation in the following form,

$$\left(u3, \theta, \zeta, \xi, hz\right) = \left[W(z), \Theta(z), Z(z), X(z), V(z)\right] \cdot \exp\left(i\,kx\ x + i\,ky\ y + nt\right) \tag{3.3}$$

where kx and ky are wave numbers in the X and Y direction and k is the resultant disturbances wave number such that

$$k = \sqrt{\left[\left(k_x\right)^2 + \left(k_y\right)^2\right]},$$

n is the frequency of any arbitrary disturbances which is a complex constant.

Now by using relation (3.3) we obtain

$$\left(D^2-a^2\right)\left[\left(\frac{\sigma}{\varepsilon}+\frac{M\sigma}{\varepsilon(1+\tau_1\sigma)}+\frac{(1+\sigma F_1)}{P_t}\right)+F\left(D^2-a^2\right)^2-\left(D^2-a^2\right)\right]W+ \\ +\frac{\alpha a^2\lambda d^2}{\nu}\left(g_0-\frac{\gamma M_0\nabla H}{\rho_0\alpha\lambda}\right)\theta+\frac{2}{\varepsilon\nu}\Omega d^3DZ-\frac{\mu eHd}{4\pi\rho_0\nu}\left(D^2-a^2\right)DV=0, \tag{3.4}$$

$$\left[\left(\frac{\sigma}{\varepsilon}+\frac{M\sigma}{\varepsilon(1+\tau_1\sigma)}+\frac{(1+\sigma F_1)}{P_t}\right)+F\left(D^2-a^2\right)^2-\left(D^2-a^2\right)\right]Z=\frac{2\Omega d}{\varepsilon\nu}, \tag{3.5}$$

$$\varepsilon\left[\left(D^2-a^2\right)-\sigma P_2\right]\mathrm{X}=-\frac{Hd}{\eta}DZ, \tag{3.6}$$

$$\varepsilon\left[\left(D^2-a^2\right)-\sigma P_2\right]\mathrm{V}=-\frac{Hd}{\eta}DW, \tag{3.7}$$

$$\left[\left(D^2-a^2\right)-\sigma E_1P_1\right]\Theta=-\left(\frac{B+\tau_1\sigma}{1+\tau_1\sigma}\right)\frac{\beta d^2}{\kappa_T}W. \tag{3.8}$$

Converting coordinates x, y and z in new units of length d, time t and $\left(d^2/\kappa_T\right)$. Let $a=kd, \sigma=nd^2/\nu$, $F=\frac{\mu'}{\rho 0d2\nu}$, $F_1=\frac{\nu'}{d^2}$, $P_t=K_1/d_2$, $P_1=\nu/\kappa_T$, $P_2=\nu/\eta$, $\tau=\frac{m}{k}$, $\tau_1=\frac{\tau\nu}{d^2}$, $E_1=(E+b\varepsilon)$, $B=\left(1+b\right)$, $x^*=x/d$, $y^*=y/d$, $z^*=z/d$, $D^*=dD$ (dropping * for convenience), where P_1 and P_t denote the Prandtl number and dimensionless medium permeability.

Now eliminate Θ, X, V and Z from (3.4) with the help of (3.5), (3.6), (3.7) and (3.8), then we get stability governing equation

$$\left(D^2-a^2\right)\left[\left(\frac{\sigma}{\varepsilon}+\frac{M\sigma}{\varepsilon(1+\tau_1\sigma)}+\frac{(1+\sigma F_1)}{P_t}\right)+F\left(D^2-a^2\right)^2-\left(D^2-a^2\right)\right]W+ \\ -\lambda a^2Rf\left(\frac{B+\tau_1\sigma}{1+\tau_1\sigma}\right)\left(\frac{1}{\left(D^2-a^2\right)-\sigma E_1P_1}\right)W+ \\ +\frac{TA}{\varepsilon^2}\frac{\left[\left(D^2-a^2\right)-\sigma P_2\right]}{\left[\left(D^2-a^2\right)-\sigma P_2\right)\left\{\begin{matrix}\left(\frac{\sigma}{\varepsilon}+\frac{M\sigma}{\varepsilon(1+\tau_1\sigma)}+\frac{(1+\sigma F_1)}{P_t}\right)+\\ +F\left(D^2-a^2\right)^2-\left(D^2-a^2\right)\end{matrix}\right\}+\frac{QD^2}{\varepsilon}\right]}\times \\ \times D^2W+\frac{Q\left(D^2-a^2\right)}{\varepsilon\left(\left(D^2-a^2\right)-\sigma P_2\right)}D^2W=0 \tag{3.9}$$

where

$$R_f = \frac{\alpha\beta d^4}{\nu\kappa_T}\left[g_0 - \frac{\gamma M_0 \nabla H}{\rho_0\alpha\lambda}\right] = \frac{\alpha\beta d^4 g_0}{\nu\kappa_T}\left[1 - \frac{\gamma M_0 \nabla H}{\rho_0\alpha\lambda g_0}\right] = R\left[1 - \frac{\gamma M_0 \nabla H}{\rho_0\alpha\lambda g_0}\right]$$

R_f is the Rayleigh number for the ferromagnetic fluid, R is the Rayleigh number for the fluid and

$T_A = \left(\frac{2\Omega d^2}{\nu}\right)^2$ is the modified Taylor number and

$Q = \frac{\mu e H^2 d^2}{4\pi\rho_0\nu\eta}$ is the Chandrasekhar number.

The perturbation in the temperature on boundaries is zero, because the boundaries are kept at constant temperature. So the appropriate condition on the boundary is

$$W = 0, \quad Z = 0, \quad at \quad z = 0 \quad \text{and} \quad z = 1,$$
$$DZ = D^2W = D^4W = 0 \quad \text{at} \quad z = 0 \quad \text{and} \quad z = 1. \tag{3.10}$$

From (3.10) it is clear that all even order derivative of W vanish on boundaries. Therefore a proper solution of (3.9) characterizing the lowest mode is

$$W = W_0 \sin \pi z \tag{3.11}$$

where W_0 is a constant.

