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MHD FREE CONVECTIVE HEAT AND MASS TRANSFER FLOW FROM A VERTICAL POROUS SURFACE WITH VARIABLE THERMAL CONDUCTIVITY, VARIABLE MASS DIFFUSIVITY AND THERMAL DIFFUSION INCLUDING VISCOUS DISSIPATION AND CHEMICAL REACTION

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An analysis is carried out to study chemically reactive, viscous dissipative effects of an incompressible and electrically conducting fluid with MHD free convection adjacent to a vertical surface with variable thermal conductivity (VTD) and variable mass diffusivity (VMD). An approximate numerical solution for the steady laminar boundary layer flow over a wall of the surface in the presence of species concentration and thermal mass diffusion has been studied. Using numerical techniques the governing boundary layer equations are solved to get the exact solution. Numerical calculations are carried out for different values of dimensionless parameters. The results are exhibited through various graphs and it is observed from the analysis of the results that the velocity field is appreciably influenced by the magnetic effect, porous effect, chemical reaction and buoyancy ratio between the species and thermal diffusion at the wall of the surface.

Key words: MHD, VTD, VMD, viscous dissipation and chemical reaction.

1. Introduction

Free convection induced by the simultaneous action of buoyancy forces resulting from thermal and mass diffusion is of considerable interest in nature. It has many industrial applications such in geophysics, oceanography, drying processes, chemical engineering and solidification of binary alloys.

Heat and mass transfer problems with chemical reaction have many applications in drying processes, evaporation at the surface of a body, energy transfer in a wet cooling tower and the flow in a desert cooler. Acrivos [1] studied the hydro dynamic flow with catalytic surface reaction. Chambre and Acrivos [2] investigated diffusion of a chemically reactive species in a laminar boundary layer flow. Combined heat and mass transfer flow with free convection over a heated surface was studied by Gebhart *et al.* [3] considering various geometries. The transient natural convective flow for an impulsively started vertical plate with heat and mass transfer was investigated by Muthukumaraswamy *et al.* [4] Chien-Hsin Chen [5] discussed the

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combined heat and mass transfer flow in MHD natural convection from a vertical surface with Ohmic heating and viscous dissipative effects. In the presence of internal heat generation / absorption Chamkha and Khaled [6] investigated the problem of MHD free convention flow coupled with heat and mass transfer. Hossain [7] made a study on viscous and Joule heating effects of MHD-free convective flow with variable plate temperature. Gorla *et al.* [8] studied Joule heating effects with MHD free convection in a micro polar fluid flow. The study of Chu *et al.* [9] was a major contribution to the study of natural convection with concentrated heat sources. Besides, due to the quick growth of electronic technology, effective cooling of electronic equipment has become warranted and thermal diffusion effect has been utilized for isotopes separation in the mixture between gases with light molecular weight (hydrogen, helium). Gebhart and Pera [10], Eckert and Drake [11] have shown that in the case of nitrogen and air which have medium molecular weight the thermal diffusion effect is found as a magnitude which cannot be neglected. In addition to the above studies, Kandasamy *et al.* [12] studied the combined heat and mass transfer flow with MHD free convection from a vertical surface with chemical reaction, Ohmic heating and viscous dissipation. Babu *et al.* [13] studied the chemical reaction and thermal radiation effects on MHD mixed convection over a vertical plate with variable fluid properties. The study of Eswaramoorthi *et al.* [14] Presented an analytical and numerical study on cross diffusion effects on magneto-convection of a chemically reacting fluid with suction/injection and convective boundary condition. Chemically reacting MHD boundary-layer flow of nanofluids over a non-linear stretching sheet with heat source/sink and thermal radiation was studied by Makinde *et al.* [15]. Nayak *et al.* [16] discussed the chemically reacting and radiating nanofluid flow past an exponentially stretching sheet in a porous medium. Arthira *et al.* [17] investigated non-linear convective in chemically reacting fluid with an induced magnetic field across a vertical porous plate in the presence of heat source/sink.