By using (3.11) with (3.9), we get

$$\begin{aligned}
R_1 = {} & \frac{1}{\lambda x}(1+x)(1+x+i\sigma_1 E_1 P_1)\left(\frac{1+i\sigma_1\pi^2\tau_1}{B+i\sigma_1\pi^2\tau_1}\right)\times \\
& \times\left[\left(\frac{i\sigma_1}{\varepsilon} + \frac{Mi\sigma_1}{\varepsilon(1+i\sigma_1\pi^2\tau_1)} + \frac{(1+i\sigma_1 F_3)}{P}\right) + F_2(1+x)^2 + (1+x)\right] + \frac{1}{\lambda x\varepsilon^2}\times \\
& \times\frac{T_{A1}(1+x+i\sigma_1 E_1 P_1)(1+x+i\sigma_1 P_2)}{(1+x+i\sigma_1 P_2)\left[\left(\frac{i\sigma_1}{\varepsilon} + \frac{Mi\sigma_1}{\varepsilon(1+i\sigma_1\pi^2\tau_1)} + \frac{(1+i\sigma_1 F_3)}{P}\right) + F_2(1+x)^2 + (1+x)\right] + \frac{Q_1}{\varepsilon}}\times \\
& \times\left(\frac{1+i\sigma_1\pi^2\tau_1}{B+i\sigma_1\pi^2\tau_1}\right) + \frac{1}{\lambda x}\frac{Q_1(1+x)(1+x+i\sigma_1 E_1 P_1)}{\varepsilon(1+x+i\sigma_1 P_2)}\left(\frac{1+i\sigma_1\pi^2\tau_1}{B+i\sigma_1\pi^2\tau_1}\right),
\end{aligned} \tag{3.12}$$

where

$$x = \frac{a^2}{\pi^2}, \quad i\sigma_1 = \frac{\sigma}{\pi^2}, \quad F_2 = \pi^2 F, \quad P = \pi^2 P_t,$$

$$F_3 = \pi^2 F_1, \quad R_1 = \frac{R_f}{\pi^4}, \quad T_{A1} = \frac{T_A}{\pi^4}, \quad Q_1 = \frac{Q}{\pi^2}.$$

4. Analytical discussion

4.1. Stationary convection

At stationary convection, when stability sets, the marginal state will be characterized by $\sigma 1=0$. Put $\sigma 1=0$ in (3.12). We get

$$R1=\frac{(1+x)^2}{B\lambda x}\left[\left\{F_2(1+x)^2+(1+x)+\frac{1}{P}\right\}+\right.$$
$$\left.+\frac{T_{A1}}{\varepsilon^2}\left\{\frac{1}{(1+x)\left\{F_2(1+x)^2+(1+x)+\frac{1}{P}\right\}+\frac{Q1}{\varepsilon}}\right\}+\frac{Q_1}{\varepsilon(1+x)}\right]. \tag{4.11}$$

Clearly, Eq.(4.11) shows the modified Rayleigh number R_1 as a function of B, F_2, T_{A1}, P, Q_1 and x parameters. Clearly, the viscoelastic parameter F_3 disappears with σ_1. It shows that for stationary convection the Rivlin-Ericksen fluid behaves like an ordinary Newtonian fluid.

To examine the effect of suspended particles, couple-stress, permeability, rotation, magnetic field and magnetization we have to study the analytical behavior of

$$\frac{dR_1}{dB},\quad \frac{dR_1}{dF_2},\quad \frac{dR_1}{dP},\quad \frac{dR_1}{dT_{A1}},\quad \frac{dR_1}{dQ_1}\quad \text{and}\quad \frac{dR_1}{dM_0},$$

$$\frac{dR_1}{dB}=-\frac{(1+x)^2}{B^2\lambda x}\left[\left\{F_2(1+x)^2+(1+x)+\frac{1}{P}\right\}+\right.$$
$$\left.+\frac{T_{A1}}{\varepsilon^2}\left\{\frac{1}{(1+x)\left\{F_2(1+x)^2+(1+x)+\frac{1}{P}\right\}+\frac{Q_1}{\varepsilon}}\right\}+\frac{Q_1}{\varepsilon(1+x)}\right]. \tag{4.12}$$

Equation (4.12) shows that suspended particles have a stabilizing effect on the thermal instability of the couple-stress Rivlin-Ericksen ferromagnetic fluid flowing through porous medium in the presence of gravity field, rotation and magnetic field if $\lambda<0$ (when gravity decreases upwards from its value at $z=0$) and destabilizing effect if $\lambda>0$ (when gravity increases upwards from its value at $z=0$)

$$\frac{dR_1}{dF_2}=\frac{(1+x)^4}{B\lambda x}\left[1-\frac{T_{A1}(1+x)}{\varepsilon^2\left[(1+x)\left\{F_2(1+x)^2+(1+x)+\frac{1}{P}\right\}+\frac{Q_1}{\varepsilon}\right]^2}\right]. \tag{4.13}$$

Equation (4.13) shows that couple-stress has a stabilizing effect on the thermal instability of the couple-stress Rivlin-Ericksen ferromagnetic fluid flowing through a porous medium in the presence of gravity field, rotation, suspended particles and magnetic field under the following condition

$$\lambda > 0, T_{A1}(1+x) < \varepsilon^2\left[(1+x)\left\{F_2(1+x)^2+(1+x)+\frac{1}{P}\right\}+\frac{Q_1}{\varepsilon}\right]^2,$$

and

$$\lambda < 0, T_{A1}(1+x) > \varepsilon^2\left[(1+x)\left\{F_2(1+x)^2+(1+x)+\frac{1}{P}\right\}+\frac{Q_1}{\varepsilon}\right]^2.$$

Equation (4.13) shows that couple-stress has a destabilizing effect on the thermal instability of the couple-stress Rivlin-Ericksen ferromagnetic fluid flowing through a porous medium in the presence of gravity field, rotation, suspended particles and magnetic field under following condition,

$$\lambda > 0, T_{A1}(1+x) > \varepsilon^2\left[(1+x)\left\{F_2(1+x)^2+(1+x)+\frac{1}{P}\right\}+\frac{Q_1}{\varepsilon}\right]^2,$$

and

$$\lambda < 0, T_{A1}(1+x) < \varepsilon^2\left[(1+x)\left\{F_2(1+x)^2+(1+x)+\frac{1}{P}\right\}+\frac{Q_1}{\varepsilon}\right]^2.$$

When rotation is not applied to the fluid, then (4.13) becomes

$$\frac{dR_1}{dF_2}=\frac{(1+x)^4}{B\lambda x}. \tag{4.14}$$

Equation (4.14) shows that couple-stress has a stabilizing effect on the thermal instability of the couple-stress Rivlin-Ericksen ferromagnetic fluid flowing through a porous medium in presence of gravity field, suspended particles and magnetic field if $\lambda>0$, (when gravity increases upwards from its value at $z=0$) and a destabilizing effect if $\lambda<0$ (when gravity decreases upwards from its value at $z=0$) when rotation is not applied to the fluid.

$$\frac{dR_1}{dP}=-\frac{(1+x)^2}{BP^2\lambda x}\left[1-\frac{T_{A1}(1+x)}{\varepsilon^2\left[(1+x)\left\{F_2(1+x)^2+(1+x)+\frac{1}{P}\right\}+\frac{Q_1}{\varepsilon}\right]^2}\right]. \tag{4.15}$$

Equation (4.15) shows that permeability has a stabilizing effect on the thermal instability of the couple-stress Rivlin-Ericksen ferromagnetic fluid flowing through a porous medium in the presence of gravity field, rotation, suspended particles and magnetic field under the following condition,

$$\lambda > 0, T_{A1}(1+x) > \varepsilon^2\left[(1+x)\left\{F_2(1+x)^2+(1+x)+\frac{1}{P}\right\}+\frac{Q_1}{\varepsilon}\right]^2,$$

and

$$\lambda < 0, T_{A1}(1+x) < \varepsilon^2\left[(1+x)\left\{F_2(1+x)^2+(1+x)+\frac{1}{P}\right\}+\frac{Q_1}{\varepsilon}\right]^2.$$

Equation (4.15) shows that permeability has a destabilizing effect on the thermal instability of the couple-stress Rivlin-Ericksen ferromagnetic fluid flowing through a porous medium in presence of gravity field, rotation, suspended particles and magnetic field under the following conditions,

$$\lambda > 0, T_{A1}(1+x) < \varepsilon^2\left[(1+x)\left\{F_2(1+x)^2+(1+x)+\frac{1}{P}\right\}+\frac{Q_1}{\varepsilon}\right]^2,$$

and

$$\lambda < 0, T_{A1}(1+x) > \varepsilon^2\left[(1+x)\left\{F_2(1+x)^2+(1+x)+\frac{1}{P}\right\}+\frac{Q_1}{\varepsilon}\right]^2.$$

In the absence of rotation (4.15) becomes,

$$\frac{dR_1}{dP}=-\frac{(1+x)^2}{BP^2\lambda x} \tag{4.16}$$

Equation (4.16) shows that when rotation is not applied to the fluid permeability has a stabilizing effect on the thermal instability of the couple-stress Rivlin-Ericksen ferromagnetic fluid flowing through a porous medium in the presence of gravity field, suspended particles and magnetic field if $\lambda<0$, (when gravity decreases upwards from its value at $z=0$) and a destabilizing effect if $\lambda>0$ (when gravity increases upwards from its value at $z=0$).

$$\frac{dR_1}{dT_{A1}}=\frac{(1+x)^2}{B\lambda x\varepsilon^2\left[(1+x)\left\{F_2(1+x)^2+(1+x)+\frac{1}{P}\right\}+\frac{Q_1}{\varepsilon}\right]} \tag{4.17}$$

Clearly, Eq.(4.17) shows that rotation has a stabilizing effect on the thermal instability of the couple-stress Rivlin-Ericksen ferromagnetic fluid flowing through a porous medium in the presence of gravity field, suspended particles and magnetic field if $\lambda>0$, (when gravity increases upwards from its value at $z=0$) and a destabilizing effect if $\lambda<0$ (when gravity decreases upwards from its value at $z=0$) or rotation has dual character.

$$\frac{dR_1}{dQ_1}=\frac{(1+x)}{B\varepsilon\lambda x}\left[1-\frac{\{(1+x)T_{A1}\}}{\varepsilon^2\left[(1+x)\left\{F_2(1+x)^2+(1+x)+\frac{1}{P}\right\}+\frac{Q_1}{\varepsilon}\right]^2}\right]. \tag{4.18}$$

Equation (4.18) shows that the magnetic field has a stabilizing effect on the thermal instability of the couple-stress Rivlin-Ericksen ferromagnetic fluid flowing through a porous medium in the presence of gravity field, rotation, suspended particles under the following conditions,

$$\lambda > 0, T_{A1}(1+x) < \varepsilon^2\left[(1+x)\left\{F_2(1+x)^2+(1+x)+\frac{1}{P}\right\}+\frac{Q_1}{\varepsilon}\right]^2,$$

and

$$\lambda < 0, T_{A1}(1+x) > \varepsilon^2\left[(1+x)\left\{F_2(1+x)^2+(1+x)+\frac{1}{P}\right\}+\frac{Q_1}{\varepsilon}\right]^2.$$

Equation (4.18) shows that the magnetic field has a destabilizing effect on the thermal instability of the couple-stress Rivlin-Ericksen ferromagnetic fluid flowing through a porous medium in the presence of gravity field, rotation, suspended particles under following conditions,

$$\lambda > 0, T_{A1}(1+x) > \varepsilon^2\left[(1+x)\left\{F_2(1+x)^2+(1+x)+\frac{1}{P}\right\}+\frac{Q_1}{\varepsilon}\right]^2,$$

and

$$\lambda < 0, T_{A1}(1+x) < \varepsilon^2\left[(1+x)\left\{F_2(1+x)^2+(1+x)+\frac{1}{P}\right\}+\frac{Q_1}{\varepsilon}\right]^2.$$

In the absence of rotation

$$\frac{dR_1}{dQ_1}=\frac{(1+x)}{B\varepsilon\lambda x}. \tag{4.19}$$

Equation (4.19) shows that when rotation is not applied the magnetic field has a stabilizing effect on the thermal instability of the couple-stress Rivlin-Ericksen ferromagnetic fluid flowing through a porous medium in the presence of gravity field if $\lambda>0$, (when gravity increases upwards from its value at $z=0$) and a destabilizing effect if $\lambda<0$ (when gravity decreases upwards from its value at $z=0$).

Replace R_1 by R_f/π^4 and R_f by $R\left[1-\frac{\gamma M_0 \nabla H}{\rho_0\alpha\lambda g_0}\right]$

$$R=\frac{\pi^4(1+x)^2}{B\lambda x}\left[\left\{F_2(1+x)^2+(1+x)+\frac{1}{P}\right\}+\right.$$
$$\left.+\frac{T_{A1}}{\varepsilon^2\left[(1+x)\left\{F_2(1+x)^2+(1+x)+\frac{1}{P}\right\}+\frac{Q_1}{\varepsilon}\right]}+\frac{Q_1}{\varepsilon(1+x)}\right]\times \tag{4.20}$$
$$\times\frac{1}{\left[1-\frac{\gamma M_0\nabla H}{\rho_0\alpha\lambda g_0}\right]}.$$

From (4.20), we obtain

$$\frac{dR}{dM_0}=\frac{\pi^4(1+x)^2}{B\lambda x}\left(\frac{\gamma\nabla H}{\rho_0\alpha\lambda g_0}\right)\left[\left\{F_2(1+x)^2+(1+x)+\frac{1}{P}\right\}+\right.$$
$$\left.+\frac{T_{A1}}{\left[(1+x)\left\{F_2(1+x)^2+(1+x)+\frac{1}{P}\right\}+\frac{Q_1}{\varepsilon}\right]}+\frac{Q_1}{\varepsilon(1+x)}\right]\frac{1}{\left[1-\frac{\gamma M_0\nabla H}{\rho_0\alpha\lambda g_0}\right]^2}. \tag{4.21}$$