No attempt has been made to analyze the combined heat and mass transfer in an MHD free convection flow past a vertical porous surface with variable thermal conductivity (VTD) and variable mass diffusivity (VMD) including thermal mass diffusion. In the present paper, a two dimensional MHD free convection flow taking into account the combined heat and mass transfer over a vertical porous surface including Ohmic heating, chemical reaction, viscous dissipation with variable thermal conductivity and variable mass diffusivity is investigated. The similarity transformation has been utilized to convert the governing partial differential equations into ordinary differential equations and then by using the Runge- Kutta and shooting method, the numerical solution of the problem is given. Numerical calculations are carried out for different values of dimensionless parameters and the results are illustrated graphically. The analysis of the results so obtained show that the velocity field is influenced appreciably by the presence of the porous parameter, magnetic parameter, chemical reaction parameter, buoyancy ratio parameter at the wall of the surface.

2. Mathematical formulation

Considered a steady two-dimensional MHD free convection boundary layer flow of a viscous and incompressible fluid over a porous vertical surface with suction or blowing.

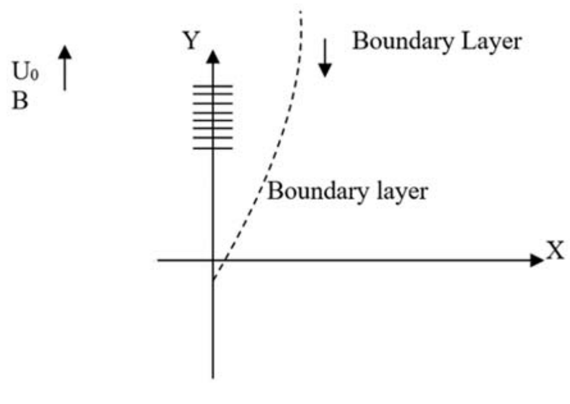

Fig.1. Schematic diagram.

As shown in the above figure, the *x*-axis is parallel to the surface and *y*-axis is taken normal to the surface. In a limited temperature range, fluid properties are assumed to be constant. In comparison to other chemical species, the concentration of diffusing species is very small and far from the wall. The chemical reactions take place in the flow with Dufour effect. All thermo physical properties are assumed to be constant except the density in the buoyancy terms of the momentum equation which is linear. Under these Boussinesq's approximations the governing equations that describe the physical situation with variable thermal conductivity and variable mass diffusivity of the fluid are

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{2.1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=v\frac{\partial^2 u}{\partial y^2}-\left(\frac{\partial B_0^2}{\rho}-\frac{v}{k}\right)u+G\beta(T-T_\infty)+G\beta^*\left(C-C_\infty\right), \tag{2.2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{\partial}{\partial y}\left(\alpha\frac{\partial T}{\partial Y}\right)-\frac{\sigma B_0^2}{\rho C_p}u^2+\frac{\mu}{\rho Cp}\left(\frac{\partial u}{\partial y}\right)^2, \tag{2.3}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=\frac{\partial}{\partial y}\left(D\frac{\partial C}{\partial Y}\right)-K_1 C+D_T\frac{\partial^2 T}{\partial Y^2} \tag{2.4}$$

where B^2 ,β, β^*, ∂ have their usual meanings. In energy transfer, the Ohmic heating (2^{nd} term of Eq.(2.3)) and viscous dissipation, (3^{rd} term of Eq.(2.3) are considered with variable thermal conductivity. In diffusion transfer, the chemical reaction term (2^{nd} term in Eq.(2.4)) is added with variable mass diffusivity. K_1 is the chemical reaction rate constant (for $K_1 < 0$ generating reactant and $K_1 > 0$ destructive reactant) and D is the effective diffusion coefficient.
The respective boundary conditions are

$$u=0,\quad v=0,\quad C=Cw,\quad T=Tw\quad \text{at}\quad y=0,$$
$$u\to uc,\quad C\to C\infty,\quad T\to T\infty,\quad \text{as}\quad y\to\infty. \tag{2.5}$$