Equation (4.21) shows that magnetization has a stabilizing effect on the thermal instability of the couple-stress Rivlin-Ericksen ferromagnetic fluid flowing through a porous medium in the presence of gravity field, rotation, suspended particles and magnetic field for each value of λ either $\lambda>0$, or $\lambda<0$ (either gravity increases upwards from its value at $z=0$ or gravity decreases upwards from its value at $z=0$).
In the absence of rotation and magnetic field

$$\frac{dR}{dM_0}=\frac{\pi^4(1+x)^2}{\lambda x}\frac{\left[\left\{F_2(1+x)^2+(1+x)+\frac{1}{P}\right\}\right]\left(\frac{\gamma\nabla H}{\rho_0\alpha\lambda g_0}\right)}{\left[1-\frac{\gamma M_0\nabla H}{\rho_0\alpha\lambda g_0}\right]^2}. \tag{4.22}$$

Equation (4.22) shows that magnetization has a stabilizing effect on the thermal instability of the couple-stress Rivlin-Ericksen ferromagnetic fluid flowing through a porous medium in the presence of gravity field for each value of λ either $\lambda>0$, or $\lambda<0$ (either gravity increases upwards from its value at $z=0$ or gravity decreases upwards from its value at $z=0$).

4.2. Stability of the system and oscillatory modes

Multiplying (3.4) by the conjugate of W, i.e. W^*, and integrating over the range of z and making use of (3.5)-(3.9) together with boundary condition (3.11), we obtain,

$$\begin{gathered}\left[\left(\frac{\sigma}{\varepsilon}+\frac{M\sigma}{\varepsilon(1+\tau_1\sigma)}+\frac{(1+\sigma F_1)}{P_t}\right)\right]I_1+I_2+FI_3+\frac{d^2}{\varepsilon}\left[\left(\frac{\sigma^*}{\varepsilon}+\frac{M\sigma^*}{\varepsilon(1+\tau_1\sigma)}+\frac{\left(1+\sigma^*F_1\right)}{P_t}\right)I_4\right.\\ \left.+I_5+FI_6+\frac{\mu_e\eta\epsilon}{4\pi\rho_0\nu}(I_7+P_2\sigma I_8)\right]+\frac{\mu_e\eta\epsilon}{4\pi\rho_0\nu}\left(I_9+P_2\sigma^*I_{10}\right)+\\ -\frac{\alpha a^2\lambda\kappa_T}{\beta\nu}\left(g_0-\frac{\gamma M_0\nabla H}{\rho_0\alpha\lambda}\right)\left(\frac{1+\sigma^*\tau_1}{B+\sigma^*\tau_1}\right)\left[I_{11}+\sigma^*E_1P_1I_{12}\right]=0\end{gathered} \tag{4.23}$$

where

$$I_1=\int\left(|DW|^2+a^2|W|^2\right)dz,\quad I_2=\int\left(\left|D^2W\right|^2+a^4|W|^2+2a^2|DW|^2\right)dz,$$

$$I_3=\int\left(\left|D^3W\right|^2+3a^2\left|D^2W\right|^2+3a^4|DW|^2+a^6|W|^2\right)dz,,\quad I_4=\int\left(|Z|^2\right)dz,$$

$$I_5=\int\left(|DZ|^2+a^2|Z|^2\right)dz,\quad I_6=\int\left(\left|D^2Z\right|^2+2a^2|DZ|^2+a^4|Z|^2\right)dz,$$

$$I7=\int\left(|DX|^2+a^2|X|^2\right)dz,\quad I_8=\int\left(|X|^2\right)dz,$$

$$I_9 = \int\left(\left|D^2V\right|^2 + a^4\left|V\right|^2 + 2a^2\left|DV\right|^2\right)dz, \quad I_{10} = \int\left(\left|DV\right|^2 + a^2\left|V\right|^2\right)dz,$$

$$I_{11} = \int\left(\left|D\Theta\right|^2 + a^2\left|\Theta\right|^2\right)dz, \quad I_{12} = \int\left(\left|\Theta\right|^2\right)dz.$$

All the above specified integrals $I_1 - I_{12}$ are definite.
Putting $\sigma = i\sigma_i$ in (4.23), equating the imaginary part, we obtain,

$$\sigma_i\left\{\left(\frac{1}{\varepsilon} + \frac{M}{\varepsilon\left(1+\tau_1^2\sigma_i^2\right)} + \frac{F_1}{P_t}\right)I_1 - \frac{d^2}{\varepsilon}\left(\frac{1}{\varepsilon} + \frac{M}{\varepsilon\left(1+\tau_1^2\sigma_i^2\right)} + \frac{F_1}{P_t}\right)I_4 + \frac{\mu e\eta\varepsilon d2}{4\pi\rho_0\nu}P_2 I_8 + \right.$$
$$-\frac{\mu_e\eta\epsilon}{4\pi\rho_0\nu}P_2 I_{10} + \frac{\alpha a^2\lambda\kappa_T}{\beta\nu}\left(g_0 - \frac{\gamma M_0\nabla H}{\rho_0\alpha\lambda}\right)\left[\left(\frac{\tau_1(B-1)}{B^2+\tau_1^2\sigma_i^2}\right)I_{11} + \right. \tag{4.24}$$
$$\left.\left. + \left(\frac{B+\tau_1^2\sigma_i^2}{B^2+\tau_1^2\sigma_i^2}\right)E_1 P_1 I_{12}\right]\right\} = 0.$$

In the absence of rotation and magnetic field (4.24) becomes,

$$\sigma_i\left\{\left(\frac{1}{\varepsilon} + \frac{M}{\varepsilon\left(1+\tau_1^2\sigma_i^2\right)} + \frac{F_1}{P_t}\right)I_1 + \frac{\mu e\eta\varepsilon d2}{4\pi\rho 0\nu}P_2 I_8 + \right.$$
$$\left. + \frac{\alpha a^2\lambda\kappa_T}{\beta\nu}\left(g_0 - \frac{\gamma M_0\nabla H}{\rho_0\alpha\lambda}\right)\left[\left(\frac{\tau_1(B-1)}{B^2+\tau_1^2\sigma_i^2}\right)I_{11} + \left(\frac{B+\tau_1^2\sigma_i^2}{B^2+\tau_1^2\sigma_i^2}\right)E_1 P_1 I_{12}\right]\right\} = 0. \tag{4.25}$$

From (4.25) the quantity in bracket will be positive definite if $g_0 > \frac{\gamma M_0\nabla H}{\rho_0\alpha\lambda}$ and $B>1$.