With the introduction of the change of variables and velocity components, we obtain

$$\psi(x,y)=\sqrt{(4ucvx)}\,'f(\eta), \tag{2.6}$$

$$\eta(x,y)=y\sqrt{\frac{uc}{4vx}}, \tag{2.7}$$

$$u=\frac{\partial\psi}{\partial Y}\quad \text{and}\quad v=-\frac{\partial\psi}{\partial x}. \tag{2.8}$$

Now with the change of new dimensionless variables for temperature and concentration as

$$\theta=T-T\infty/Tw-T\infty, \tag{2.9}$$

$$\varphi = C - C\infty / Cw - C\infty, \tag{2.10}$$

Eqs (2.2) to (2.4) convert to

$$f_{\eta\eta\eta} + 3ff_{\eta\eta} - 2f_{\eta}^{2} + 4(\theta + N\varphi) - 4(M + K_{2})f_{\eta} = 0, \tag{2.11}$$

$$\theta_{\eta\eta} + 3P_{r}.f\theta_{\eta} - 4\Pr f_{\eta}\theta + 4(M + K_{2})E_{c}\Pr f_{\eta}^{2} + Ec\Pr f_{\eta\eta}^{2} = 0, \tag{2.12}$$

$$\varphi_{\eta\eta} + 3Scf\varphi_{\eta} - 4\Pr f_{\eta}\varphi - 4v\varphi = 0, \tag{2.13}$$

with boundary conditions

$$\begin{aligned} &\eta = 0 \;\Rightarrow\; f(0) = 0, \quad f'(0) = 1, \quad \theta(0) = 1, \quad \varphi(0) = 1, \\ &\eta \to \infty \;\Rightarrow\; f'(\infty) = 0, \quad \theta(\infty) = 0, \quad \varphi(\infty) = 0 \end{aligned} \tag{2.14}$$

where

$M = \dfrac{\sigma B_0^2 x}{\rho U_c}$ – magnetic parameter,

$\Pr = \dfrac{\nu C_P}{K}$ – Ptandtl number,

$K_2 = \dfrac{v_1 x}{K u_C}$ – permeability parameter,

$Sc = \dfrac{\nu}{D} =$ – Schmidt number,

$N = \beta * \dfrac{(C_W - C_\infty)}{\beta(T_W - T_\infty)}$ – buoyancy ratio parameter,

$E_C = \dfrac{U^2}{C_P(T_W - T_\infty)}$ – Eckert number,

$u_c = g\beta_x\sqrt{(T_w - T_\infty)}$ – free stream velocity,

$v = \dfrac{K_1 x}{u_c}$ – chemical reaction parameter.

3. Method of solution

The solution of the set of ordinary differential equations (2.11) to (2.13) corresponding to the respective boundary conditions (2.14) are obtained numerically by MATLAB ode 45 solver with the shooting technique. The values of f', θ, and ϕ are known at the end points., i.e. at $\eta \to \infty$.

4. Skin friction, Nusselt number and Sherwood number

The important physical parameters for this type of boundary layer flow such as the skin friction coefficient, Nusselt number and Sherwood number are defined as follows.
(i) The skin friction at the plate for a known velocity field in a non-dimensional form in terms of velocity gradient is given by

$$C_f = -\left(\frac{du}{dy}\right) y = 0 = -f_{\eta\eta}(0). \tag{4.1}$$

(ii) For the known temperature field the rate of heat transfer coefficient can be obtained in a non-dimensional form in terms of the Nusselt number as

$$Nu = \frac{-x\left(\frac{dT}{dy}\right)}{T_w - T_\infty} y = 0 \quad = -\theta_\eta(0). \tag{4.2}$$

(iii) The rate of mass transfer coefficient in terms of the non-dimensional Sherwood number for the concentration field is given by

$$Sh = \frac{-x\left(\frac{dC}{dy}\right)_{y=0}}{C_w - C_\infty} = -\varphi_\eta(0). \tag{4.3}$$

5. Results and discussion

In order to validate the results various numerical values are assigned to the physical parameters encountered in the problem and are displayed through graphical representations.

The graph of mass concentration distribution with the effect of various values of buoyancy ratio parameter and chemical reaction parameter at the wall of the sheet is displayed in Fig.2. It is observed from the figure the uniform magnetic field and porous field lead to a fall in the concentration of the fluid along the wall of the surface for increasing values of the buoyancy parameter. Thus, the chemical reaction parameter decelerates the concentration profile of the fluid along the wall of the surface. It implies physically that the effect of the destructive reaction on the concentration profiles is much more pronounced than that of the generative reaction.

Figure 3 is the representation of dimensionless concentration along the wall of the surface for various values of the chemical reaction parameter √. It is seen from the figure that an increase in the chemical reaction parameter values leads to a fall in the concentration distribution of the fluid along the wall of the surface of the boundary layer.

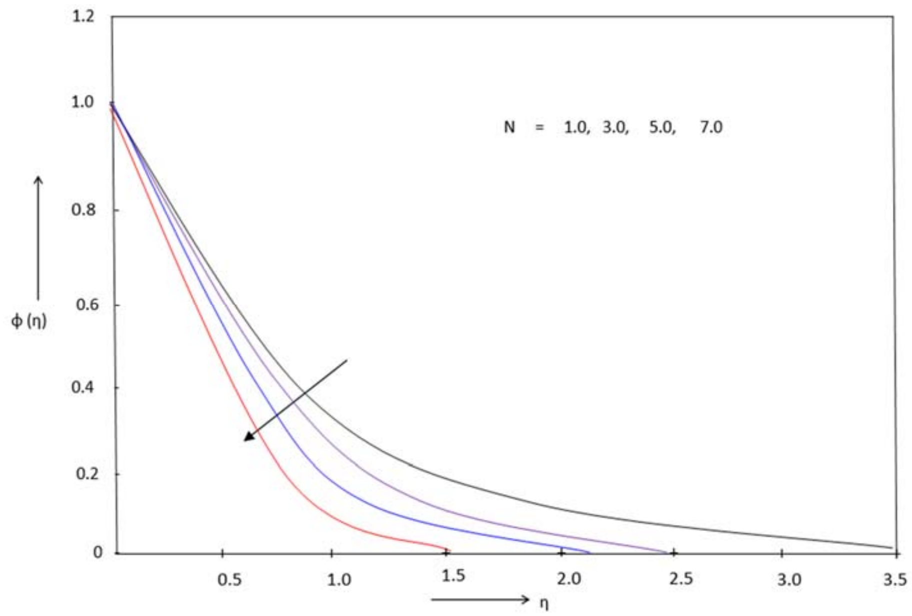

Fig.2. Effects of buoyancy ratio parameter N over concentration profiles for $\Pr = 0.71,\ Sc = 0.72\ ,\ Mn = 2.0,\ Ec = 0.02,\ K_2 = 2.0,\ \gamma = 1.0$.