It means $\sigma_i = 0$, modes are non oscillatory or oscillatory modes are not allowed and the principle of exchange of stability is satisfied if $g_0 > \frac{\gamma M_0\nabla H}{\rho_0\alpha\lambda}$ and $B>1$.

5. Numerical computations

Dispersion equation (4.11) is analyzed also. The critical Rayleigh number R_1 and Rayleigh number for the ferromagnetic fluid is calculated for different values of the couple-stress F_2, rotation T_{A1}, suspended particles B, permeability P, magnetic field Q_1 and magnetization M_0.

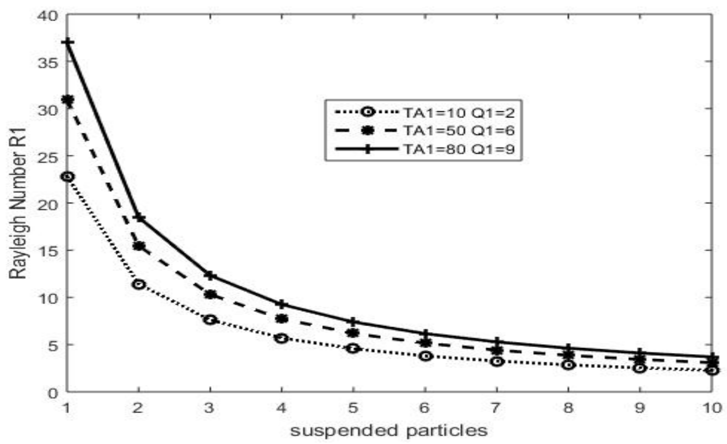

Fig.2. Change of R_1 with suspended particles B for $\lambda = 10$, $x = 1$, $T_{A1} = [10, 50, 80]$, $Q_1 = [2, 6, 9]$.

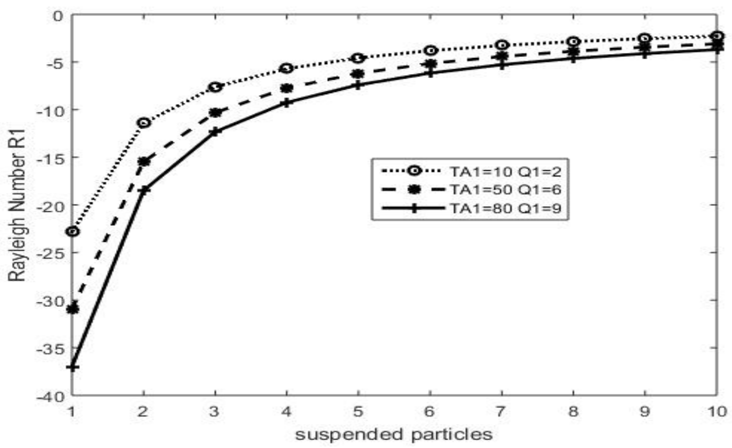

Fig.3. Change of R_1 with suspended particles B for $\lambda = -10$, $x = 1$, $T_{A1} = [10, 50, 80]$, $Q_1 = [2, 6, 9]$.

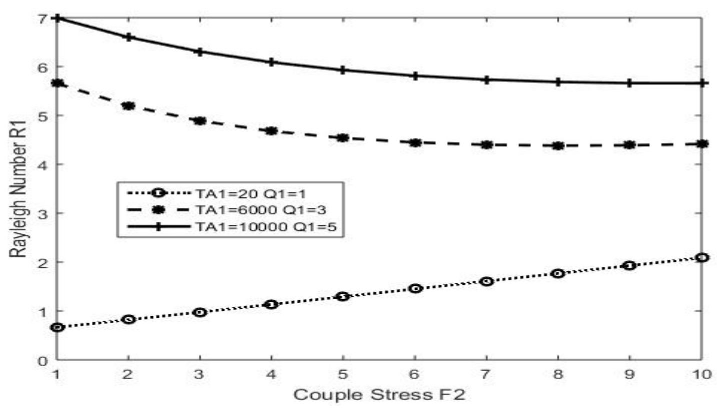

Fig.4. Change of R_1 with couple stress F_2 for $\lambda = 10$, $x = 1$, $T_{A1} = [20, 6000, 10000]$, $Q_1 = [1, 3, 5]$.

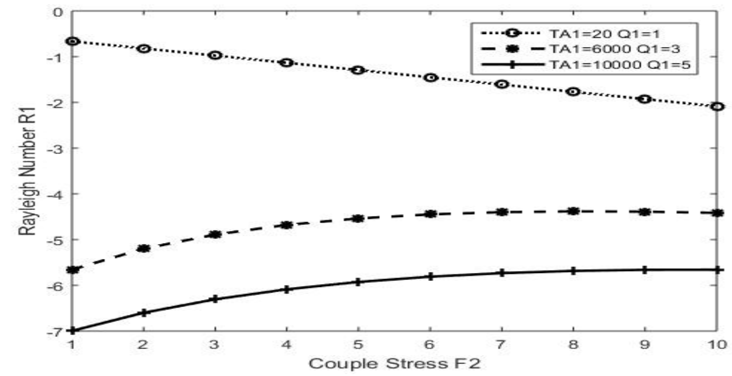

Fig.5. Change of R_1 with couple stress F_2 for $\lambda = -10$, $x = 1$, $T_{A1} = [20, 6000, 10000]$, $Q_1 = [1, 3, 5]$.

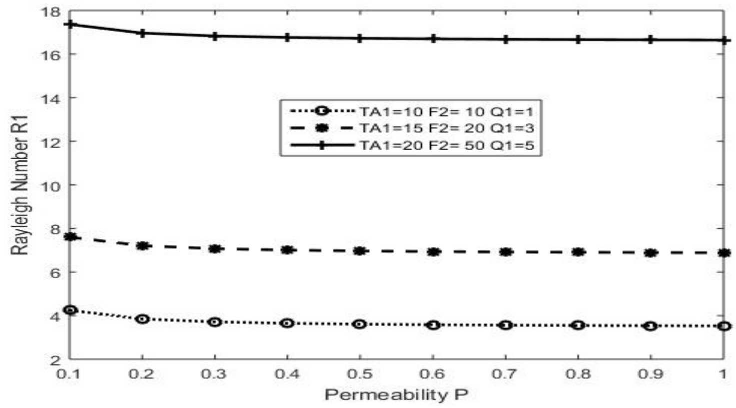

Fig.6. Change of R_1 with permeability P for $\lambda = 10$, $x = 1$, $T_{A1} = [10,15,20]$, $Q_1 = [1, 3, 5]$, $F_2 = [10,20,50]$.