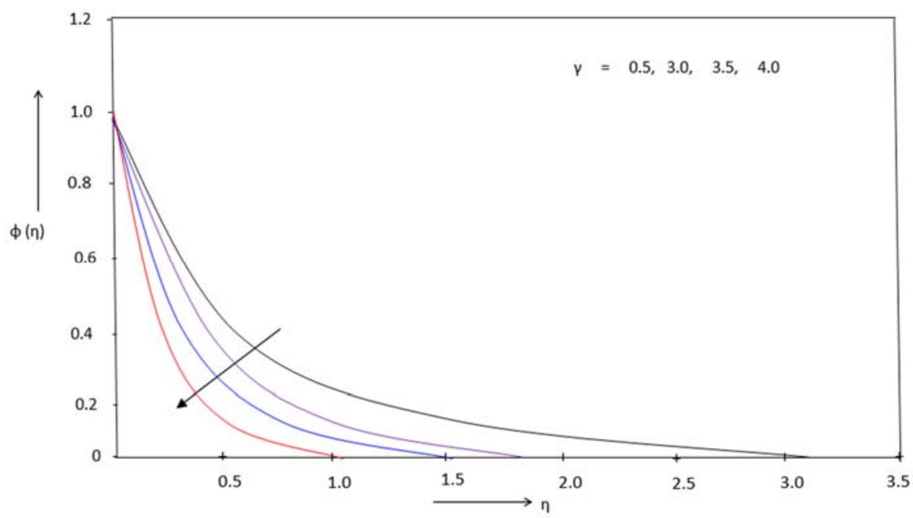

Fig.3. Effect of chemical reaction parameter γ over concentration profiles for $\Pr = 0.71,\ Sc = 0.72, Mn = 2,\ Ec = 0.02,\ K_2 = 2,\ N = 1.0$.

The effect of the permeability parameter K_2 on the rate of mass transfer is shown in Fig.4. The figure shows that for increasing values of K_2, there is a decrease in the rate of mass transfer diffusion.

Figure 5 is the graph of dimensionless temperature profiles for different values of the magnetic parameter $M = 1, 3, 5, 7$. It is seen from the graph that in the case of a uniform chemical reaction, the temperature of the fluid is a decreasing function of the magnetic parameter. In particular, the temperature of the fluid gradually changes from a higher value to a lower value only where the magnetic field is stronger and higher than the chemical species concentration effect. For $M = 7.0$, large distortion of the temperature field is caused. A negative value of the temperature distribution is seen in the outer boundary region for $M = 7.0,\ \surd = 1.0\ ,\ N = 1.0$.

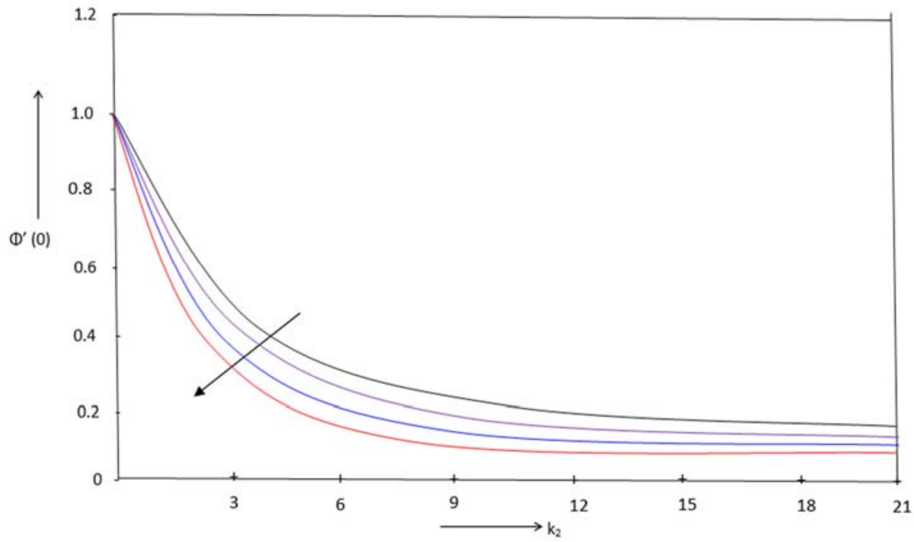

Fig.4. Effect of permeability parameter K_2 on rate of mass transfer.

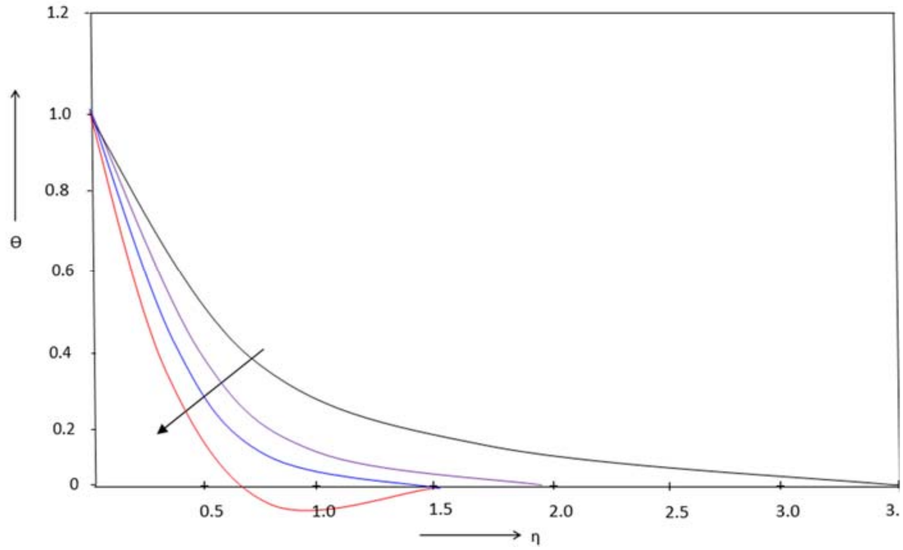

Fig.5. Effect of magnetic parameter $Mn = 1, 3, 5, 7$ when $Sc = 0.72$, $\gamma = 1.0$, $N = 1.0$, $Pr = 0.71$, $Ec = 0.02$ over temperature profiles.

This type of physical behavior is due to the combined effect of the buoyancy ratio between the chemical species ,the thermal diffusion and the strength of the magnetic field along with the uniform chemical reaction.

The effect of the permeability parameter on temperature profiles is shown in the Fig.6. It is seen from the figure that in case of a uniform chemical reaction , the temperature of the fluid is a decreasing function of K_2. Due to a large porous effect the temperature of the fluid changes gradually from a higher region to a lower region. For $K_2 = 5.0$ and $K_2 = 7.0$, the negative nature of the temperature profile is seen in the outer boundary layer region.

Figure 7 is the presentation of the rate of heat transfer for various values of magnetic parameter M. The figure shows that the rate of heat transfer gradually decelerates and reaches the saturation point.

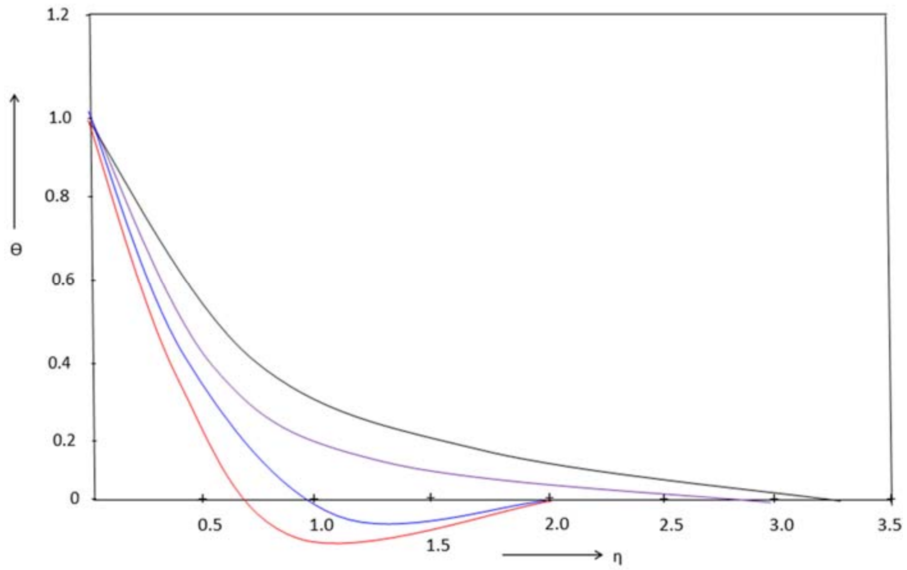

Fig.6. Effect of porous parameter $K_2 = 1, 3, 5, 7$ on temperature profiles.