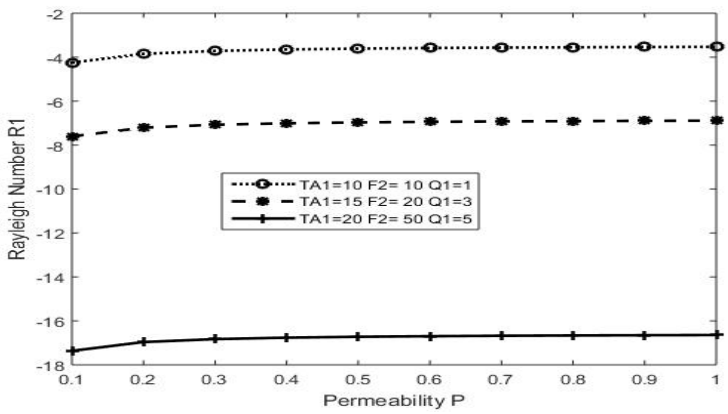

Fig.7. Change of R_1 with permeability P for $\lambda = -10$, $x = 1$, $T_{A1} = [10,15,20]$, $Q_1 = [1, 3, 5]$, $F_2 = [10,20,50]$.

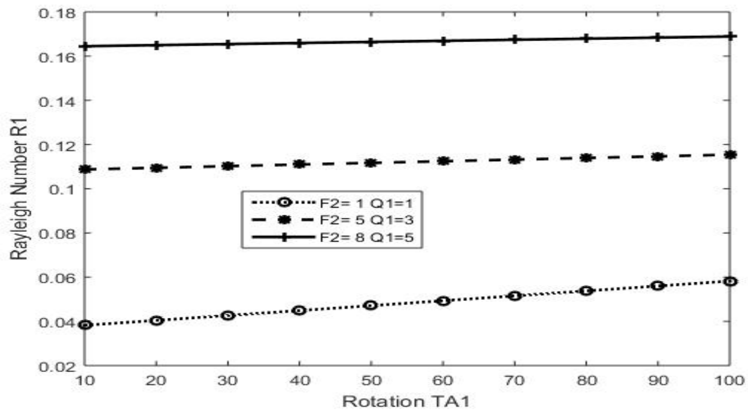

Fig.8. Change of R_1 with rotation T_{A1} for $\lambda = 100,\ x = 1, b = 10,\ F_2 = [1,5,8], Q_1 = [1,3,5]$.

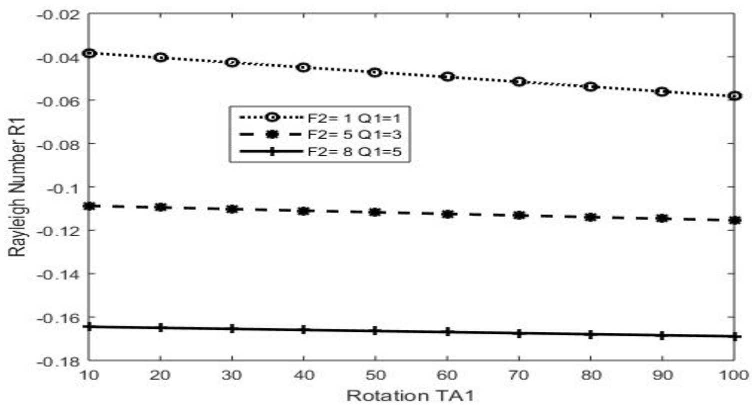

Fig.9. Change of R_1 with rotation T_{A1} for $\lambda = -100,\ x = 1,\ P = 0.5, \varepsilon = 0.5,\ \ b = 10, F_2 = [1,5,8],\ \ Q_1 = [1,3,5]$.

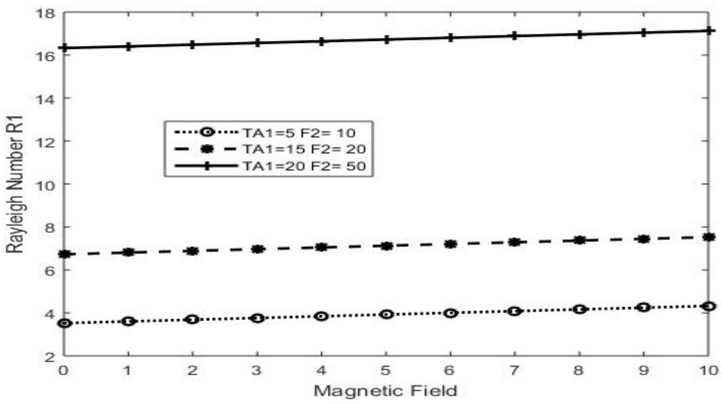

Fig.10. Change of R_1 with magnetic field Q_1 for $\lambda = 10,\ x = 1,\ P = 0.5, \varepsilon = 0.5,\ F_2 = [10,\ 20,\ 50]$, $T_{A1} = [5,15,20]$.

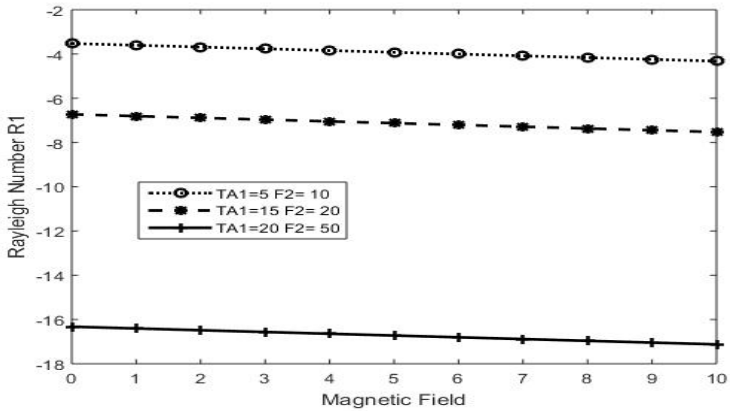

Fig.11. Change of R_1 with magnetic field Q_1 for $\lambda = -10$, $x = 1$, $P = 0.5, \varepsilon = 0.5$, $F_2 = [10,\ 20,\ 50]$, $T_{A1} = [5, 15, 20]$.