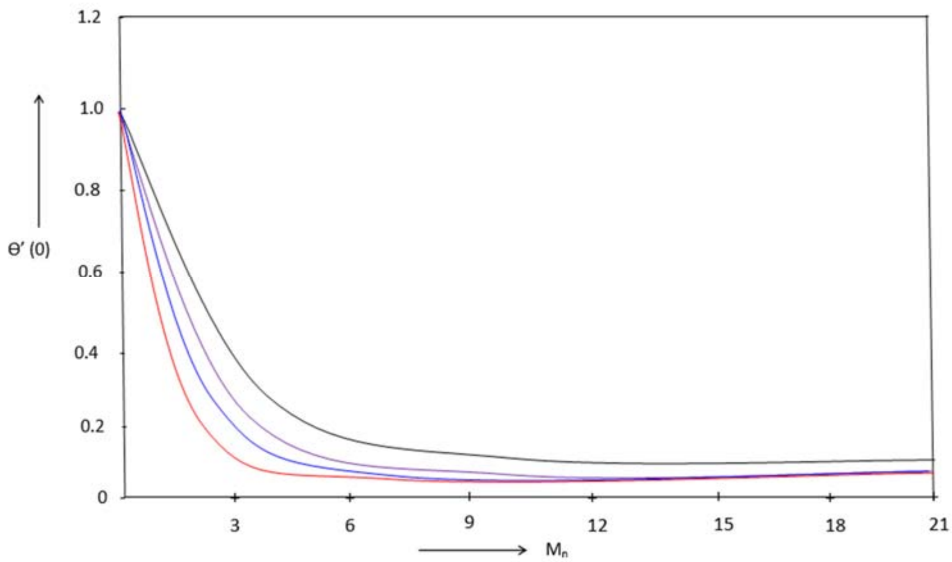

Fig.7. Effect of magnetic parameter on the rate of heat transfer.

6. Conclusions

In the presence of uniform chemical reaction at the wall of the sheet along the boundary layer, the fluid flow decelerates the concentration of the fluid with increase of buoyancy ratio between species and thermal diffusion. All the facts are clearly due to the combined effects of chemical reaction, porosity and Ohmic heating at the wall of the surface.

Increase of chemical reaction decelerates the concentration profile of the fluid due to the uniform effect of magnetic and porous field and buoyancy ratio. This type of result encounter the consumption reaction of the chemical reaction parameter.

The temperature distribution across the boundary layer through the fluid decreases with increase of magnetic parameter. This physical behaviour is due to the combined effect of buoyancy ratio between diffusive species and thermal diffusion and the strength of porous medium with magnetic field along with uniform chemical reaction.

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Nomenclature

B_o^2 – magnetic field strength
C – species concentration in the fluid
C_∞ – species concentration far away from the surface
C_w – species concentration near the surface
C_P – specific heat at constant pressure in $J / Kg\ K$
D – chemical molecular diffusivity m^2 / s
Ec – Eckert number
f – density of the fluid kg / m^3
K_2 – permeability parameter
K – thermal conductivity
M – magnetic parameter
N – buoyancy ration parameter
Pr – Prandtl number
Sc – Schmidt number
T – temperature of the fluid
T_w – temperature of the wall
T_∞ – temperature of the far away from the wall
u_c – free stream velocity
u, v – velocity components along x – and y – directions
α – thermal diffusivity m^2 / s
v – kinematic viscosity m^2 / s
$\surd$ – chemical reaction parameter

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