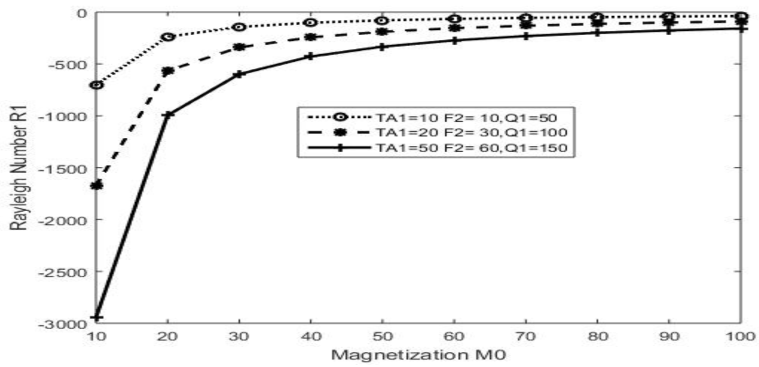

Fig.12. Change of R_1 with magnetic field M_0 for $\lambda = 5$, $x = 1, P = 0.5, \varepsilon = 0.5$, $\gamma = 1, \rho_0 = 1$, $g = 9.8$, $\alpha = 1$, $T_{A1} = [10, 20, 50], Q_1 = [50, 100, 150]$, $F_2 = [10, 30, 60]$.

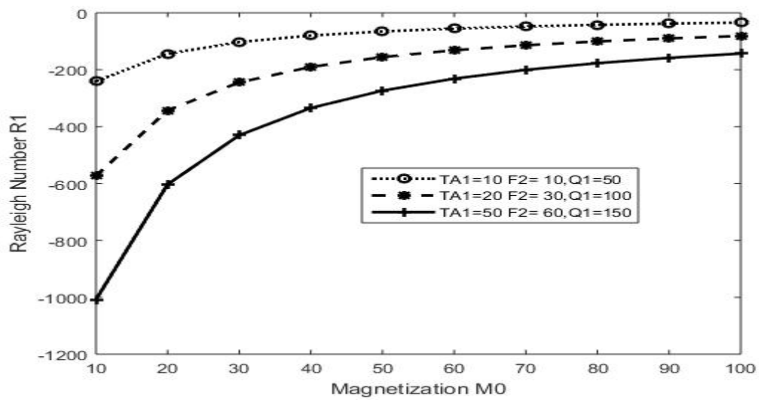

Fig.13. Change of R_1 with magnetic field M_0 for $\lambda = -5$, $x = 1, P = 0.5, \varepsilon = 0.5, \gamma = 1$, $\rho_0 = 1$, $g = 9.8$, $\alpha = 1$, $T_{A1} = [10, 20, 50]$, $Q_1 = [50, 100, 150]$, $F_2 = [10, 30, 60]$.

In Figs 2 and 3 a graph has been plotted between critical Rayleigh number R_1 and suspended particles parameter B for different values for rotation and magnetic field and couple-stress. which shows R_1 decreases with an increase in B for $\lambda>0$ and R_1 increases with increase in B for $\lambda<0$. So suspended particles have a destabilizing effect on the system for $\lambda>0$ and a stabilizing effect on the system for $\lambda<0$.

In Fig.4, a graph has been plotted for the critical Rayleigh number R_1 and couple-stress parameter F_2 for $T_{A1}=[20,\ 6000,\ 10000]$, $Q_1=[1,\ 3,\ 5]$ and $\lambda=10>0$, which shows R_1 increases with an increases in F_2 for $T_{A1}=20$ so the couple-stress has a stabilizing effect on the system for $\lambda>0$, but a reverse effect for $T_{A1}=6000$ and 10000 because stabilization condition fails for $T_{A1}=6000$ and 10000.

In Fig.5, a graph has been plotted for the critical Rayleigh number R_1 and couple-stress parameter F_2 for $T_{A1}=[20,\ 6000,\ 10000]$, $Q_1=[1,\ 3,\ 5]$ and $\lambda=-10<0$, which shows R_1 increases with an increases in F_2 for $T_{A1}=20$ so the couple-stress has a stabilizing effect on the system for $\lambda<0$, but a reverse effect for $T_{A1}=6000$ and 10000 because stabilization condition fails for $T_{A1}=6000$ and 10000 .

In Fig.6, a graph has been plotted for the critical Rayleigh number R_1 and permeability of the medium P for $F_2=[10,\ 20,\ 50]$, $T_{A1}=[10,\ 15,\ 20], Q_1=[1,\ 3,\ 5]$ and $\lambda=10>0$, which shows R_1 decreases with an increase in P. So permeability has a destabilizing effect on the system for $\lambda>0$.

In Fig.7, a graph has been plotted for the critical Rayleigh number R_1 and permeability of the medium P for $F_2=[10,\ 20,\ 50]$, $T_{A1}=[10,\ 15,\ 20]$, $Q_1=[1,\ 3,\ 5]$ and $\lambda=-10<0$, which shows R_1 increases with an increase in P. So permeability has a stabilizing effect on the system for $\lambda<0$.

In Fig.8, a graph has been plotted for the critical Rayleigh number R_1 and rotation T_{A1} for $F_2=[1,\ 5,\ 8]$, $Q_1=[1,\ 3,\ 5]$ and $\lambda=100>0$, which shows R_1 increases with an increase in T_{A1}. So rotation has a stabilizing effect on the system for $\lambda=100>0$.

In Fig.9, a graph has been plotted for the critical Rayleigh number R_1 and rotation T_{A1} for $F_2=[1,\ 5,\ 8]$, $Q_1=[1,\ 3,\ 5]$ and $\lambda=-100<0$, which shows R_1 decreases with an increase in T_{A1}. So rotation has a destabilizing effect on the system for $\lambda=-100<0$.

In Fig.10, a graph has been plotted for the critical Rayleigh number R_1 and magnetic field for $F_2=[10,\ 20,\ 50]$, $T_{A1}=[5,\ 15,\ 20]$ and $\lambda=10>0$, which shows R_1 increases with increases in Q_1. So magnetic field has stabilizing effect on the system for $\lambda=10>0$.

In Fig.11, a graph has been plotted for the critical Rayleigh number R_1 and magnetic field for $F_2=[10,\ 20,\ 50]$, $T_{A1}=[5,\ 15,\ 20]$ and $\lambda=-10<0$, which shows R_1 decreases with an increase in Q_1. So magnetic field has a destabilizing effect on the system for $\lambda=-10<0$.

In Fig.12, a graph has been plotted for the Rayleigh number R and magnetization of the ferromagnetic fluid M_0 for $F_2=[10,\ 30,\ 60]$, $T_{A1}=[10,\ 20,\ 50]$, $Q_1=[50,\ 100,\ 150]$ *and* $\lambda=5>0$, which shows R increases with an increase in M_0. So magnetization has stabilizing effect on the system for $\lambda>0$.

In Fig.13, a graph has been plotted for the Rayleigh number R and magnetization of the ferromagnetic fluid M_0 for $F_2=[10,\ 30,\ 60]$, $T_{A1}=[10,\ 20,\ 50]$, $Q_1=[50,\ 100,\ 150]$ and $\lambda=-5<0$, which shows R increases with increases in M_0. So magnetization has a stabilizing effect on the system for $\lambda<0$.

6. Conclusion

In the present paper, the effect of different parameters such as suspended particles, couple-stress, viscoelasticity, permeability, rotation, magnetic field and magnetization on the thermal instability of the couple-stress Rivlin-Ericksen ferromagnetic fluid flowing through a porous medium in the presence of variable gravity field in a porous medium has been investigated.

Results of the investigation are as follows:

Suspended particles always have a destabilizing effect for $\lambda > 0$ and a stabilizing effect for $\lambda < 0$.

Couple-stress has a stabilizing effect on the thermal instability of the couple-stress Rivlin-Ericksen ferromagnetic fluid flowing through a porous medium in the presence of suspended particles, varying gravity, rotation and magnetic fields under the following conditions,

$$\lambda > 0, T_{A1}(1+x) < \left[(1+x)\left\{ F_2 (1+x)^2 + (1+x) + \frac{1}{P} \right\} + \frac{Q_1}{\varepsilon} \right]^2,$$

and

$$\lambda < 0, T_{A1}(1+x) > \left[(1+x)\left\{ F_2 (1+x)^2 + (1+x) + \frac{1}{P} \right\} + \frac{Q_1}{\varepsilon} \right]^2.$$

Couple-stress has a destabilizing effect on the thermal instability of the couple-stress Rivlin-Ericksen ferromagnetic fluid flowing through a porous medium in the presence of varying gravity, rotation and magnetic fields under the following conditions,

$$\lambda > 0, T_{A1}(1+x) > \left[(1+x)\left\{ F_2 (1+x)^2 + (1+x) + \frac{1}{P} \right\} + \frac{Q_1}{\varepsilon} \right]^2,$$

and

$$\lambda < 0, T_{A1}(1+x) < \left[(1+x)\left\{ F_2 (1+x)^2 + (1+x) + \frac{1}{P} \right\} + \frac{Q_1}{\varepsilon} \right]^2.$$

Couple-stress has a stabilizing effect if $\lambda > 0$ and a destabilizing effect if $\lambda < 0$ on the thermal instability of the couple-stress Rivlin-Ericksen ferromagnetic fluid flowing through a porous medium in the presence of varying gravity, rotation and magnetic fields.

Rotation has always a stabilizing effect if $\lambda > 0$ and a destabilizing effect if $\lambda < 0$.

Permeability has a stabilizing effect on the thermal instability of the couple-stress Rivlin-Ericksen ferromagnetic fluid flowing through a porous medium in the presence of suspended particles, varying gravity, rotation and magnetic fields under the following conditions,

$$\lambda > 0, T_{A1}(1+x) > \left[(1+x)\left\{ F_2 (1+x^2 + (1+x) + \frac{1}{P} \right\} + \frac{Q_1}{\varepsilon} \right]^2,$$

and

$$\lambda < 0, T_{A1}(1+x) < \left[(1+x)\left\{ F_2 (1+x)^2 + (1+x) + \frac{1}{P} \right\} + \frac{Q_1}{\varepsilon} \right]^2.$$

Permeability has a destabilizing effect on the thermal instability of the couple-stress Rivlin-Ericksen ferromagnetic fluid flowing through a porous medium in the presence of suspended particles, varying gravity, rotation and magnetic fields under the following conditions,

$$\lambda > 0, T_{A1}(1+x) < \left[(1+x)\left\{ F_2 (1+x)^2 + (1+x) + \frac{1}{P} \right\} + \frac{Q_1}{\varepsilon} \right]^2,$$

and

$$\lambda < 0, T_{A1}(1+x) > \left[(1+x)\left\{F_2(1+x)^2+(1+x)+\frac{1}{P}\right\}+\frac{Q_1}{\varepsilon}\right]^2.$$

In the absence of rotation permeability of the porous medium has a stabilizing effect if $\lambda > 0$ and a destabilizing effect if $\lambda < 0$.

The magnetic field has a stabilizing effect on the thermal instability of the couple-stress Rivlin-Ericksen ferromagnetic fluid flowing through a porous medium in presence of gravity field, rotation, suspended particles under the following conditions,

$$\lambda > 0, T_{A1}(1+x) < \varepsilon^2\left[(1+x)\left\{F_2(1+x)^2+(1+x)+\frac{1}{P}\right\}+\frac{Q_1}{\varepsilon}\right]^2,$$

and

$$\lambda <> 0, T_{A1}(1+x)\varepsilon^2\left[(1+x)\left\{F_2(1+x)^2+(1+x)+\frac{1}{P}\right\}+\frac{Q_1}{\varepsilon}\right]^2.$$

The magnetic field has a destabilizing effect on the thermal instability of the couple-stress Rivlin-Ericksen ferromagnetic fluid flowing through a porous medium in presence of gravity field, rotation, suspended particles under the following conditions,

$$\lambda > 0, T_{A1}(1+x) > \varepsilon^2\left[(1+x)\left\{F_2(1+x)^2+(1+x)+\frac{1}{P}\right\}+\frac{Q_1}{\varepsilon}\right]^2,$$

and

$$\lambda < 0, T_{A1}(1+x) < \varepsilon^2\left[(1+x)\left\{F_2(1+x)^2+(1+x)+\frac{1}{P}\right\}+\frac{Q_1}{\varepsilon}\right]^2.$$

In the absence of rotation, the magnetic field has a stability effect on the fluid layer if $\lambda>0$ and a destabilizing effect if $\lambda<0$.

Magnetization always has a stabilizing effect on the fluid layer.

The Rivlin-Ericksen fluid behaves like an ordinary Newtonian fluid for stationary convection.
The presence of the viscoelastic parameter, either rotation or magnetic field introduces oscillatory modes and the principle of exchange is not satisfied.

Acknowledgements

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Nomenclature

a – wave number
D – derivative with respect to z
d – depth of layer $[m]$
g – gravity field $\left[m/s^2\right]$
H – magnetic field vector having components $(0,0,H)$
p – pressure of the fluid
N_s – number density of suspended particles

Q – Chandrashekhar number
$q(u_1,u_2,u_3)$ – velocity of the fluid
$q_s(l,r,s)$ – velocity of suspended particles
T – temperature $[K]$
T_A – Taylor number
α – coefficient of thermal expansion
β – uniform temperature gradient $[K/m]$
ρ – density
Ω – rotation field vector having components $(0,0,\Omega)$
κ_T – thermal diffusivity
ν – kinematic viscosity
ν' – kinematic viscoelasticity

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